E. Krasnoshchekov, A. Sukomel

PROBLEMS IN HEAT TRANSFER

MIR PUBLISHERS MOSCOW

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LIST OF SYMBOLS

r, R—radius, m
d, D—diameter, m
l, L—length, m
δ—thickness, m
h, H—height, m
u—perimeter, m
F—surface (area), m^2
f—cross-sectional area, m^2
τ—time, h, s
t—temperature, °C
T—temperature, K
Δt—temperature difference, °C
δt—fluid temperature difference in the direction of flow
p—pressure, Pa
Δp—pressure drop, Pa
G—rate of fluid flow, kg/s
w—velocity, m/s
Q—heat flux, W
q—rate of heat flow, W/m^2
q_l—heat-transfer rate of release from a length of 1 m, W/m
q_v—heat-transfer rate from a volume of 1 m^3, W/m^3
α—heat-transfer coefficient, W/m$^2\cdot$°C
k—overall heat-transfer coefficient, W/m$^2\cdot$°C
C—radiation factor, W/m$^2\cdot$K^4
ε—emissivity
E—emissive power, W/m^2
λ—thermal conductivity, W/m$\cdot$°C
c—heat capacity, J/kg$\cdot$°C
r—heat of vapourization, J/kg
i—enthalpy, J/kg
ρ—density, kg/m^3
a—thermal diffusivity, m^2/s
μ—dynamic viscosity, Pa$\cdot$s
ν—kinematic viscosity, m^2/s
β—volumetric expansion coefficient, K^{-1}
g—acceleration of gravity, m/s^2

PREFACE

The third edition of the book of problems in heat transfer includes new problems and typical calculations associated with heat transfer in nuclear power plants.

Answers are given to all problems. As a rule, the first problem of a group is solved in the book in detail.

Typical problems are solved with the aid of the relationships found in the book: V. V. Isachenko, V. A. Osipova, A. S. Sukomel, "Heat Transfer", Mir Publishers, Moscow, 1977.

The Appendices contain reference material, sufficient for solving all problems given.

Chapter One

STEADY-STATE CONDUCTION

1-1. Calculate the rate of heat flow through a plane homogeneous wall the thickness of which is considerably smaller than its width and height, assuming the wall to be made from: (a) steel ($\lambda = 40$ W/m·°C); (b) concrete ($\lambda =$ $= 1.1$ W/m·°C); (c) diatomite brick ($\lambda = 0.11$ W/m·°C). In all the three cases wall thickness $\delta = 50$ mm. The temperatures of the wall surfaces are maintained constant:

$$t_{w1} = 100°\text{C} \quad \text{and} \quad t_{w2} = 90°\text{C}.$$

Answer

For a steel wall $q = 8000$ W/m^2;
for a concrete wall $q = 220$ W/m^2;
for a wall laid from diatomite brick $q = 22$ W/m^2.

1-2. The rate of heat flow through a plane wall of thickness $\delta = 50$ mm, $q = 70$ W/m^2.

Determine the difference between the temperatures of wall surfaces and the numerical values of the temperature gradient through the wall, if it is made from: (a) brass ($\lambda =$ $= 70$ W/m·°C); (b) red brick ($\lambda = 0.7$ W/m·°C); (c) cork ($\lambda = 0.07$ W/m·°C).

Answer

For a brass wall $\Delta t = 0.05°$C and $|\text{grad}\ t| = 1°$C/m;
for a brick wall $\Delta t = 5°$C and $|\text{grad}\ t| = 100°$C/m;
for a cork wall $\Delta t = 50°$C and $|\text{grad}\ t| = 1000°$C/m.

1-3. Determine the loss of heat Q, W, through a wall laid from red brick of length $l = 5$ m, height $h = 4$ m and thickness $\delta = 0.250$ m, if the temperatures of wall surfaces are maintained equal to $t_{w1} = 110°$C and $t_{w2} = 40°$C. The thermal conductivity of red brick $\lambda = 0.70$ W/m·°C.

Answer

The loss of heat $Q = 3920$ W.

1-4. Determine the thermal conductivity of the material of a wall 40 mm thick, if at a temperature difference between surface temperatures $\Delta t = 20°C$ the rate of heat flow $q = 145\ W/m^2$.

Answer

The thermal conductivity $\lambda = 0.29\ W/m \cdot °C$.

1-5. It is necessary to insulate a flat surface so that the loss of heat from unit area of this surface per unit time will not exceed 450 W/m^2. The temperature of the surface underneath the layer of insulation $t_{w1} = 450°C$, and the temperature of the external surface of the insulating layer $t_{w2} = 50°C$.

Determine the thickness of insulation for two cases:

(a) the insulation is made from sovelite, for which

$$\lambda = 0.09 + 0.0000874t, \quad W/m \cdot °C;$$

(b) the insulation is made from asbestos cement, for which

$$\lambda = 0.109 + 0.000146t, \quad W/m \cdot °C.$$

Answer

(a) $\delta = 100$ mm;
(b) $\delta = 130$ mm.

1-6. A plane wall is laid from fireclay (chamotte) brick, and wall thickness $\delta = 250$ mm. The temperatures of wall surfaces $t_{w1} = 1350°C$ and $t_{w2} = 50°C$. The thermal conductivity of fireclay brick is a function of temperature, $\lambda = 0.838\,(1 + 0.0007t)\ W/m \cdot °C$.

Calculate and represent graphically the distribution of temperature through the wall.

Answer

x, mm . . .	0	50	100	125	150	200	225	250
t, °C.	1350	1145	920	800	670	390	230	50

Temperature distribution in the wall is shown in Fig. 1-1.

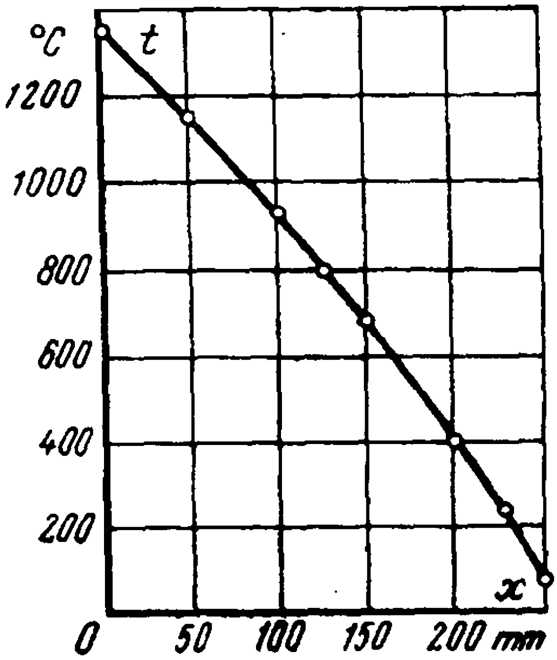

Fig. 1-1. Solution to problem 1-6

Solution

With linear dependence of thermal conductivity on temperature the rate of heat flow

$$q=\frac{\lambda_m}{\delta}(t_{w1}-t_{w2}),\quad \text{W/m}^2,$$

where

$$\lambda_m=\lambda_0\left(1+\beta_\lambda\frac{t_{w1}+t_{w2}}{2}\right),\quad \text{W/m}\cdot{}^\circ\text{C}.$$

In the case considered

$$\lambda_m=0.838\left(1+0.0007\frac{1350+50}{2}\right)=1.25\ \text{W/m}\cdot{}^\circ\text{C}$$

and

$$q=\frac{1.25}{0.25}(1350-50)=6500\ \text{W/m}^2.$$

At any distance x from the surface of the wall the temperature is determined from the following formula

$$t_x=\sqrt{\left(\frac{1}{\beta_\lambda}+t_{w1}\right)^2-\frac{2qx}{\lambda_0\beta_\lambda}}-\frac{1}{\beta_\lambda},\ {}^\circ\text{C}.$$

Substituting the known value of λ_0 and the calculated value of the rate of heat flow q into above equation, we find that

$$t_x=\sqrt{\left(\frac{1}{0.0007}+1350\right)^2-\frac{2\times 6500x}{0.838\times 0.0007}}-\frac{1}{0.0007},$$

whence

$$t_x=\left(\sqrt{7.74-22.2x}-1.43\right)\times 10^3,\quad {}^\circ\mathrm{C}.$$

Substituting in the equation obtained the values of x expressed in metres, we find the corresponding values of the wall temperature.

1-7. The surface temperatures of a 200 mm thick fireclay wall ($\delta = 200$ mm) are: $t_{w1} = 1000^\circ$C and $t_{w2} = 200^\circ$C. The thermal conductivity of fireclay (chamotte) varies depending on temperature, obeying the following equation

$$\lambda = 0.813 + 0.000582t,$$

$$\mathrm{W/m\cdot{}^\circ C}.$$

Show that in the event of linear dependence of thermal conductivity on temperature the rate of heat flow q, W/m^2, can be calculated from the formula for the constant thermal conductivity, taken at a mean wall temperature.

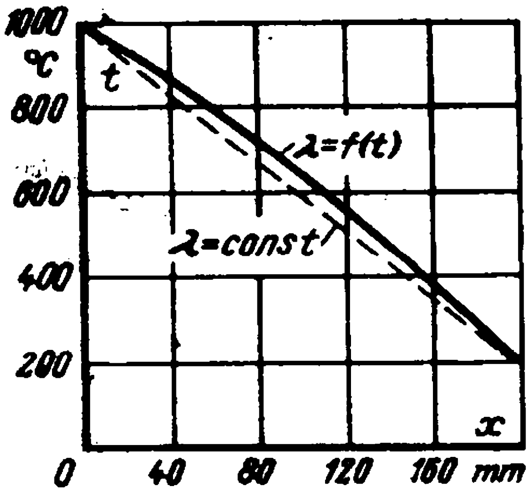

Fig. 1-2. Solution to problem 1-7

Find the error admitted in determining the temperature at points $x = 57.5$ mm; 110 mm; 157.5 mm, if the calculation is based on the mean thermal conductivity for the given temperature interval, and plot the graph representing temperature distribution in the wall.

Answer

$$q = 4650\ \mathrm{W/m^2}.$$

The temperature distribution through the wall is shown in Fig. 1-2.

	x, mm	0	57.5	110	157.7	200
t_x, °C	$\lambda = \lambda_m = \text{const}$	1000	770	560	370	200
	$\lambda = 0.813 + 5.82 \times 10^{-4} t$	1000	800	600	400	200

1-8. The flat wall of a tank, of an area $F = 5$ m², is covered with two layers of a heat-insulating material. The steel wall of the tank is 8 mm thick and its thermal conductivity $\lambda_1 = 46.5$ W/m·°C. The first layer of the insulation is made from an insulating material novoasbosurite of thickness $\delta_2 = 50$ mm and the thermal conductivity of this insulating material is determined from the equation

$$\lambda_2 = 0.144 + 0.00014t, \quad \text{W/m}\cdot{}^\circ\text{C}.$$

The second layer of insulation of thickness $\delta_3 = 10$ mm is lime plaster whose thermal conductivity $\lambda_3 = 0.698$ W/m·°C.

The temperature of the inside surface of the tank wall $t_{w1} = 250$°C and the temperature of the outside surface of insulation $t_{w4} = 50$°C.

Calculate the amount of heat transferred through the wall, the boundary temperatures for the layers of insulation, and plot the graph of temperature distribution.

Answer

The heat flux through the wall $Q = 3170$ W; the boundary temperatures for the layers of insulation are $t_{w2} = 249.9$°C and $t_{w3} = 59$°C.

1-9. The brickwork of a furnace is built up of layers laid of fireclay and red brick and the space between the two layers of brickwork is filled with crushed diatomite brick (Fig. 1-3). The fireclay layer is 120 mm thick (δ_1); the thickness of the diatomite filling $\delta_2 = 50$ mm, and the thickness of the red brick layer $\delta_3 = 250$ mm. The thermal conductivity of the three materials are equal respectively:

$$\lambda_1 = 0.93; \quad \lambda_2 = 0.13 \text{ and } \lambda_3 = 0.7 \text{ W/m}\cdot{}^\circ\text{C}.$$

What should be the thickness of the red brick layer δ_3, if the brickwork is to be laid without the diatomite filling between the two layers, so that the heat flux through the brickwork remains constant?

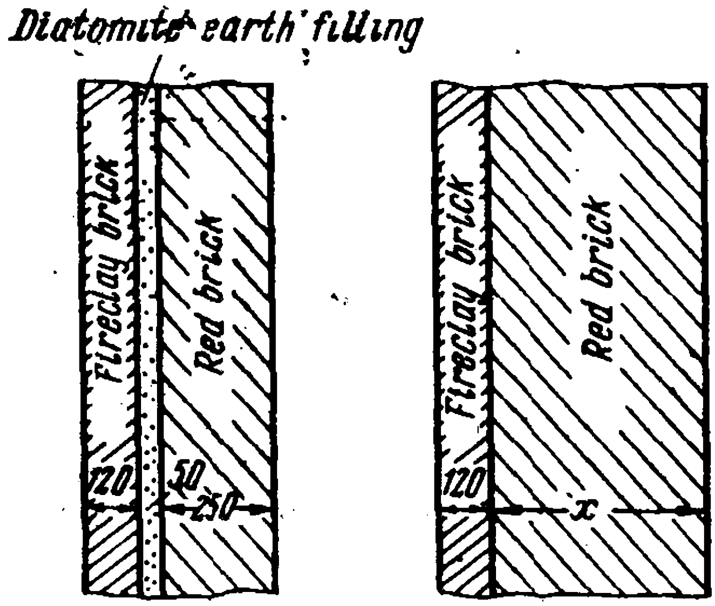

Fig. 1-3. Solution to problem 1-9

Answer

The layer of red brick must be 500 mm thick.

1-10. The wall of a steam boiler furnace which is not fitted with waterwalls is made of a layer of foamed fireclay

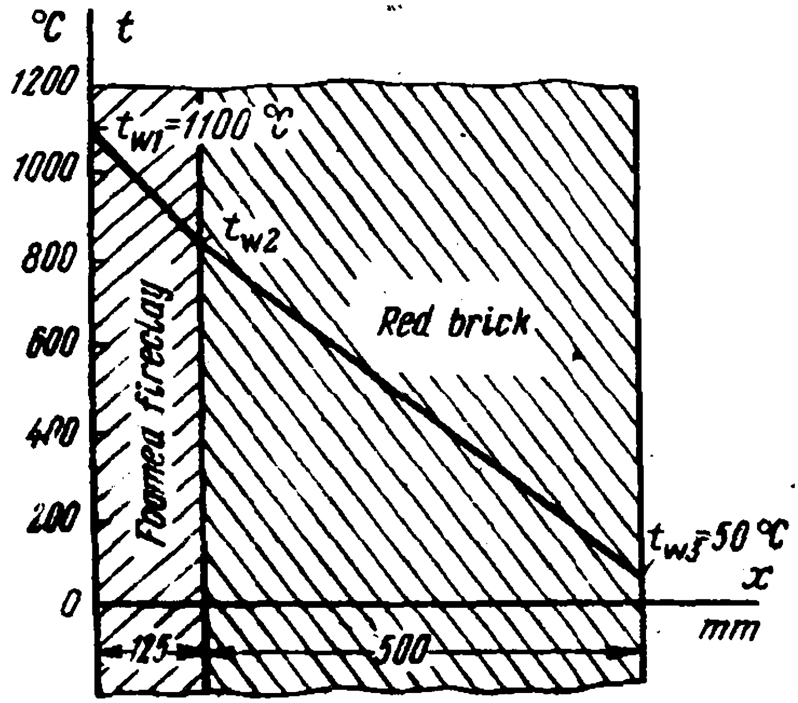

Fig. 1-4. Solution to problem 1-10

of thickness $\delta_1 = 125$ mm and a layer of red brick 500 mm thick (δ_2). The two layers fit each other tightly. The wall temperature inside the boiler furnace $t_{w1} = 1100°C$ and the wall outside temperature $t_{w3} = 50°C$ (Fig. 1-4). The thermal conductivity of foamed fireclay $\lambda_1 = 0.28 + 0.00023t$ and that of red brick, $\lambda_2 = 0.7$ W/m

Calculate the amount of heat lost from 1 m² of the furnace wall by conduction and the temperature at the layer interface.

Answer

The loss of heat or the rate of heat flow through the wall $q = 1090$ W/m². The temperature at the layer interface $t_{w2} = 828°C$.

1-11. The thickness of the layer of red brick in the wall of the boiler furnace setting considered in problem 1-10 is

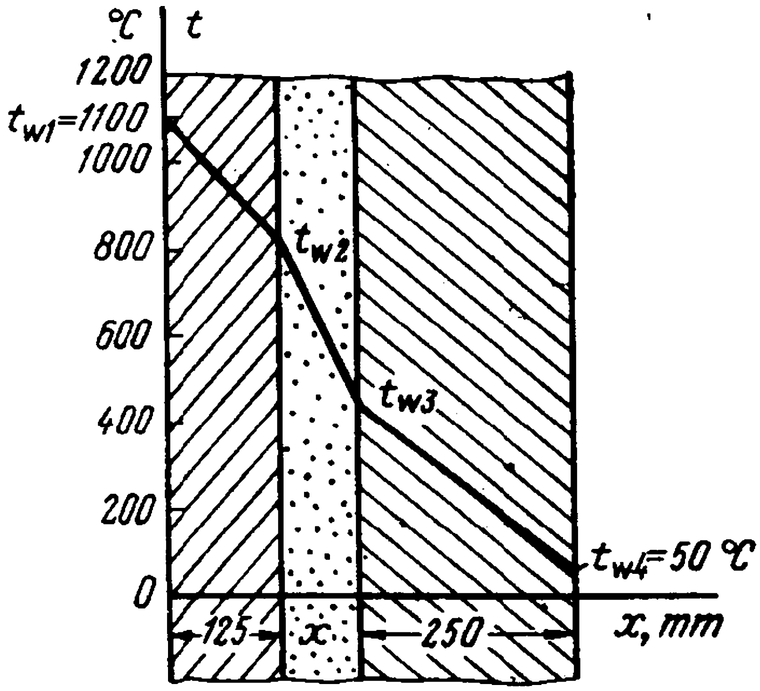

Fig. 1-5. Solution to problem 1-11

to be reduced 2 times and the space between the two layers of the setting is to be filled with crushed diatomite (Fig. 1-5), whose thermal conductivity

$$\lambda = 0.113 + 0.00023t, \quad \text{W/m}\cdot°\text{C}.$$

What should be the thickness of the diatomite filling to ensure the same loss of heat with the inside and outside wall temperatures remaining as before, in problem 1-10?

Answer

The diatomite filling must be 94 mm thick.

Solution

Since the heat loss $q = 1090$ W/m² must remain unchanged, the temperature at the interface between the two layers of the diatomite filling and red brick is found from the equation

$$t_{w3} = t_{w1} + q\frac{\delta_2}{\lambda_2} = 50 + 1090\,\frac{0.25}{0.7} = 439\ ^\circ\text{C}.$$

The mean thermal conductivity of the diatomite filling is determined from the equation

$$\lambda_m = a + b\,\frac{t_{w3} + t_{w2}}{2} = 0.113 + 0.00023\left(\frac{828 + 439}{2}\right) = 0.259\ \text{W/m}\cdot{}^\circ\text{C}$$

Then, the thickness of the diatomite filling will be:

$$\delta = \frac{\Delta t_{\text{fil}}}{q}\,\lambda_m = \frac{828 - 439}{1090}\,0.259 = 0.0936\ \text{m};\quad \delta \approx 94\ \text{mm}.$$

1-12. The walls of a drying chamber are built up of a layer of red brick of thickness $\delta_1 = 250$ mm and a layer of felt. The outside surface temperature of the red brick layer $t_{w1} = 110$°C and the outside surface temperature of the felt layer $t_{w3} = 25$°C.

The thermal conductivities of red brick $\lambda_1 = 0.7$ W/m·°C and of felt $\lambda_2 = 0.0465$ W/m·°C.

Calculate the temperature at the interface between the two layers, and the thickness of the felt layer on condition that the heat loss from 1 m² of the chamber wall does not exceed $q = 110$ W/m².

Answer

The temperature at the interface between the two layers $t_{w2} = 70.7$°C. The thickness of the layer of felt $\delta_2 \approx 19$ mm.

1-13. In the instrument used to determine the thermal conductivity of various materials the specimen of the tested material is placed between the hot and cold surfaces (Fig. 1-6).

The specimen is a disk having a diameter $d = 120$ mm and thickness $\delta = 20$ mm.

The temperature of the hot surface of the instrument $t_{w1} = 180$°C and the temperature of the cold surface $t_{w2} = 30$°C. After steady-state conduction sets in the heat flux through the specimen $Q = 50.6$ W. Due to the presence of protective heaters there is no heat flow in the radial direction.

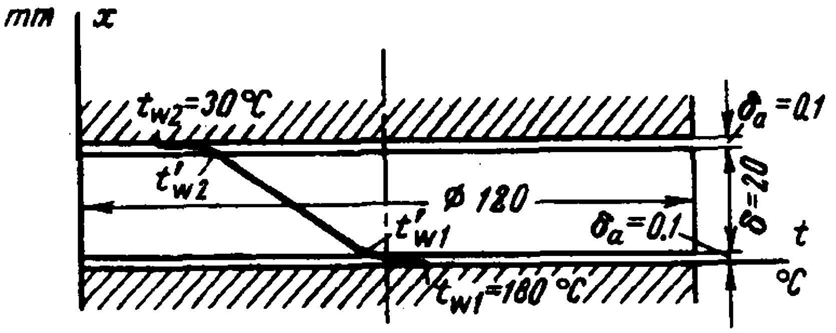

Fig. 1-6. Solution to problem 1-13

Due to poor matching, air clearances of thickness $\delta_a = 0.1$ mm formed in the instrument between the cold and hot surfaces and the tested specimen. Calculate the relative error admitted in determining the thermal conductivity of the tested material, Δ_λ, if the formed clearances are not accounted for when treated the test results. Reduce the thermal conductivity of the air in the clearances to the temperatures of corresponding surfaces, t_{w1} and t_{w2}.

Answer

The relative error admitted in determining λ is $\Delta_\lambda \approx 21$ per cent.

1-14. Calculate the loss of heat by steady conduction through unit area of the brick setting of a steam boiler in the vicinity of the water economizer, and the wall temperatures, if the thickness of the brick setting $\delta = 250$ mm, flue gas temperature $t_{f1} = 700$°C and the temperature of air in the boiler room $t_{f2} = 30$°C. The local coefficient of heat transfer from the flue gas to the surface of the setting $\alpha_1 = 23$ W/m^2·°C and the local coefficient of heat transfer from the setting to the air $\alpha_2 = 12$ W/m^2·°C. The thermal conductivity of the boiler setting $\lambda = 0.7$ W/m·°C.

Answer

The loss of heat (rate of heat flow) $q = 1385$ W/m^2. The surface temperatures of the setting, inside and outside, $t_{w1} = 640$°C and $t_{w2} = 145.5$°C.

1-15. Calculate the rate of heat flow through 1 m^2 of the clean heating surface of a steam boiler and surface temperatures, if the following values are given: the flue gas temperature $t_{f1} = 1000$°C; the boiling water temperature $t_{f2} = 200$°C; the local coefficients of heat transfer from the flue gas to the wall (heating surface) $\alpha_1 = 100$ W/m$^2\cdot$°C, and from the wall to the boiling water $\alpha_2 = 5000$ W/m$^2\cdot$°C.

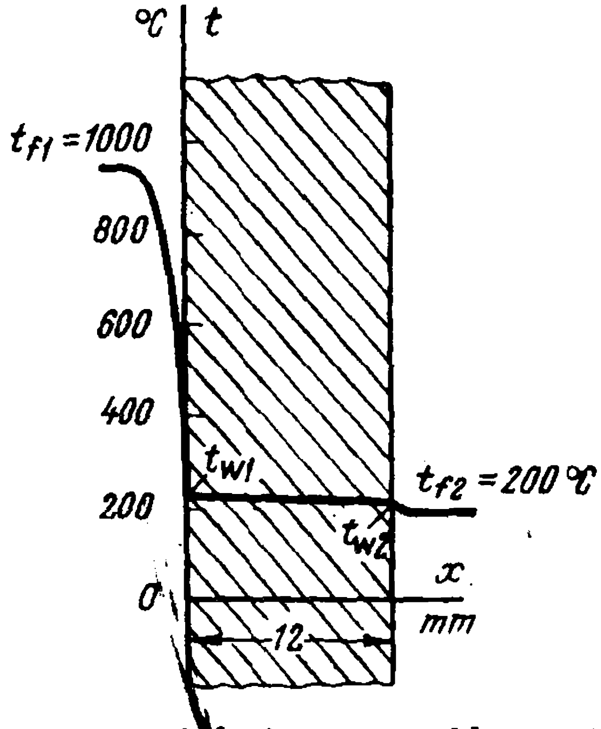

Fig. 1-7. Solution to problem 1-15

The thermal conductivity of the material from which the boiler heating surface is made $\lambda = 50$ W/m·°C and the wall thickness $\delta = 12$ mm.

Answer

$$q = 76\,500 \text{ W/m}^2.$$

Heating surface temperatures are: $t_{w1} = 235$°C and $t_{w2} = 215$°C (Fig. 1-7).

1-16. Solve problem 1-15 on condition that in the course of service the heating surface of the steam boiler, exposed to the flue gas, becomes covered with a layer of soot of thickness $\delta_s = 1$ mm ($\lambda_s = 0.08$ W/m·°C), and the surface on the water side is covered with a layer of scale of thickness $\delta_{sc} = 2$ mm ($\lambda_{sc} = 0.8$ W/m·°C).

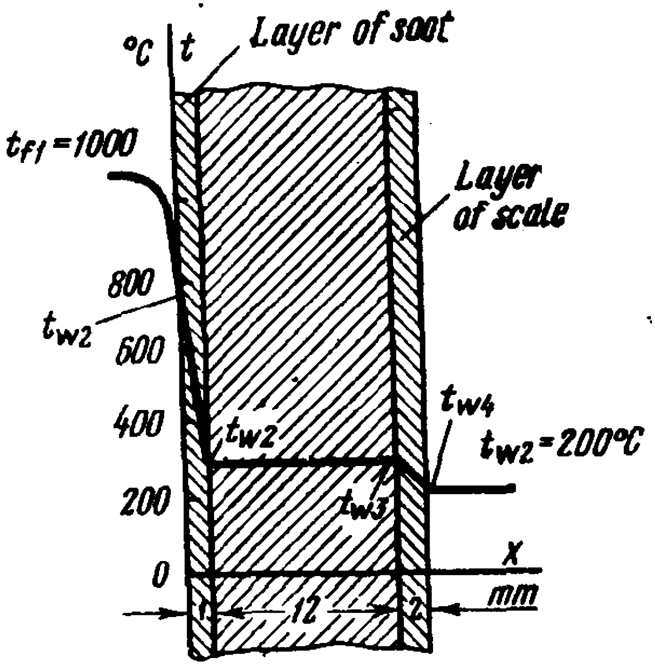

Fig. 1-8. Solution to problem 1-16

Calculate the rate of heat flow through 1 m² of the contaminated boiler heating surface and the surface temperatures of corresponding layers t_{w1}, t_{w2}, t_{w3} and t_{w4} (Fig. 1-8). Compare calculation results with the answer to problem 1-15 and determine the reduction in the rate of heat flow through unit area of the heating surface.

Answer

The rate of heat flow $q = 31\,500$ W/m². The surface temperatures are as follows: $t_{w1} = 685°C$, $t_{w2} = 291°C$, $t_{w3} = 283°C$, $t_{w4} = 206°C$.

The reduction in the rate of heat flow due to contamination of the boiler heating surface $\Delta q \approx 58.9$ per cent.

1-17. Determine the rate of heat flow through 1 m² of a brick wall of thickness $\delta = 510$ mm with a thermal conductivity $\lambda = 0.8$ W/m·°C. Air temperature inside the room $t_{f1} = 18°C$; the local coefficient of heat transfer from the air to the inside surface of the wall $\alpha_1 = 7.5$ W/m²·°C; outside air temperature $t_{f2} = -30°C$; the local coefficient of heat transfer from the outside surface of the wall, exposed to the wind, to the surrounding air $\alpha_2 = 20$ W/m²·°C. Calculate also wall surface temperatures t_{w1} and t_{w2}.

Answer

The rate of heat flow through the wall $q = 58.6$ W/m². The wall surface temperatures $t_{w1} = 10.2$°C and $t_{w2} = -27.1$°C.

1-18. Solve problem 1-17, assuming the wall to be covered on the outside with a layer of heat insulation 50 mm thick and with a thermal conductivity $\lambda_{ins} = 0.08$ W/m·°C. Compare the losses of heat through the non-insulated and insulated walls.

Answer

The amount of heat lost from 1 m² of the insulated wall $q = 33.2$ W/m². The wall surface temperatures are: $t_{w1} = 13.6$°C and $t_{w3} = -28.3$°C. The loss of heat from a non-insulated wall is 76.5 per cent greater than from a heat-insulated wall.

1-19. Calculate the rate of heat flow q, W/m², in a plate-type air heater and the surface temperatures of plates, if given are the mean temperature of the flue gas $t_{f1} = 315$°C and the mean air temperature $t_{f2} = 135$°C, and also respectively the local coefficients of heat transfer $\alpha_1 = 23$ W/m²·°C and $\alpha_2 = 30$ W/m²·°C. The plates of the air heater are 2 mm thick. The thermal conductivity of the material from which the plates are made $\lambda = 50$ W/m·°C.

Answer

$$q = 2200 \text{ W/m}^2; \quad t_{w1} = 208.5^\circ\text{C}; \quad t_{w2} = 208.4^\circ\text{C};$$

$$t_{w1} \approx t_{w2} \approx 208^\circ\text{C}.$$

1-20. The brickwork of a furnace is built up of a layer of fireclay brick with a thermal conductivity $\lambda = 0.84\,(1 + 0.695 \times 10^{-3}\,t)$ W/m·°C; the brickwork is 250 mm thick.

Determine the loss of heat from one square metre of brickwork surface, q W/m², and the wall surface temperatures, if the temperature of the gas in the furnace $t_{f1} = 1200$°C and the room temperature $t_{f2} = 30$°C, and the local coefficient of heat transfer from the hot gas to the brickwork $\alpha_1 = 30$ W/m²·°C, and the local coefficient of heat transfer

from the brickwork to surroundings $\alpha_2 = 10$ W/m²·°C.
Answer

$$q = 3530 \text{ W/m}^2.$$

Solution

With the dependence of the thermal conductivity of fireclay brick on temperature given, the loss of heat can be calculated from the following formula

$$q = \frac{\lambda}{\delta}(t_{w1} - t_{w2}) =$$

$$\frac{1}{\delta} = \left\{\lambda_0 + \lambda_0\beta_\lambda\left[\frac{t_{f1}+t_{f2}}{2} - \frac{q}{2}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right)\right]\right\} \times$$

$$\times\left[t_{f1} - t_{f2} - q\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)\right]$$

or with the aid of the method of successive approximations (also known as the step-by-step or iteration method). The solution with the aid of the method of successive approximations is given below.

Let us assume a mean wall temperature $\bar{t}_w = 650$°C. At this temperature the thermal conductivity of fireclay (chamotte) brick $\lambda_m = 0.84\,(1 + 0.695 \times 10^{-3} \times 650) = 1.12$ W/m·°C. Determine now the overall heat transfer coefficient

$$k = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{30} + \frac{0.25}{1.12} + \frac{1}{10}} = 2.81 \text{ W/m}^2\cdot{}^\circ\text{C}$$

and the rate of heat flow

$$q = k\,(t_{f1} - t_{f2}) = 2.81\,(1200 - 30) = 3290 \text{ W/m}^2.$$

Making use of the determined rate of heat flow, calculate the surface temperatures for the brickwork:

$$t_{w1} = t_{f1} - q\frac{1}{\alpha_1} = 1200 - \frac{3290}{30} = 1091 \text{ °C};$$

$$t_{w2} = t_{f2} - q\frac{1}{\alpha_2} = 30 + \frac{3290}{10} = 359 \text{ °C}.$$

Determine the mean temperature of the brickwork and specify the magnitude of the thermal conductivity:

$$t_w = 0.5\,(1091 + 359) = 725°\mathrm{C};$$

$$\lambda_m = 0.84\,(1 + 0.695 \times 10^{-3} \times 725) = 1.265\ \mathrm{W/m\cdot°C}$$

$$k = \frac{1}{\frac{1}{30} + \frac{0.25}{1.265} + \frac{1}{10}} = 3.02\ \mathrm{W/m^2\cdot°C}.$$

The rate of heat flow will then be

$$q = 3.02\,(1200 - 30) = 3530\ \mathrm{W/m^2}.$$

Using this new value of the rate of heat flow, calculate the wall surface temperatures t_{w1} and t_{w2}:

$$t_{w1} = 1200 - 3530\,\frac{1}{30} = 1082\ °\mathrm{C};$$

$$t_{w2} = 30 + 3530\,\frac{1}{10} = 383\ °\mathrm{C}.$$

Determine the mean surface temperature and thermal conductivity:

$$\bar{t}_w = 0.5\,(1082 + 383) = 732°\mathrm{C};$$

$$\lambda_m = 0.84\,(1 + 0.695 \times 10^{-3} \times 732) = 1.267\ \mathrm{W/m\cdot°C}.$$

Inasmuch as the obtained mean thermal value of conductivity coincides practically with the previously assumed value, no further calculations are necessary and it may be assumed that

$$q = 3530\ \mathrm{W/m^2}.$$

1-21. In the slag-tap bottom furnace of a steam boiler the gas temperature must be maintained equal to $t_{f1} = 1300°\mathrm{C}$ and the temperature of air in the boiler room $t_{f2} = 30°\mathrm{C}$. The furnace walls are made of a layer of a refractory material of thickness $\delta_1 = 250$ mm with a thermal conductivity $\lambda_1 = 0.28\,(1 + 0.833 \times 10^{-3}\,t)\ \mathrm{W/m\cdot°C}$ and a layer of diatomite brick with a thermal conductivity $\lambda_2 = 0.113\,(1 + 0.206 \times 10^{-3}\,t)\ \mathrm{W/m\cdot°C}$.

The coefficient of heat transfer from the gases, or products of combustion, to the setting $\alpha_1 = 30\ \mathrm{W/m^2\cdot°C}$ and the coefficient of heat transfer by radiation from the outside surface of the furnace setting to surroundings $\alpha_2 = 10$ W/m$^2\cdot$°C.

What should be the thickness of the diatomite layer of the setting so that the losses of heat to surroundings by radiation should not exceed 750 W/m^2?

Answer

The diatomite layer of the setting must be 132 mm thick ($\delta_2 = 132$ mm).

1-22. The loops of a steam superheater are made from tubes fabricated from heat-resistant steel; the diameter of the tubes $d_1/d_2 = 32/42$ mm and the thermal conductivity of the steel $\lambda = 14$ W/m$\cdot$°C. The temperature of the outside surface of a tube $t_{w2} = 580$°C and of the inside surface $t_{w1} = 450$°C.

Calculate the rate of heat flow through the wall of the tube per unit length, q_l W/m.

Answer

$$q_l = 42\,100 \text{ W/m}.$$

1-23. A steam pipeline, 150/160 mm in diameter, is covered with a layer of heat-insulating material of thickness

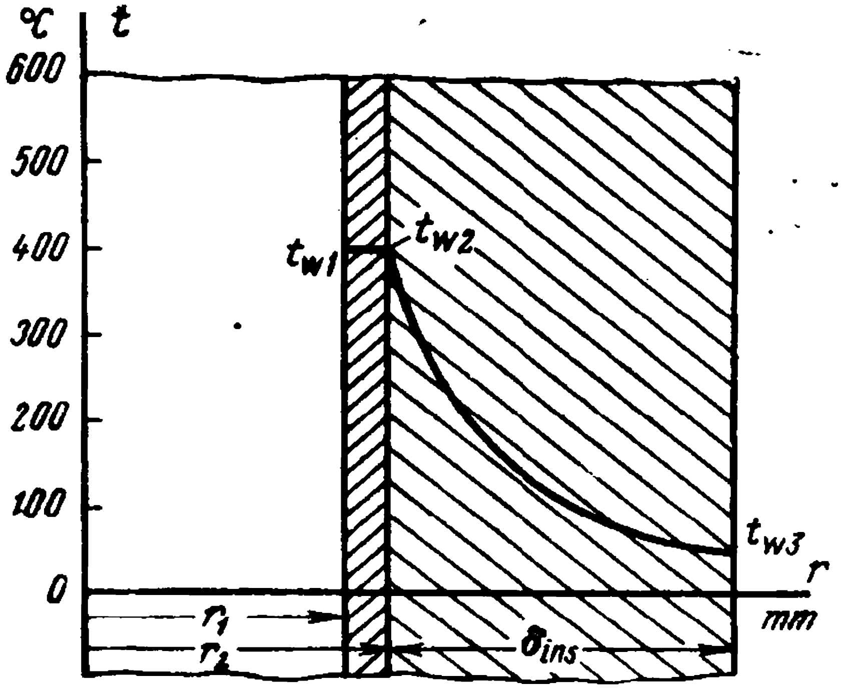

Fig. 1-9. Solution to problem 1-23

$\delta_{ins} = 100$ mm; the thermal conductivity of the tube $\lambda_1 = 50$ W/m·°C, and that of the insulating material $\lambda_2 = 0.08$ W/m·°C. The temperature of the pipeline inside surface $t_{w1} = 400$°C and the temperature of the outside surface of the insulating layer $t_{w3} = 50$°C (Fig. 1-9).

Find the loss of heat from length of 1 m of the pipeline and the temperature at the interface between the tube and insulation.

Answer

The loss of heat from length of 1 m of the steam pipeline $q_l = 216$ W/m. The temperature at the interface between the wall of the pipeline and the insulation $t_{w2} \approx 400$°C.

1-24. A steel pipeline of a diameter $d_1/d_2 = 100/110$ mm with a thermal conductivity $\lambda_1 = 50$ W/m·°C is covered with two layers of insulation of equal thickness $\delta_2 = \delta_3 = 50$ mm. The temperature of the tube inside surface $t_{w1} = 250$°C and the temperature of the outside surface of insulation $t_{w4} = 50$°C (Fig. 1-10).

Determine the loss of heat through the insulation from length of 1 m of the pipeline and the temperature at the interface between the two layers of insulation, if the first layer of insulation applied on the surface of the tube is made from a material with a thermal conductivity $\lambda_2 = 0.06$ W/m·°C and the second layer—from an insulating material with a thermal conductivity $\lambda_3 = 0.12$ W/m·°C.

Answer

The loss of heat from length of 1 m of the pipeline $q_l = 89.5$ W/m. The temperature at the interface between the two layers of insulation $t_{w3} = 97$°C.

1-25. How will change the loss of heat from length of 1 m of the pipeline considered in problem 1-24, if the two layers of insulation are changed places, i.e. if the layer with a greater thermal conductivity is placed directly on the tube, with all other conditions of the problem remaining unchanged.

Answer

The loss of heat shall increase and amount to $q_l = 105.5$ W/m. The temperature at the interface between the two layers of insulation $t_{w3} = 159$°C (Fig. 1-10).

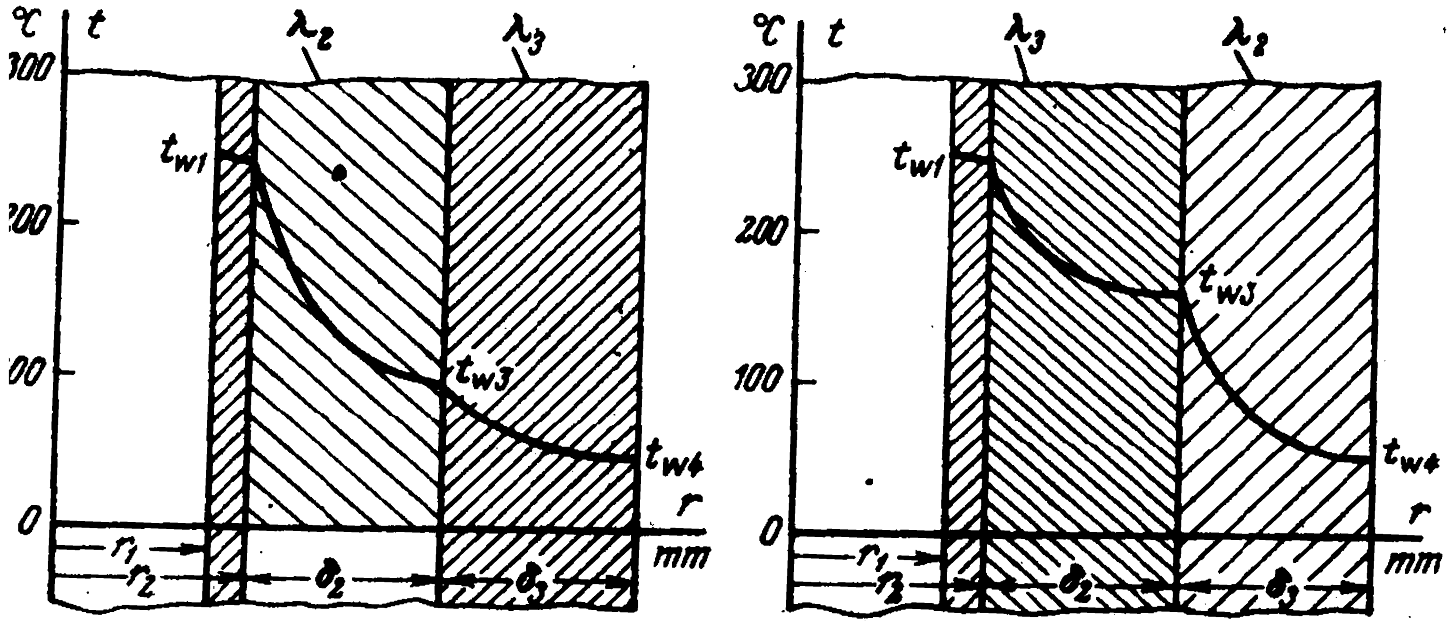

Fig. 1-10. Solution to problems 1-24 and 1-25

1-26. A steam pipeline $d_1/d_2 = 160/170$ mm in diameter is covered with a layer of insulation of thickness $\delta = 100$ mm with a thermal conductivity depending on temperature in the following way: $\lambda_{ins} = 0.062\,(1 + 0.363 \times 10^{-3}t)$ W/m·°C.

Determine the amount of heat loss from length of 1 m of pipeline and the temperature of its inner surface, if the temperature of the outer pipeline surface $t_{w2} = 300$°C and the temperature of the outer surface of the insulation should not exceed 50°C.

Answer

The loss of heat from length of 1 m of the steam pipeline $q_l = 205$ W/m. The inner pipeline surface temperature $t_{w1} = 300$°C.

1-27. A reinforced-concrete smoke stack (Fig. 1-11) with an inner diameter $d_2 = 800$ mm and an outer diameter $d_3 = 1300$ mm must be lined with a refractory on the inside.

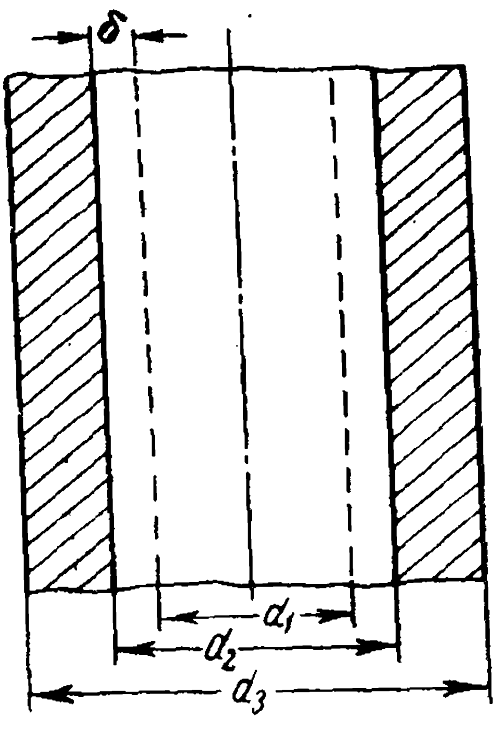

Fig. 1-11. Solution to problem 1-27

Determine the thickness of the refractory lining and the temperature of the outer surface of the smoke stack, t_{w3}, under conditions that the loss of heat from length of 1 m of the smoke stack must not exceed 2000 W/m and the temperature of the inner surface of the reinforced-concrete smoke stack, t_{w2}, should not exceed 200°C. The temperature of the inner surface of the lining refractory $t_{w1} = 425$°C; the thermal conductivity of the lining $\lambda_1 = 0.5$ W/m·°C; the thermal conductivity of concrete $\lambda_2 = 1.1$ W/m·°C.

Answer

$\delta = 120$ mm. The temperature of the tube outer surface $t_{w3} = 59$°C.

1-28. Under the conditions assumed in problem (1-27) determine the thickness of the refractory lining δ, if it is made from fireclay brick. Carrying out calculations, take into account that thermal conductivity λ depends on temperature by formula

$$\lambda = 0.84 + 0.0006t, \text{ W/m}\cdot{}^\circ\text{C}.$$

Answer

The refractory lining must be 206 mm thick.

1-29. In the instrument used to determine the thermal conductivity of materials by the "heated filament" method (Fig. 1-12) the annular clearance between the platinum filament and quartz tube is filled with the transformer oil being tested. The diameter of the platinum filament $d_1 = 0.12$ mm and its length $l = 90$ mm; the inner and outer diameters of the quartz tube are $d_2 = 1$ mm and $d_3 = 3$ mm; the thermal conductivity of quartz $\lambda = 1.4$ W/m·°C.

Calculate the thermal conductivity λ_f and the mean temperature t_f of the transformer oil, if, with a rate of heat flow through the circular layer of oil $Q = 1.8$ W, the temperature of the platinum filament $t_{w1} = 106.9$°C and the temperature of the outer surface of the quartz tube $t_{w3} = 30.6$°C.

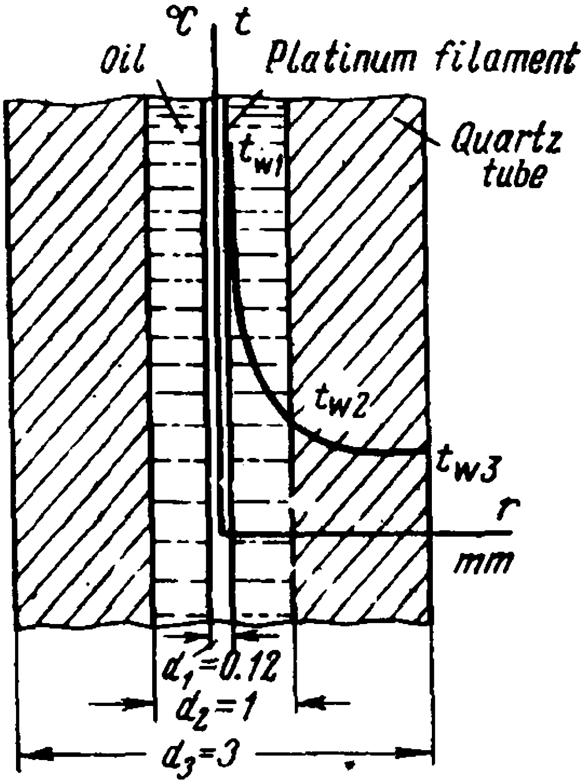

Fig. 1-12. Solution to problem 1-29

Answer

The thermal conductivity of transformer oil $\lambda_f \doteq 0.0915$ W/m·°C at oil temperature $t_f = 70$°C.

1-30. Calculate the admissible current intensity for a copper wire 2 mm in diameter covered with rubber insulation of thickness $\delta = 1$ mm, on condition that the maximum temperature of the insulation should not exceed 60°C and the temperature of the outer insulation surface—40°C. The thermal conductivity of rubber $\lambda = 0.15$ W/m·°C. The electric resistance of the copper wire $R = 0.005$ Ohm/m.

Answer

The admissible current intensity $I = 79.8$ A.

1-31. Determine the heating surface of a convection steam superheater built-up from heat-resistant steel tubes $d_1/d_2 = 32/40$ mm in diameter. The thermal conductivity of steel $\lambda = 39.5$ W/m·°C. The throughput of the superheater $Q = 61.1$ kg/s (of steam). At the superheater inlet the pressure of the dry saturated steam is 9.8 MPa. At the superheater outlet the temperature of the superheated steam $t_s = 500$°C.

The local coefficient of heat transfer from the flue gas to the wall $\alpha_2 = 81.5$ W/m²·°C and from the wall to the steam $\alpha_1 = 1163$ W/m²·°C; the mean flue gas temperature $t_f = 900$°C. The hydraulic resistance of the superheater is ignored.

Answer

The heating surface of the superheater, related to an outside tube diameter, $F = 1090$ m².

1-32. Solve problem (1-31) ignoring the curvature of the wall (assuming a plane wall). Compare the heating surface obtained with the answer to problem 1-31.

Answer

The heating surface $F = 1055$ m².

1-33. Find the heating surface of a sectional water-to-water heater of capacity $Q = 1500$ kW, on condition that the mean temperature of the heating water $t_{f1} = 115$°C and the mean temperature of the water being heated $t_{f2} = 77$°C. The heating surface is built up of brass tubes $d_1/d_2 = 14/16$ mm in diameter with a thermal conductivity $\lambda_w = 120$ W/m·°C. The inner surface of the brass tubes is covered with a layer of scale of thickness $\delta_{sc} = 0.2$ mm with a thermal conductivity $\lambda_{sc} = 2$ W/m·°C. The local coefficient of heat transfer on the side of the heating water is $\alpha_1 = 10\,000$ W/m²·°C and on the side of the heated water— $\alpha_2 = 4000$ W/m²·°C. Since the ratio of diameters $d_2/d_1 < 1.8$, use the calculation formula derived for a plane wall.

Answer

The heating surface $F = 18.1$ m².

1-34. Calculate the loss of heat from length of 1 m of a non-insulated, bare, pipeline, $d_1/d_2 = 150/165$ mm in diameter, laid in the open air if the pipeline carries water at a mean temperature $t_{f1} = 90°C$, and the ambient air temperature $t_{f2} = -15°C$. The thermal conductivity of the tube material $\lambda = 50$ W/m·°C. The local coefficient of heat transfer from the water to the wall of the tube $\alpha_1 = 1000$ W/m²·°C and that from the tube to the surrounding air $\alpha_2 = 12$ W/m²·°C.

Also determine the temperatures of the tube inner and outer surfaces (Fig. 1-13).

Answer

$$q_l = 652 \text{ W/m}, \quad t_{w1} = 89.8°C, \quad t_{w2} = 89.6°C.$$

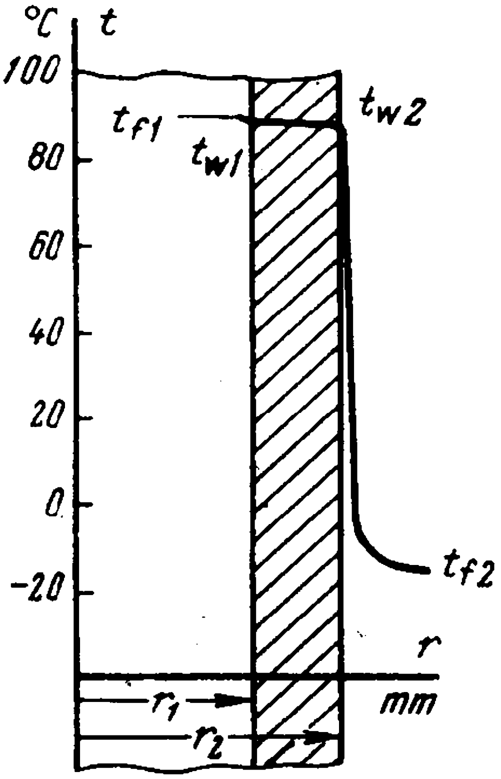

Fig. 1-13. Solution to problem 1-34

1-35. Determine the loss of heat from length of 1 m of the pipeline considered in the foregoing problem, provided the pipeline is covered with a layer of insulation 60 mm thick (Fig. 1-14). The thermal conductivity of the insulating material $\lambda_1 = 0.15$ W/m·°C. The local coefficient of heat transfer from the surface of the insulation to the surrounding air $\alpha_2 = 8$ W/m²·°C. All other conditions remain as in problem 1-34. Also calculate the temperatures of the outer tube surface t_{w2} and of the outer surface of the insulation t_{w3}.

Answer

$$q_l = 145 \text{ W/m}; \quad t_{w2} = 89.9°C; \quad t_{w3} = 5.3°C.$$

1-36. A pipeline, $d_1/d_2 = 25/29$ mm in diameter ($\lambda_1 = 50$ W/m·°C), covered with a layer of torfolium insulation 25 mm thick (δ_2) with thermal conductivity $\lambda_2 = 0.06$ W/m·°C carries saturated steam at pressure of 980 k·Pa.

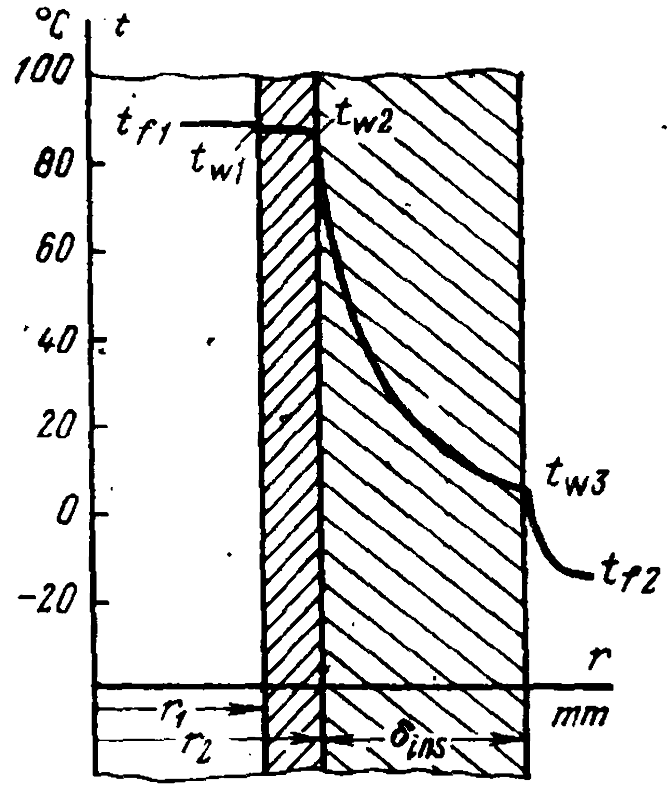

Fig. 1-14. Solution to problem 1-35

Determine the daily loss of heat, expressed in kilograms of steam, from a section of the pipeline 30 m long and the temperature of the insulation outer surface, if the local coefficient of heat transfer from steam to the wall $\alpha_1 = 2000$ W/m$^2\cdot$°C, and the local coefficient of heat transfer from the outer surface of the insulation to surroundings $\alpha_2 = 10$ W/m$^2\cdot$°C. The temperature of the surrounding air $t_{f2} = 10$°C.

Answer

The daily loss of heat, expressed in kilograms of steam, $G = 105$ kg/day. The temperature of the outer surface of insulation $t_{w3} = 61.2$°C.

1-37. A tubular air heater rated at 2.78 kg of air per second is built-up of tubes $d_1/d_2 = 43/49$ mm in diameter. The thermal conductivity of the tubes $\lambda = 50$ W/m·°C. The hot gas flows inside the tubes and the outside surface of the tubes is exposed to air in crossflow. The mean temperature of the flue gas $t_{f1} = 250$°C and the mean temperature of the air being heated $t_{f2} = 145$°C. The difference between the air inlet and outlet temperatures $\delta t = 250°C$. The local coefficient of heat transfer from the flue gas to the wall $\alpha_1 = 45$ W/m$^2\cdot$°C and that from the wall to the air $\alpha_2 = 25$ W/m$^2\cdot$°C.

Calculate the overall, or total, coefficient of heat transfer and determine the heating surface of the air heater. Calculate, using the formulae derived for a plane (flat) and cylindrical walls. Compare calculation results.

Answer

1. Calculations with the aid of the formula derived for a cylindrical wall give an overall heat-transfer coefficient $k_l = 0.75$ W/m$^2\cdot$°C, and a heating surface $F = 412$ m^2.

2. Calculations with the aid of the formula derived for a plane wall give an overall heat-transfer coefficient $k = 16$ W/m$^2\cdot$°C, and a heating surface $F = 418$ m^2.

1-38. Find the change in the thermal capacity of the air heater and the rate of air flow in problem 1-37, if in the course of service the surface of the heater exposed to the flue gas becomes covered with a layer of soot of thickness $\delta_s = 1$ mm; the thermal conductivity of soot $\lambda_s = 0.08$ W/m$\cdot$°C. All other conditions of problem 1-37 remain the same.

Calculate with the aid of the formula for a plane wall and compare with the corresponding variant of problem 1-37.

Answer

Under the conditions assumed, the thermal capacity of the air heater $Q = 587$ kW. The drop in thermal capacity $\Delta Q = 16.7$ per cent. The rate of air flow through the heater $G = 2.23$ kg/s.

1-39. An oil pipeline, $d_1/d_2 = 44/51$ mm in diameter, is covered with a layer of concrete of thickness $\delta_2 = 80$ mm. The thermal conductivity of pipeline material $\lambda_1 = 50$ W/m$\cdot$°C and the thermal conductivity of concrete $\lambda_2 = 1.28$ W/m$\cdot$°C. The mean temperature of the oil filling the section of pipeline considered $t_{f1} = 120°C$; ambient air temperature $t_{f2} = 20°C$. The local coefficient of heat transfer from oil to the wall $\alpha_1 = 100$ W/m$^2\cdot$°C and that from the surface of concrete to air $\alpha_2 = 10$ W/m$^2\cdot$°C.

(a) Calculate the loss of heat from length of 1 m of a bare pipeline and from the pipeline covered with concrete.

(b) What should the magnitude of the thermal conductivity of the insulation be to ensure a loss of heat from the pipeline covered with insulation of any thickness, not exceeding the loss of heat from a bare pipeline?

Answer

(a) The loss of heat from unit length of the bare pipeline $q_l = 142.5$ W/m. The loss of heat from a concrete-covered pipeline $q_l = 249$ W/m.

(b) For the heat loss from the insulated pipeline to be less than the amount of heat loss from a bare pipeline, at any thickness of the layer of insulation, the thermal conductivity of the insulation must be equal to or less than 0.26 W/m·°C.

1-40. What should the thickness of sovelite insulation be, δ_{ins}, with a thermal conductivity $\lambda_{ins} = 0.08$ W/m·°C so that the loss of heat from length of 1 m of the insulated pipeline would be three times less than the amount of heat loss from a bare pipeline, under the conditions assumed in problem 1-39?

Answer

The sovelite insulation must be 75 mm thick, $\delta_{ins} = 75$ mm.

Hints.

In the foregoing problem, the loss of heat from length of 1 m of a non-isolated, bare pipeline was found to be $q_l = 142$ W/m. For an isolated pipeline the formula for the loss of heat from length of 1 m will take the form

$$q_{l\,ins} = \frac{q_l}{3} = \frac{\pi (t_{f1} - t_{f2})}{1/\alpha_1 d_1 + {}^1/_2\lambda_1 \ln d_2/d_1 + {}^1/_2\lambda_{ins} \ln d_3/d_2 + 1/\alpha_2 d_3}\,.$$

Solving the above equation in respect to $\log d_3/d_2$, we find:

$$\log \frac{d_3}{d_2} = \frac{\Delta t \pi 2\lambda_{ins}}{2.3 q_{l\,ins}} \left[1 - \frac{q_{l\,ins}}{\pi \Delta t} \left(\frac{1}{\alpha_1 d_1} + \frac{2.3}{2\lambda_1} \log \frac{d_2}{d_1} + \frac{1}{\alpha_2 d_3} \right) \right]. \quad \text{(a)}$$

It is most convenient to solve equations of this type graphically, denoting the left-hand side of the equation by

$$Y_1 = \log d_3/d_2$$

and the right-hand side by

$$Y_2=\frac{\Delta t\pi 2\lambda_{ins}}{2.3q_{l\,ins}}\left[1-\frac{q_{l\,ins}}{\pi\Delta t}\left(\frac{1}{\alpha_1 d_1}+\frac{2.3}{2\lambda_1}\log\frac{d_2}{d_1}+\frac{1}{\alpha_2 d_3}\right)\right].$$

Assuming several values of d_3, we find graphically the value of the root that would satisfy equation (a).

1-41. Dry saturated steam flows through a pipe $d_1/d_2 = 18/20$ mm in diameter. To reduce the loss of heat to surroundings, the tube must be insulated. Is it expedient to insulate the tube with asbestos the thermal conductivity of which $\lambda = 0.11$ W/m·°C, if the local coefficient of heat transfer from the outer surface of the insulation to surroundings $\alpha = 8$ W/m^2·°C?

Answer

Since the critical diameter of the insulation is larger than the outside diameter of the tube ($d_{cr.\,ins} > d_2$), it is inexpedient to use asbestos to insulate the tube, and an insulating material with a smaller thermal conductivity must be employed.

1-42. It is necessary to insulate the shell of a heat-exchanger. The outside diameter of the shell $d_{out} = 300$ mm and its outer surface temperature $t_w = 280$°C, which can be assumed remaining the same after the layer of insulation is applied on the shell. The temperature of the outer surface of the insulation must not exceed 30°C, and the loss of heat from length of 1 m of the heat-exchanger shell must not be greater than 200 W/m. The local coefficient of heat transfer from the outer surface of the insulation to the surrounding air $\alpha = 8$ W/m^2·°C.

Is it expedient to use slag wool as an insulating material, the thermal conductivity of this material depending on temperature, obeying the following formula $\lambda = 0.06 + 0.000145t$ W/m·°C? If expedient, what should the thickness of the layer of this insulating material be to suit the preset conditions?

Answer

1. The critical diameter of the insulation $d_{cr} = 20.5$ mm which is considerably smaller than the outside diameter of the shell, therefore, the use of such an insulating material is advantageous.

2. To ensure the preset conditions, the layer of insulation applied on the shell should be 136 mm thick.

1-43. An electrical conductor having a diameter $d_1 = 1.5$ mm is at a temperature $t_{w1} = 70°C$ and is cooled by a stream of air whose temperature $t_f = 15°C$. The local coefficient of heat transfer from the surface of the conductor (wire) to the air $\alpha_1 = 16$ W/m²·°C.

Determine the surface temperature t'_{w1} of the conductor, if the latter is covered with rubber insulation of thickness $\delta = 2$ mm, leaving the intensity of the current carried by the conductor unchanged. The thermal conductivity of rubber $\lambda = 0.15$ W/m·°C. The local coefficient of heat transfer from the surface of the rubber insulation to the stream of air $\alpha_2 = 8.2$ W/m²·°C.

Answer

The temperature of the insulated wire $t'_{w1} = 44°C$. Thus, the use of an insulating material with $d_{cr} = 37 \text{ mm} > d_1$ leads to a greater rate of heat transfer from the surface and results in a lower conductor temperature.

1-44. Determine the thickness of the rubber insulation of the electric conductor considered in problem 1-43, making it possible for the conductor to carry a current of maximum intensity, with the temperature of the conductor remaining unchanged, $t_{w1} = 70°C$.

Answer

$$\delta = 17.75 \text{ mm}.$$

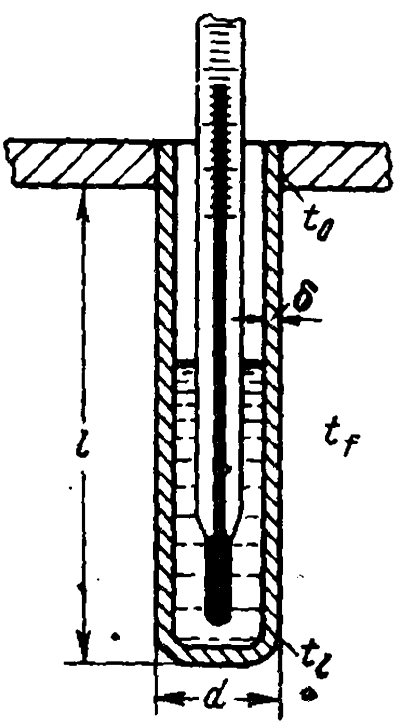

Fig. 1-15. Solution to problem 1-45

1-45. The temperature of air in a reservoir is measured with the aid of a mercury-in-glass thermometer placed into a protective well (a steel tube) filled with oil (Fig. 1-15). The thermometer shows the temperature at the end of the protective well, $t_l = 84°C$.

How large is the measurement error due to the transfer of heat by conduction along the protective well, if the temperature at the base of the protective well $t_0 = 40°C$; the well is 120 mm long; its thickness $\delta = 1.5$ mm; the thermal conductivity of the material of the protective well $\lambda = 55.8$ W/m·°C, and the local coefficient of heat transfer from the air to the protective well $\alpha = 23.3$ W/m²·°C?

Answer

The actual temperature of the air $t_f \doteq 100°C$; $t_f - t_l = 16°C$.

Solution

To solve the problem, let us make use of the approximate formula derived for a rod of finite length:

$$\frac{\vartheta_l}{\vartheta_0} = \frac{1}{\operatorname{ch}(ml)},$$

where

$$\frac{\vartheta_l}{\vartheta_0} = \frac{t_f - t_l}{t_f - t_0}; \quad m = \sqrt{\frac{\alpha u}{\lambda f}}, \quad 1/m;$$

the perimeter of the protective well $u = \pi d$, its cross section $f = \pi d \delta$, whence $u/f \approx 1/\delta$;
then

$$m = \sqrt{\frac{23.3}{55.8 \times 0.0015}} = 16.7, \quad 1/\text{m};$$

$$ml = 16.7 \times 0.12 = 2.$$

From tables we find that ch (2.0) = 3.76, consequently,

$$\frac{t_f - t_l}{t_f - t_0} = \frac{1}{3.76} = 0.266$$

and the temperature of air in the reservoir

$$t_f = \frac{t_l - 0.266 t_0}{1 - 0.266} = \frac{84 - 0.266 \times 40}{0.734} = 100\ °C.$$

The measurement error

$$t_f - t_l = 100 - 84 = 16°C.$$

1-46. What temperature will the thermometer indicate and by how much will the measurement error diminish, if the protective well of problem 1-45 is made from stainless steel with a thermal conductivity $\lambda = 23.3$ W/m·°C of length $l = 160$ mm and thickness $\delta = 0.8$ mm, and if the temperature at the base of the protective well will be increased to $t_0 = 70°C$ due to better insulation of the spot where the well is inserted into the reservoir?

Answer

$$t_l = 99.8°C; \quad t_f - t_l = 0.2°C.$$

1-47. To ensure better cooling of the outside surface of a semiconductor refrigerator, the outside surfaces of the refrigerator side walls are provided with vertical aluminium fins (Fig. 1-16). In the plan the refrigerator chamber is square-shaped. The width of the side walls $b = 800$ mm; height $h = 1000$ mm; the height and thickness of the fins are respectively $l = 30$ mm and $\delta = 3$ mm. Each wall has 40 fins.

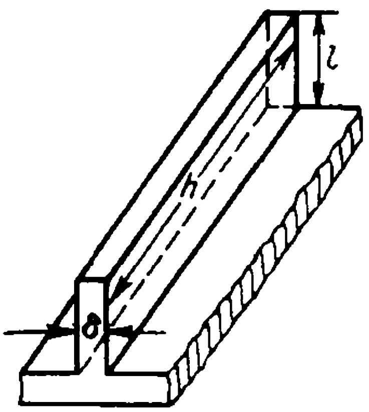

Fig. 1-16. Solution to problem 1-47

The temperature at the base of a fin $t_0 = 30°C$; the temperature of surroundings $t_f = 20°C$; the thermal conductivity of aluminium $\lambda = 202\ W/m\cdot°C$; the local coefficient of heat transfer from a finned surface to the surrounding air $\alpha = 7\ W/m^2\cdot°C$*.

Calculate the temperature at the end of a fin t_l and the quantity of heat transferred from the four side walls, $Q_{f\cdot w}$. Also calculate the amount of heat that would be lost from non-finned walls to surroundings under the same conditions, Q_w.

Answer

$$t_l = 29.8°C; \quad Q_{f\cdot w} = 848\ W; \quad Q_w = 223\ W.$$

1-48. A heating unit is made in the form of a vertical tube fitted with vertical rectangular-section steel fins (Fig. 1-17). The tube height $h = 1200$ mm; outside diameter $d_2 = 60$ mm; the fins are 50 mm long, and their thickness $\delta = 3$ mm. There is a total number of 20 fins.

The temperature at the base of a fin $t_0 = 80°C$; the temperature of surroundings $t_f = 18°C$. The local coefficient of heat transfer from the fins and from the plane, non-finned surface of the tube to the surrounding air $\alpha = 9.3$ W/m^2·°C; the thermal conductivity of the tube wall $\lambda = 55.7$ W/m·°C.

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* In solving the problem, the coefficient of heat transfer from the surface between fins (plane, non-finned surface) is assumed equal to the coefficient of heat transfer from the surface of fins.

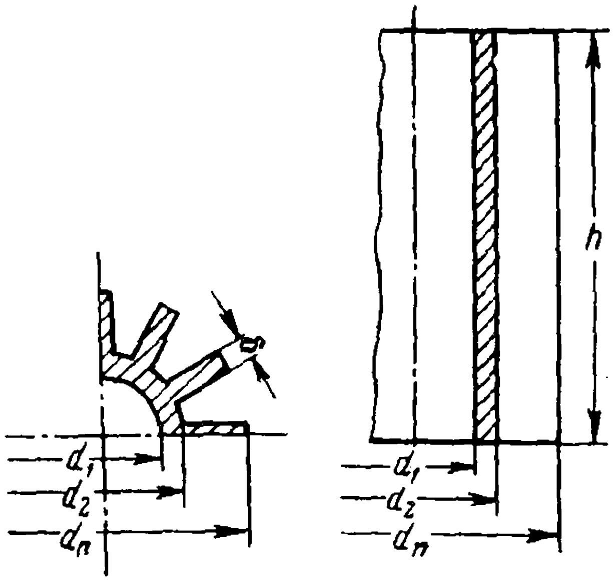

Fig. 1-17. Solution to problem 1-48

Calculate the amount of heat transferred from the finned wall to surroundings.

Answer

The amount of heat lost into surroundings from the fins $Q_f = 1270$ W. The amount of heat lost from the plane, non-finned surface of the tube, arranged between the fins, $Q_w = 88$ W. The amount of heat lost from the entire finned wall $Q_{f \cdot w} = 1358$ W.

1-49. A water economizer of the ЦКТИ system is built-up of round finned iron tubes of an outside diameter $d = 76$ mm. The diameter of the circular fins $D = 200$ mm and fin thickness $\delta = 5$ mm.

Determine the quantity of heat that will be transferred from the flue gas to the external surface of one economizer tube and the temperature at the tip of a fin, if flue gas temperature $t_f = 400$°C and the temperature at the base of fins $t_0 = 180$°C, the length of the tube exposed to the hot flue gas $l = 3$ m and the number of fins placed along this length $n = 150$.

The local coefficient of heat transfer from the hot flue gas to the finned surface $\alpha = 46.5$ W/m²·°C; the thermal conductivity of iron $\lambda = 52.4$ W/m·°C.

Answer

The amount of heat transferred through the fins $Q_f = 50$ kW. The quantity of heat transferred through the plane, non-finned surface of the economizer tube $Q_w = 5.5$ kW; the total amount of heat transferred from the flue gas to a finned tube $Q_{f.w} = 55.5$ kW.

Solution

If the flow of heat from the tip of a fin is ignored, the formulae for the excess temperature at the tip of a fin and the quantity of heat transferred through one fin take the appearance:

$$\vartheta_2 = \vartheta_1 \frac{I_0(mr_2)\,K_1(mr_2) + I_1(mr_2)\,K_0(mr_2)}{I_0(mr_1)\,K_1(mr_2) + I_1(mr_2)\,K_0(mr_1)};$$

$$Q_{f1} = 2\pi r_1 \lambda \delta m \vartheta_1 \psi,$$

where

$$\psi = \frac{I_1(mr_2)\,K_1(mr_1) - I_1(mr_1)\,K_1(mr_2)}{I_0(mr_1)\,K_1(mr_2) + I_1(mr_2)\,K_0(mr_1)}.$$

In our case

$$m = \sqrt{\frac{\alpha u}{\lambda f}} \approx \sqrt{\frac{2\alpha}{\lambda\delta}} = \sqrt{\frac{2 \times 46.5}{52.4 \times 0.005}} = 18.9 \text{ 1/m};$$

$$r_1 = d/2 = 76/2 = 38 \text{ mm}; \quad r_2 = 100 \text{ mm};$$

$$mr_1 = 18.9 \times 0.038 = 0.719; \quad mr_2' = 18.9 \times 0.1025 = 1.94,$$

where the flow of heat from the tip of a fin was accounted for approximately by increasing r_2 by one half of fin thickness: $r_2' = r_2 + \delta/2 = 0.10 + 0.0025 = 0.1025$ m.

Substituting the obtained values of mr_1 and mr_2' into the expression for the excess temperature at the end of a fin, we obtain:

$$\vartheta_2 = \vartheta_1 \frac{I_0(1.94)\,K_1(1.94) + I_1(1.94)\,K_0(1.94)}{I_0(0.719)\,K_1(1.94) + I_1(1.94)\,K_0(0.719)} =$$

$$= (400 - 180)\frac{2.1926 \times 0.153 + 1.509 \times 0.1305}{1.1336 \times 0.153 + 1.509 \times 0.643} = 102.5;$$

whence the temperature at the end (tip) of a fin

$$t_l = t_f - \vartheta_2 = 400 - 102.5 = 297.5°\text{C}$$

To determine the amount of heat transferred through one fin, we calculate the function

$$\psi = \frac{I_1(1.94)\,K_1(0.719) - I_1(0.719)\,K_1(1.94)}{I_0(0.719)\,K_1(1.94) + I_1(1.94)\,K_0(0.719)} =$$

$$= \frac{1.509 \times 1.014 - 0.3836 \times 0.153}{1.1336 \times 0.153 + 1.509 \times 0.643} = 1.295$$

and

$$Q_{f1} = 2\pi \times 0.038 \times 52.4 \times 18.9 \times 0.005 \times 220 \times$$
$$\times 1.295 = 337 \text{ W};$$

for 150 fins $Q_f = nQ_{f1} = 150 \times 337 = 50$ kW.

The amount of heat lost from the plane, non-finned surface arranged between the fins

$$Q_w = \alpha\vartheta_1 2\pi r_1 (l - n\delta) = 46.5 \times 220 \times 2\pi \times$$
$$\times 0.038 \times (3 - 150 \times 0.005) = 5.5 \text{ kW}.$$

The total amount of heat transferred from the finned tube to the surrounding air is

$$Q_{f\cdot w} = Q_f + Q_w = 50 + 5.5 = 55.5 \text{ kW}.$$

1-50. Solve problem 1-49 by means of the simplified solution method, using the relation for straight fins and making use of the graph shown in Fig. 1-18 [13].

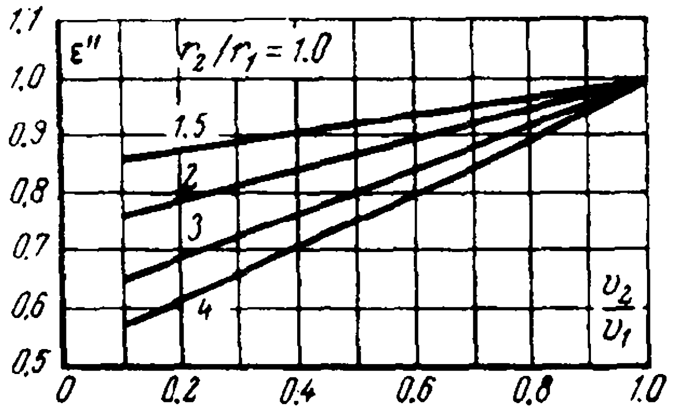

Fig. 1-18. Solution to problem 1-50

Answer

$$Q_{f1} = 341 \text{ W}.$$

Solution

Determine the height of a straight fin

$$h = r_2 - r_1 + \frac{\delta}{2} = 100 - 38 + \frac{5}{2} = 64.5 \text{ mm}.$$

From problem 1-49 we know that $m = 18.9$, then

$$mh = 18.9 \times 0.0645 = 1.22.$$

Determine the ratio of the excess temperatures at the tip and base of a fin

$$\frac{\vartheta_2}{\vartheta_1} = \frac{1}{\mathrm{ch}\,(mh)} = \frac{1}{\mathrm{ch}\,(1.22)} = 0.543;$$

$$\frac{r_2}{r_1} = \frac{100}{38} = 2.64.$$

Knowing the ratios ϑ_2/ϑ_1 and r_2/r_1, the graph in Fig. 1-18 is used to find the correction factor $\varepsilon'' = 0.836$. It is the ratio of the amounts of heat lost from unit area of a circular and a straight fin of equal thickness:

$$\varepsilon'' = \frac{q_{f1}}{q} = \frac{Q_{f1}}{F_{f1}} \times \frac{F}{Q}.$$

The amount of heat transferred to a straight fin of length $l = 1$ m and with a cross-sectional area $f = \delta l = 5 \times \times 10^{-3} \times 1 = 0.005$ m^2,

$$Q = \lambda m f \vartheta \,\mathrm{th}\,(mh) = 52.3 \times 18.9 \times 1 \times 0.005 \times \times 220 \times \mathrm{th}\,(1.22) = 978 \text{ W}.$$

The surface of such a fin

$$F = 2hl = 0.0645 \times 1 \times 2 = 0.129 \text{ m}^2;$$

$$q = \frac{Q}{F} = \frac{978}{0.129} = 7580 \text{ W/m}^2.$$

The surface of a circular fin

$$F_{f1} = 2\pi(0.1^2 - 0.038^2) = 0.0537 \text{ m}^2.$$

The unknown amount of heat transferred to the circular fin

$$Q_{f1} = \varepsilon'' q F_{f1} = 0.836 \times 7580 \times 0.0537 = 341 \text{ W}.$$

1-51. An air heater comprises component elements made of oval-shaped iron tubes. The profile of the fins is trapezoidal, and they are arranged along the generating line on the inner surface of a tube, as illustrated in Fig. 1-19.

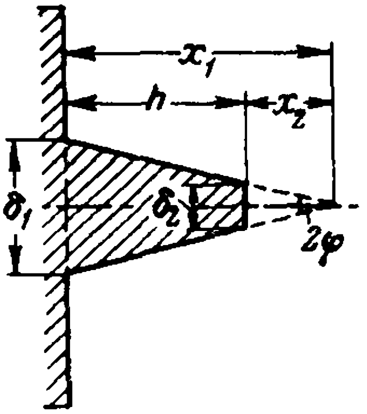

Fig. 1-19. Solution to problem 1-51

Determine the amount of heat lost from the surface of a fin of a tube $l = 2500$ mm. The height of the fin $h = 30$ mm; the thickness at the surface of the tube $\delta_1 = 3$ mm; the thickness at the end or tip of the fin $\delta_2 = 1$ mm. The thermal conductivity of iron $\lambda = 52.3$ W/m·°C.

The temperature at the base of the fin $t_0 = 450$°C; air temperature $t_f = 350$°C. The coefficient of heat transfer from the surface of the fin to the air, flowing through the heater, $\alpha = 23.3$ W/m²·°C. Also determine the temperature at the end of the fin.

Use exact formulae to solve the problem. The heat dissipated from the tip of the fin is accounted for by increasing its height by one half of its thickness.

Answer

$$Q_f = 312 \text{ W}; \quad t_l = 435\text{°C}.$$

Solution

If the heat transferred from the tip of the fin to surroundings is ignored, the formulae for the excess temperature at the tip of the fin and the amount of heat transferred take the form:

$$\vartheta_2 = \vartheta_1 \frac{I_0(2\sqrt{Z_2})\,K_1(2\sqrt{Z_2}) + I_1(2\sqrt{Z_2})\,K_0(2\sqrt{Z_2})}{I_0(2\sqrt{Z_1})\,K_1(2\sqrt{Z_2}) + I_1(2\sqrt{Z_2})\,K_0(2\sqrt{Z_1})};$$

$$Q_f = \psi \frac{\alpha \delta_1 l \vartheta_1}{\sqrt{Z_1}\tan\varphi},$$

where

$$\psi = \frac{I_1(2\sqrt{Z_1})\,K_1(2\sqrt{Z_2}) - I_1(2\sqrt{Z_2})\,K_1(2\sqrt{Z_1})}{I_0(2\sqrt{Z_1})\,K_1(2\sqrt{Z_2}) + I_1(2\sqrt{Z_2})\,K_0(2\sqrt{Z_1})}$$

and

$$Z_1 = \frac{\alpha}{\lambda \tan\varphi} x_1; \quad Z_2 = \frac{\alpha}{\lambda \tan\varphi} x_2.$$

In the problem considered

$$\tan \varphi = \frac{0.5(\delta_1 - \delta_2)}{h} = \frac{0.5(3-1)}{30} = \frac{1}{30};$$

$$x_1 = \frac{0.5\delta_1}{\tan \varphi} = 0.5 \times 3 \times 30 = 45 \text{ mm};$$

$$x_2 = x_1 - h = 45 - 30 = 15 \text{ mm};$$

$$Z_1 = \frac{\alpha x_1}{\lambda \tan \varphi} = \frac{23.3 \times 0.045}{52.3} 30 = 0.59;$$

$$Z_2 = \frac{23.3 \times 0.015}{52.3} 30 = 0.2;$$

$$2\sqrt{Z_1} = 2\sqrt{0.59} = 1.54; \quad 2\sqrt{Z_2} = 2\sqrt{0.2} = 0.894.$$

Substituting the values obtained into the expressions for the excess temperature at the tip of the fin, we obtan:

$$\vartheta_2 = \vartheta_1 \frac{I_0(0.894)\,K_1(0.894) + I_1(0.894)\,K_0(0.894)}{I_0(1.54)\,K_1(0.894) + I_1(0.894)\,K_0(1.54)} =$$

$$= (450 - 350)\,\frac{1.2130 \times 0.717 + 0.497 \times 0.487}{1.7 \times 0.717 + 0.497 \times 0.204} = 85\ ^\circ\text{C}.$$

In the case under consideration the temperature at the tip of the fin $t_l = 85 + 350 = 435$°C. To determine the amount of heat transferred from the fin to surroundings, calculate the function ψ:

$$\psi = \frac{I_1(1.54)\,K_1(0.894) - I_1(0.894)\,K_1(1.54)}{I_0(1.54)\,K_1(0.894) + I_1(0.894\,K_0(1.54)} =$$

$$= \frac{1.023 \times 0.717 - 0.497 \times 0.263}{1.7 \times 0.717 + 0.497 \times 0.204} = 0.458;$$

then

$$Q_f = \psi \frac{\alpha \delta_1 l \vartheta_1}{\sqrt{Z_1} \tan \varphi} = 0.458 \frac{23.3 \times 0.003 \times 2.5 \times 100}{\sqrt{0.59} \times \frac{1}{30}} = 312\ W.$$

1-52. Solve problem 1-51 with the aid of the simplified calculation method for trapezoidal section fins, using the graph shown in Fig. 1-20 [13].

Answer

$$Q_f = 312 \text{ W}.$$

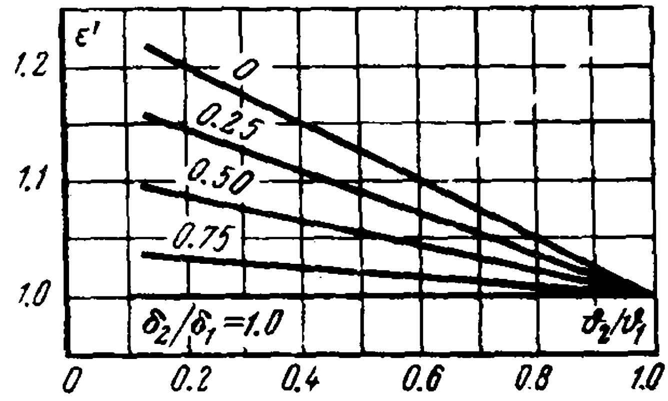

Fig. 1-20. Solution to problem 1-52

1-53. An electric heater is made from nichrome wire 2 mm in diameter (d) and 10 m long (l).

Cold air is blown at the heater, at a temperature $t_f = 20°C$.

Calculate the rate of heat flow from length of 1 m of the heater, and also the temperatures at the surface, t_w, and central line, t_0, of the nichrome wire, if the intensity of the current flowing through the heater is equal to 25 A. The specific resistance of nichrome $\rho = 1.1$ Ohm·mm²/m; the thermal conductivity of nichrome $\lambda = 17.5$ W/m·°C, and the local coefficient of heat transfer from the surface of the heater to the air $\alpha = 46.5$ W/m²·°C.

Answer

$$q_l = 218.5 \text{ W/m};$$

$$t_0 = 770°\text{C};\ t_w = 769°\text{C}.$$

Solution

The electric resistance of the heater

$$R = \frac{\rho l}{\pi r^2} = \frac{1.1 \times 10}{3.14 \times 1} = 3.5 \text{ Ohm}.$$

The amount of heat released by the heater

$$Q = I^2 R = 25^2 \times 3.5 = 2185 \text{ W}.$$

The rate of heat flow from length of 1 m of the nichrome wire

$$q_l = \frac{Q}{l} = \frac{2185}{10} = 218.5 \text{ W/m}.$$

The surface temperature of the wire is determined from the conditions for heat transfer:

$$t_w = t_f + \frac{q_l}{\pi d \alpha} = 20 + \frac{218.5}{3.14 \times 0.002 \times 46.5} = 769\ ^\circ\text{C}.$$

The temperature at the axis of the nichrome wire is determined from the conditions for heat transfer in the presence of inner heat sources:

$$t_0 = t_w + \frac{q_l}{4\pi\lambda} = 769 + \frac{218.5}{4 \times 3.14 \times 17.5} = 770\ ^\circ\text{C}.$$

1-54. A nichrome rod 5 mm in diameter and 420 mm long carries an electric current. The potential difference across the ends of the rod, $U = 10$ V.

On the surface of the rod water boils at a pressure $p = 5 \times 10^5$ Pa.

Determine the volumetric rate of heat liberation from the inner heat sources, q_v, W/m³, the rate of heat flow on the rod, q, W/m², the rate of heat flow from unit length of the rod, q_l, W/m, and the temperatures at the surface and central line of the conductor, if the local coefficient of heat transfer from the surface of the rod to the boiling water $\alpha = 44\,400$ W/m²·°C. The specific resistance of nichrome $\rho = 1.17$ Ohm·mm²/m. The thermal conductivity of nichrome $\lambda = 17.5$ W/m·°C.

Answer

$$q_v = 4.83 \times 10^8\ \text{W/m}^3;\quad q = 6.08 \times 10^5\ \text{W/m}^2;$$
$$q_l = 9540\ \text{W/m};\quad t_w = 164.7\,^\circ\text{C};\quad t_0 = 208.2\,^\circ\text{C}.$$

1-55. The fuel element of a nuclear reactor is made from a mixture of uranium carbide and graphite in the form of a cylindrical rod with a diameter $d = 12$ mm.

The volumetric rate of heat liberation from the heat sources $q_v = 3.88 \times 10^8$ W/m³. The heat sources can be assumed to be uniformly distributed over the entire volume. The thermal conductivity of the material from which the rod is made $\lambda = 58$ W/m·°C.

Determine the temperature and the rate of heat flow from the surface of a fuel element, if maximum surface temperature is 2000°C.

Answer

$$t_w = 1940\,^\circ\text{C};\quad q = 1.16 \times 10^6\ \text{W/m}^2.$$

1-56. The permissible continuous load for edge-mounted steel bus bars of 100 × 3 mm cross-sectional area should not exceed 300 A. At an ambient air temperature $t_f = 25°$ C the maximum temperature of the bus bar must not exceed 70°C.

Calculate the bus-bar surface temperature and determine what should be the local coefficient of heat transfer from the surface of the bus bar for the surface temperature of the bus bar would not exceed the maximum admissible value ($t_0 = 70°$C).

The thermal conductivity of steel $\lambda = 64$ W/m·°C and the specific resistance of steel $\rho = 0.13$ Ohm·mm²/m.

Answer

$$t_w \approx t_0 = 70°\text{C}; \quad \alpha = 4.3 \text{ W/m}^2\cdot°\text{C}.$$

1-57. An electric heater made from a constantan band, measuring 1 × 6 mm and 1 m long, carries a current of 20 A. The voltage drop across the heater amounts to 200 V.

Determine the surface temperature of the constantan band and its mid thickness temperature, if its local coefficient of heat transfer $\alpha = 1000$ W/m²·°C, the surrounding temperature $t_f = 100°$C, and the thermal conductivity of constantan $\lambda = 20$ W/m·°C.

Answer

$$t_w = 433°\text{C}; \quad t_0 = 437°\text{C}.$$

1-58. A stainless steel tube with an inner diameter $d_1 = 7.6$ mm and an outer diameter $d_2 = 8$ mm is heated with electric current, connecting the tube directly into an electric circuit.

All the heat evolved in the wall of the tube is transferred from the tube through its inner surface.

Calculate the volumetric rate of heat liberation from the inner heat sources and the temperature drop across the wall of the tube, if the latter carries a current of 250 A intensity.

The specific resistance and the thermal conductivity of steel are respectively $\rho = 0.85$ Ohm·mm²/m, $\lambda = 18.6$ W/m·°C.

Answer

$$q_v = 2.22 \times 10^9 \text{ W/m}^3; \quad t_{w2} - t_{w1} \approx 2.4°\text{C}.$$

Solution

The electric resistance per unit length of the tube

$$R_l = \frac{\rho}{\pi (r_2^2 - r_1^2)} = \frac{0.85}{3.14\,(4^2 - 3.8^2)} = 0.174 \text{ Ohm/m}.$$

The rate of heat flow per unit length of the tube

$$q_l = I^2 R_l = 250^2 \times 0.174 = 10\,870 \text{ W/m}.$$

The volumetric rate of heat liberation from the inner heat sources

$$q_v = \frac{q_l}{\pi (r_2^2 - r_1^2)} = \frac{10\,870}{3.14\,(4^2 - 3.8^2)\,10^{-6}} = 2.22 \times 10^9 \text{ W/m}^3.$$

The temperature drop across the wall of the tube

$$t_{w2} - t_{w1} = \frac{q_l r_2^2}{4\pi\lambda\,(r_2^2 - r_1^2)} \left[2 \ln \frac{r_2}{r_1} + \left(\frac{r_1}{r_2}\right)^2 - 1\right] =$$

$$= \frac{q_v r_2^2}{4\lambda} \left[2 \ln \frac{r_2}{r_1} + \left(\frac{r_1}{r_2}\right)^2 - 1\right] =$$

$$= \frac{2.22 \times 10^9 \times 0.004^2}{4 \times 18.6} \left[2 \times 2.3 \ln \frac{4}{3.8} + \left(\frac{3.8}{4}\right)^2 - 1\right] \approx 2.4 \text{ °C}.$$

1-59. A stainless steel tube is heated with electric current by direct connection into an electric circuit. The tube is 500 mm long, its outer and inner diameters are respectively $d_2 = 12.4$ mm and $d_1 = 12$ mm.

All the heat evolved in the wall of the tube is transferred through its outer surface.

Determine the temperature drop across the wall and the intensity of the current flowing through the tube, if the heat flux transferred from the outer surface of the tube $Q = 14$ kW.

The specific resistance and the thermal conductivity of the tube material are respectively

$$\rho = 0.85 \text{ Ohm·mm}^2/\text{m} \text{ and } \lambda = 18.6 \text{ W/m·°C}.$$

Answer

$$t_{w1} - t_{w2} = 4.0\text{°C}; \quad I = 502 \text{ A}.$$

1-60. A plate with uniformly distributed inner heat sources is cooled symmetrically from both surfaces. The temperature of the cooling fluid, or coolant, is t_f. Under these conditions the temperatures of the surfaces $t_{w1} = t_{w2} = t_w$, and the maximum temperature at the middle of the plate is equal to t_0.

Determine the temperatures of the surfaces of the plate t_{w1} and t_{w2}, if the transfer of heat from one of the surfaces is terminated, i.e., under the condition that $q_{w2} = 0$.

Answer

$$t_{w1} = t_f + 2(t_w - t_f) = 2t_w - t_f;$$

$$t_{w2} = t_f + 4t_0 - 2(t_w + t_f) = 4t_0 - (2t_w + t_f).$$

1-61. Acting in a 6 mm plate, made from a material with a thermal conductivity $\lambda = 20$ W/m·°C, are uniformly distributed inner sources of heat q_v. The temperatures of plate surfaces are respectively $t_{w1} = 120$°C and $t_{w2} = 127.2$°C.

Determine the relative coordinate x_0/s and the maximum temperature in the plate t_0, and also the rates of heat transfer from the plate surfaces, q_{w1} and q_{w2}, if $q_v = 5 \times 10^7$, 2×10^7, 8×10^6, and 4×10^6 W/m^3.

Answer

At $q_v = 5 \times 10^7$ W/m^3: $x_0/s = 0.58$;

$$t_0 \approx 135°\text{C};\quad q_{w2} = 1.26 \times 10^5 \text{ and } q_{w1} = 1.74 \times 10^5 \text{ W/m}^2.$$

At $q_v = 2 \times 10^7$ W/m^3: $x_0/s = 0.7$;

$$t_0 \approx 130°\text{C};\quad q_{w2} = 3.6 \times 10^4 \text{ and } q_{w1} = 8.4 \times 10^4 \text{ W/m}^2.$$

At $q_v = 8 \times 10^6$ W/m^3: $x_0/s = 1.0$;

$$t_0 = t_{w2} = 127.2\ °\text{C},\ q_{w2} = 0, \text{ and } q_{w1} = q_v s = 4.8 \times 10^4 \text{ W/m}^2.$$

At $q_v = 4 \times 10^6$ W/m^3: $x_0/s = 1.5$;

$$q_{w2} = -1.2 \times 10^4 \text{ W/m}^2 \text{ and } q_{w1} = 3.6 \times 10^4 \text{ W/m}^2.$$

Temperature distribution profiles are shown in Fig. 1-21.

Solution

If the origin of coordinates is placed as shown in Fig. 1-21 the equation for the temperature field in the plate takes the form:

$$t - t_{w1} = \frac{q_v x}{2\lambda}(2x_0 - x),$$

where

$$x_0/s = \frac{1}{2} + \frac{\lambda}{q_v s^2}(t_{w2} - t_{w1}),$$

and the maximum temperature

$$t_0 = t_{w1} + \frac{q_v x_0^2}{2\lambda}.$$

At $q_v = 5 \times 10^7$ W/m^3

$$\frac{x_0}{s} = \frac{1}{2} + \frac{20}{5\times10^7(6\times10^{-3})^2}(127.2-120) = 0.58;$$

$$x_0 = 0.58s = 0.58\times 6 = 3.48\ \text{mm};$$

$$t_0 = 120 + \frac{5\times10^7(3.48\times10^{-3})^2}{2\times20} \approx 135\ ^\circ\text{C};$$

$$q_{w1} = q_v x_0 = 5\times10^7\times3.48\times10^{-3} = 1.74\times10^5\ \text{W/m}^2;$$

$$q_{w2} = q_v(s-x_0) = 5\times10^7(6-3.48)10^{-3} = $$
$$= 1.26\times10^5\ \text{W/m}^2.$$

For other values of q_v calculations are identical.

At $q_v = 8 \times 10^6$ W/m^3 $x_0/s = 1$, i.e., the maximum temperature is at the plate surface, $t_0 = t_{w2} = 127.2$°C, $q_{w2} = 0$, and the heat liberated in the plate is transferred through the other surface: $q_{w1} = q_v x_0 = q_v s$.

At $q_v = 4 \times 10^6$ W/m^3 the temperature has a fictitious maximum arranged outside of the plate ($x_0 > s$), and heat is transferred to one of the plate surfaces from the outside, i.e., heat is being transferred through the wall: $q_{w2} = q_v(s - x_0) = -1.2 \times 10^4$ W/m^2. The heat removed from the plate through the other surface, $q_{w1} = q_v x_0 = q_v s + |q_2| = 2.4 \times 10^4 + 1.2 \times 10^4 = 3.6 \times 10^4$ W/m^2.

Temperature profiles in the plate are shown in Fig. 1-21.

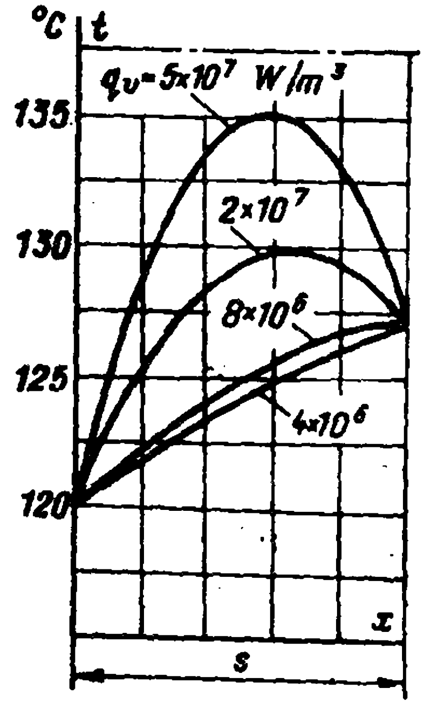

Fig. 1-21. Solution to problem 1-61

1-62. Determine the value of t_0 and the coordinate x_0 of the maximum temperature in a plate with uniformly distributed heat sources having a volumetric rate of heat liberation $q_v = 8 \times 10^6$ W/m^3. The plate thickness $s = 10$ mm; the thermal conductivity of the plate material $\lambda = 20$ W/m·°C. The surface temperatures of the plate are respectively $t_{w1} = 80$°C and $t_{w2} = 86$°C.

Answer

$t_0 \approx 88.5$°C, the coordinate x_0, measured from the surface which is at a temperature t_{w1}, is equal to 6.5 mm.

1-63. Acting in a plate of thickness $s = 5$ mm are uniformly distributed heat sources with a volumetric rate of heat liberation $q_v = 2.7 \times 10^7$ W/m^3. The thermal conductivity of the plate material $\lambda = 25$ W/m·°C. The local coefficients of heat transfer from the surfaces of the plate to the fluid to which it is exposed $\alpha_1 = 3000$ W/m^2·°C and $\alpha_2 = 1500$ W/m^2·°C and the temperatures of the fluid are respectively $t_{f1} = 130$°C and $t_{f2} = 140$°C.

Determine the coordinate and the magnitude of the maximum temperature in the plate, x_0 and t_0, and also the temperatures at the surfaces of the plate, t_{w1} and t_{w2}.

Answer

$$x_0 = 3.5 \text{ mm}; \quad t_0 = 168.1\text{°C}; \quad t_{w1} = 161.5\text{°C};$$

$$t_{w2} = 167\text{°C}.$$

Solution

With $q_v = \text{const}$ and $\lambda = \text{const}$, a non-symmetrical temperature field and boundary conditions of the third kind the relative coordinate or the maximum temperature in the plate

$$\frac{x_0}{s} = \frac{\frac{1}{2} + \frac{\lambda}{q_v s^2}(t_{f2} - t_{f1}) + \frac{\lambda}{\alpha_2 s}}{1 + \frac{\lambda}{s}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)},$$

where x_0 is measured from the surface which is exposed to the fluid at a temperature t_{f1}.

In the case considered

$$\frac{x_0}{s}=\frac{\frac{1}{2}+\frac{25}{2.7\times 10^7\,(5\times 10^{-3})^2}\,(140-130)+\frac{25}{1.5\times 10^3\times 5\times 10^{-3}}}{1+\frac{25}{5\times 10^{-3}}\left(\frac{1}{3\times 10^3}+\frac{1}{1.5\times 10^3}\right)}=0.7;$$

$$x_0 = 0.7 \times 5 = 3.5 \text{ mm}.$$

The surface temperatures of the plate

$$t_{w1}=t_{f1}+\frac{q_{w1}}{\alpha_1}=t_{f1}+\frac{q_v x_0}{\alpha_1}=$$

$$=130+\frac{2.7\times 10^7\times 3.5\times 10^{-3}}{3\times 10^3}=161.5^\circ\text{C};$$

$$t_{w2}=t_{f2}+\frac{q_{w2}}{\alpha_2}=t_{f2}+\frac{q_v\,(s-x_0)}{\alpha_2}=$$

$$=140+\frac{2.7\times 10^7\,(5-3.5)\times 10^{-3}}{1.5\times 10^3}=167^\circ\text{C}.$$

The maximum temperature

$$t_0=t_{w1}+q_{w1}\frac{x_0}{2\lambda}=t_{w1}+\frac{q_v x_0^2}{2\lambda}=$$

$$=161.5+\frac{2.7\times 10^7\,(3.5\times 10^{-3})^2}{2\times 25}=168.1\ ^\circ\text{C}.$$

1-64. A plate with uniformly distributed inner heat sources having a volumetric rate of heat liberation q_v, W/m^3, is exposed on both sides to a flowing fluid. The thickness of the plate is s, m, and the thermal conductivity of its material is λ, W/m·°C. The temperature of the fluid on one side of the plate is t_{f1}, °C, and the local coefficient of heat transfer from this surface to the fluid is equal to α_1, W/m^2·°C.

Determine the temperature of fluid on the opposite surface of the plate, t_{f2}, at which the rate of heat flow from this surface will be equal to zero ($q_{w2} = 0$).

Answer

$$t_{f2}=t_{f1}+q_v s\left(\frac{s}{2\lambda}+\frac{1}{\alpha_1}\right).$$

1-65. A plate with uniformly distributed inner heat sources with a volumetric rate of heat liberation q_v, W/m^3, is exposed on both sides to a flowing fluid. The local coefficients of heat transfer from the surfaces of the plate to the fluid and the temperatures of the fluid are respectively α_1 and α_2, W/m²·°C, t_{f1} and t_{f2},°C. The thickness of the plate is s, m, and the thermal conductivity of its material is λ, W/m·°C.

(a) Determine the relation between the temperature difference $(t_{f2} - t_{f1})$ and the local heat-transfer coefficients α_1 and α_2 for the case when the maximum temperature is at the middle of the plate, $x_0/s = 1/2$.

(b) Determine the value of the temperature difference $(t_{f2} - t_{f1})$ at $x_0/s = 1/2$, if $\alpha_2 = 2\alpha_1$.

(c) Find the expression for x_0/s when $t_{f2} = t_{f1}$ and $\alpha_2 = 2\alpha_1$.

Answer

(a) $t_{f2} - t_{f1} = \frac{q_v s}{2}\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2}\right)$.

(b) $t_{f2} - t_{f1} = \frac{q_v s}{4\alpha_1}$.

(c) $\frac{x_0}{s} = \frac{\alpha_1 s + \lambda}{2\alpha_1 s + 3\lambda}$.

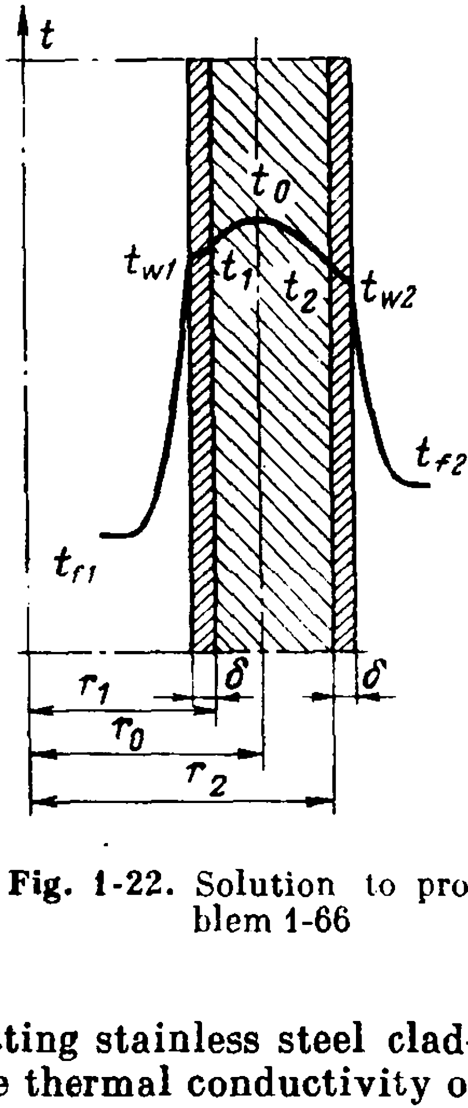

Fig. 1-22. Solution to problem 1-66

1-66. Calculate the distribution of temperature in the cross section of a fuel element made in the form of a long hollow cylinder (Fig. 1-22) having an inner diameter $d_1 = 16$ mm and outer diameter $d_2 = 26$ mm. The fuel element is made from uranium with a thermal conductivity $\lambda = 31$ W/m·°C. The inner and outer surfaces of the fuel element are covered with tightly fitting stainless steel claddings of thickness $\delta = 0.5$ mm. The thermal conductivity of stainless steel $\lambda_{cl} = 21$ W/m·°C. The volumetric rate of heat liberation in the fuel element is assumed to be uniform over the cross section and equal to $q_v = 5 \times 10^7$ W/m³.

The fuel element is being cooled with carbon dioxide (CO_2) flowing along an inner and outer channels. The bulk temperature of the coolant (CO_2) in the inner channel $t_{f1} = 200$°C and in the outer channel, $t_{f2} = 240$°C. The local coefficients of heat transfer from the surfaces of the claddings to the gaseous coolant are equal respectively to $\alpha_1 = 520$ W/m²·°C and $\alpha_2 = 560$ W/m²·°C.

Determine by calculation the maximum temperature for the fuel element t_0, the surface temperatures of the claddings t_{w1} and t_{w2} and the uranium surface temperatures t_1 and t_2.

Answer

$t_0 = 463°C$; $t_{w1} = 457°C$; $t_{w2} = 455°C$; $t_1 = 459°C$; $t_2 = 458°C$. Temperature distribution through the fuel element is shown in Fig. 1-22.

Solution

To calculate the temperature distribution, we must find the neutral-section radius r_0. Since the magnitude of the neutral-section radius r_0 depends on the rate of heat transfer (removal) from the surfaces of the uranium fuel element and known are the local coefficients of heat transfer from the surfaces of the claddings, α_1 and α_2, first determine the effective heat-transfer coefficients α_{ef1} and α_{ef2}, accounting for the thermal resistance of the claddings:

$$\frac{1}{\alpha_{ef1}d_1} = \frac{1}{\alpha_1(d_1-2\delta)} + \frac{1}{2\lambda_{cl}}\ln\frac{d_1}{(d_1-2\delta)} =$$

$$= \frac{1}{520(16-1)\times 10^{-3}} + \frac{1}{2\times 21}\ln\frac{16}{16-1} = 0.1298;$$

$$\alpha_{ef1} = \frac{1}{0.1298\times 16\times 10^{-3}} = 482 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$\frac{1}{\alpha_{ef2}} = \frac{d_2}{\alpha_2(d_2+2\delta)} + \frac{d_2}{2\lambda_{cl}}\ln\frac{d_2+2\delta}{d_2} =$$

$$= \frac{26\times 10^{-3}}{560(26+1)\times 10^{-3}} + \frac{26\times 10^{-3}}{2\times 21}\ln\frac{26+1}{26} = 1.747\times 10^{-3};$$

$$\alpha_{ef2} = \frac{1}{1.747\times 10^{-3}} = 573 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The neutral-section radius

$$r_0 = \sqrt{\frac{(t_{f2}-t_{f1}) + \frac{q_v}{2}\left[\frac{r_1}{\alpha_{ef1}} + \frac{r_2}{\alpha_{ef2}} + \frac{1}{2\lambda}(r_2^2-r_1^2)\right]}{\frac{q_v}{2}\left(\frac{1}{\alpha_{ef1}r_1} + \frac{1}{\alpha_{ef2}r_2} + \frac{1}{\lambda}\ln\frac{r_2}{r_1}\right)}} =$$

$$=\sqrt{\frac{(240-220)+2.5\cdot10^{7}\left[\frac{8\cdot10^{-3}}{482}+\frac{13\cdot10^{-3}}{573}+\frac{1}{2\cdot31}(13^{2}-8^{2})10^{-6}\right]}{2.5\cdot10^{7}\left(\frac{1}{482\cdot8\cdot10^{-3}}+\frac{1}{573\cdot13\cdot10^{-3}}+\frac{1}{31}\ln\frac{13}{8}\right)}}=10^{-3}\sqrt{103.9}=10.2\times10^{-3}\ \text{m}.$$

The rate of heat flow on the inner surface of the uranium fuel element is determined from the relationship

$$q_1 2\pi r_1 = q_v\pi\,(r_0^2-r_1^2),$$

$$q_1=\frac{q_v r_1}{2}\left(\frac{r_0^2}{r_1^2}-1\right)=\frac{5\times10^{7}\times8\times10^{-3}}{2}\left(\frac{10.2^{2}}{8^{2}}-1\right)=$$

$$=1.25\times10^{5}\ \text{W/m}^2.$$

The temperature at inner surface of uranium fuel element

$$t_1=t_{f1}+\frac{q_1}{\alpha_{ef1}}=200+\frac{1.25\times10^{5}}{482}=200+259=459^\circ\text{C}.$$

The rate of heat flow on the inner surface of the cladding

$$q_{w1}=q_1\frac{d_1}{d_1-2\delta}=1.25\times10^{5}\frac{16}{15}=1.335\times10^{5}\ \text{W/m}^2.$$

The temperature at the inner surface of the cladding

$$t_{w1}=t_{f1}+\frac{q_{w1}}{\alpha_1}=200+\frac{1.335\times10^{5}}{520}=200+257=457^\circ\text{C}.$$

The rates of heat flow q_2 and q_{w2} and the temperatures t_2 and t_{w2} for the outer surface of the uranium fuel element are determined in a similar way

$$q_2=\frac{q_v r_2}{2}\left(1-\frac{r_0^2}{r_2^2}\right)=\frac{5\times10^{7}\times13\times10^{-3}}{2}\left(1-\frac{10.2^{2}}{13^{2}}\right)=$$

$$=1.25\times10^{5}\ \text{W/m}^2;$$

$$t_2=t_{f2}+\frac{q_2}{\alpha_{ef2}}=240+\frac{1.25\times10^{5}}{573}=240+218=458^\circ\text{C};$$

$$q_{w2}=q_2\frac{d_2}{d_2+2\delta}=1.25\times10^{5}\frac{26}{27}=1.205\times10^{5}\ \text{W/m}^2;$$

$$t_{w2}=t_{f2}+\frac{q_{w2}}{\alpha_2}=240+\frac{1.205\times10^{5}}{560}=240+215=455^\circ\text{C}.$$

The distribution of temperature over the cross section of the uranium fuel element is determined from equation

$$t = t_1 + \frac{q_v}{4\lambda}\left[2r_0^2 \ln \frac{r}{r_1} - (r^2 - r_1^2)\right],$$

and the maximum temperature is found from the condition with $r = r_0$ $t = t_0$ and, hence,

$$t_0 = t_1 + \frac{q_v}{4\lambda}\left[2r_0^2 \ln \frac{r_0}{r_1} - (r_0^2 - r_1^2)\right] =$$

$$= 459 + \frac{5 \times 10^7}{4 \times 31}\left[2 \times 10.2^2 \ln \frac{10.2}{8} - (10.2^2 - 8^2)\right] 10^{-6} =$$

$$= 459 + 4.2 \approx 463°\text{C}.$$

Temperature distribution is shown in Fig. 1-22.

1-67. Determine the maximum temperature of a fuel element under conditions of problem 1-66, if (a) the inner channel ceased to be cooled for some reason or other; (b) the outer channel ceased to be cooled.

Answer

(a) $t_1 = t_0 \approx 610°\text{C}$; (b) $t_2 = t_0 \approx 904°\text{C}$, which is inadmissible both for the uranium element and for its cladding.

Solution

(a) If the inner channel is no more cooled, then $q_1 = 0$ and the maximum temperature shall be at the point $r_0 = r_1$. Under these conditions

$$q_2 = \frac{q_v r_2}{2}\left(1 - \frac{r_1^2}{r_2^2}\right) = \frac{5 \times 10^7 \times 13 \times 10^{-3}}{2}\left(1 - \frac{8^2}{13^2}\right) =$$

$$= 2.02 \times 10^5 \text{ W/m}^2;$$

$$t_2 = t_{f2} + \frac{q_2}{\alpha_{ef2}} = 240 + \frac{2.02 \times 10^5}{573} = 240 + 353 = 593°\text{C};$$

$$t_0 = t_1 = t_2 + \frac{q_v}{4\lambda}\left[(r_2^2 - r_1^2) - 2r_1^2 \ln \frac{r_2}{r_1}\right] =$$

$$= 593 + \frac{5 \times 10^7}{4 \times 31}\left[(13^2 - 8^2) - 2 \times 8^2 \ln \frac{13}{8}\right] 10^{-6} =$$

$$= 593 + 17.4 \approx 610\,°\text{C}.$$

(b) If cooling of the outer channel is terminated, then $q_2 = 0$ and $r_0 = r_2$. Then,

$$q_1 = \frac{q_v r_1}{2}\left(\frac{r_2^2}{r_1^2} - 1\right) = \frac{5 \times 10^7 \times 8 \times 10^{-3}}{2}\left(\frac{13^2}{8^2} - 1\right) =$$

$$= 3.28 \times 10^5 \text{ W/m}^2;$$

$$t_1 = t_{f1} + \frac{q_1}{\alpha_{ef1}} = 200 + \frac{3.28 \times 10^5}{482} = 200 + 680 = 880°\text{C};$$

$$t_0 = t_2 = t_1 + \frac{q_v}{4\lambda}\left[2r_2^2 \ln\frac{r_2}{r_1} - (r_2^2 - r_1^2)\right] =$$

$$= 880 + \frac{5 \times 10^7}{4 \times 31}\left[2 \times 13^2 \ln\frac{13}{8} - (13^2 - 8^2)\right] 10^{-6} =$$

$$= 880 + 23.8 \approx 904\,°\text{C}.$$

1-68. A hollow cylindrical fuel element with an inner diameter $d_1 = 14$ mm and outer diameter $d_2 = 24$ mm is made from uranium with a thermal conductivity $\lambda = 31$ W/m·°C. Both surfaces of the fuel element are covered with tightly fitting stainless steel claddings with $\lambda_{cl} = 21$ W/m·°C. The thickness of a cladding is 0.5 mm. The volumetric rate of heat liberation in the uranium fuel element is assumed to be uniform over the cross section and equal to $q_v = 2 \times 10^8$ W/m^3.

The fuel element is cooled with water flowing along an inner and outer coolant channels. The bulk temperature of the cooling water in the inner channel $t_{f1} = 200$ °C and in the outer channel $t_{f2} = 220$ °C. The local coefficients of heat transfer from the surfaces of the claddings to the cooling water are respectively $\alpha_1 = 8200$ W/m^2·°C and $\alpha_2 = 7800$ W/m^2·°C.

Determine the maximum temperature in the cross section of the fuel element, t_0, the rate of heat flow and the surface temperatures of the claddings, q_{w1}, q_{w2}, t_{w1} and t_{w2}, and the rates of heat flow and the surface temperatures for the uranium, q_1, q_2, t_1 and t_2.

Answer

$$t_0 = 308\,°\text{C}; \quad q_{w1} = 6.05 \times 10^5 \text{ W/m}^2;$$

$$q_{w2} = 4.44 \times 10^5 \text{ W/m}^2;$$

$$t_{w1} \approx 274\,°\text{C}; \quad t_{w2} \approx 277\,°\text{C}; \quad q_1 = 5.62 \times 10^5 \text{ W/m}^2;$$

$$q_2 = 4.63 \times 10^5 \text{ W/m}^2; \quad t_1 \approx 287\,°\text{C}; \quad t_2 \approx 288\,°\text{C}.$$

1-69. Determine the maximum temperature of a fuel element under the conditions given in problem 1-68, if cooling of the inner channel is terminated for some reason or other.

Answer

$$t_1 = t_0 = 404\,^\circ\mathrm{C}.$$

Chapter Two

TRANSIENT CONDUCTION

2-1. A rubber plate of thickness $2\delta = 20$ mm is heated to a temperature of $t_0 = 140°C$, then exposed to the ambient air at a temperature $t_f = 15°C$.

Determine the temperatures at the middle and at the surface of the plate in a time interval $\tau = 20$ min after chilling of the plate is initiated.

The thermal conductivity of rubber $\lambda = 0.175$ W/m·°C. The thermal diffusivity of rubber $a = 0.833 \times 10^{-7}$ m^2/s.

The local coefficient of heat transfer from the surface of the rubber plate to the surrounding air $\alpha = 65$ W/m^2·°C.

Answer

$$t_{x=0} = 47.5\,°C; \quad t_{x=\delta} = 25.4\,°C.$$

Solution

The temperatures at the middle and at the surface of a plate of infinite length undergoing cooling (heating) in surroundings at a constant temperature can be determined with the aid of the graphs $\Theta_{x=0} = f_1$ (Bi, Fo), shown in Fig. 2-1, and $\Theta_{x=\delta} = f_2$ (Bi, Fo), illustrated in Fig. 2-2. In the case considered

$$\mathrm{Bi} = \frac{\alpha\delta}{\lambda} = \frac{65 \times 0.01}{0.175} = 3.73;$$

$$\mathrm{Fo} = \frac{a\tau}{\delta^2} = \frac{0.833 \times 10^{-7} \times 1200}{(0.01)^2} = 1.0.$$

Using the above found values of the dimensionless terms Bi and Fo, we find from the graph in Fig. 2-1 $\Theta_{x=0} = 0.26$ and from the graph in Fig. 2-2 $\Theta_{x=\delta} = 0.083$.

The dimensionless temperature

$$\Theta = \frac{t-t_f}{t_0-t_f},$$

hence,

$$t_{x=0} = t_f + \Theta_{x=0}(t_0 - t_f) =$$
$$= 15 + 0.26\,(140 - 15) = 47.5\,^{\circ}\mathrm{C}$$

and

$$t_{x=\delta} = t_f + \Theta_{x=\delta}(t_0 - t_f) =$$
$$= 15 \times 2 \times 0.083\,(140 - 15) = 25.4\,^{\circ}\mathrm{C}.$$

2-2. Assuming the conditions of problem 2-1, determine the temperature at a distance $x = \delta/2 = 5$ mm from the middle of the rubber plate. Also calculate the dimensionless temperatures at the middle and at the surface of the plate and compare calculation results with the values of $\Theta_{x=0}$ and $\Theta_{x=\delta}$, obtained in solving problem 2-1.

Answer

$$t_{x=\delta/2} = 41^{\circ}\mathrm{C}; \quad \Theta_{x=0} = 0.257; \quad \Theta_{x=\delta} = 0.082.$$

Solution

The dimensionless temperature of an infinite plate cooled in surroundings at a constant temperature is expressed by equation

$$\Theta = \sum_{n=1}^{\infty} \frac{2\sin\varepsilon_n}{\varepsilon_n + \sin\varepsilon_n \cos\varepsilon_n} \cos\left(\varepsilon_n \frac{x}{\delta}\right) \exp\left(-\varepsilon_n^2\,\mathrm{Fo}\right). \qquad (2\text{-}1)$$

Since in the case considered the dimensionless term $\mathrm{Fo} = 1 > 0.3$ (see problem 2-1), one can use only the first term of the series, then

$$\Theta = N \exp\left(-\varepsilon_1^2\,\mathrm{Fo}\right) \cos\left(\varepsilon_1 \frac{x}{\delta}\right)$$

and the dimensionless temperatures at the middle and at the surface of the infinite plate will respectively be equal to:

$$\Theta_{x=0} = N \exp\left(-\varepsilon_1^2 \mathrm{Fo}\right);$$
$$\Theta_{x=\delta} = P \exp\left(-\varepsilon_1^2 \mathrm{Fo}\right).$$

The values of N, P, ε_1 and ε_1^2 as a function of Bi are given in Table 2-1.

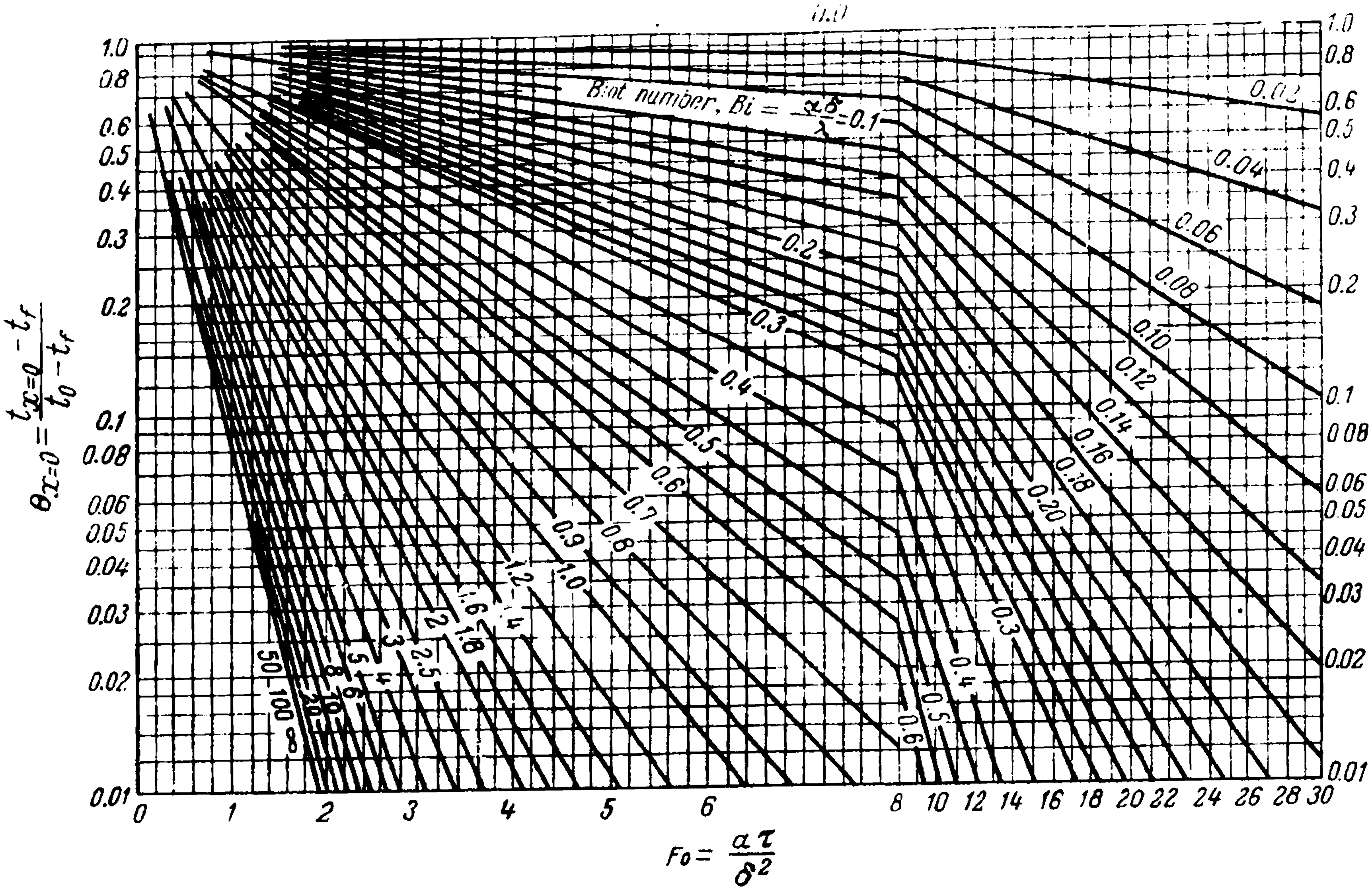

Fig. 2-1. $\Theta = f_1(\text{Fo}, \text{Bi})$ for the middle of a thin plate [24]

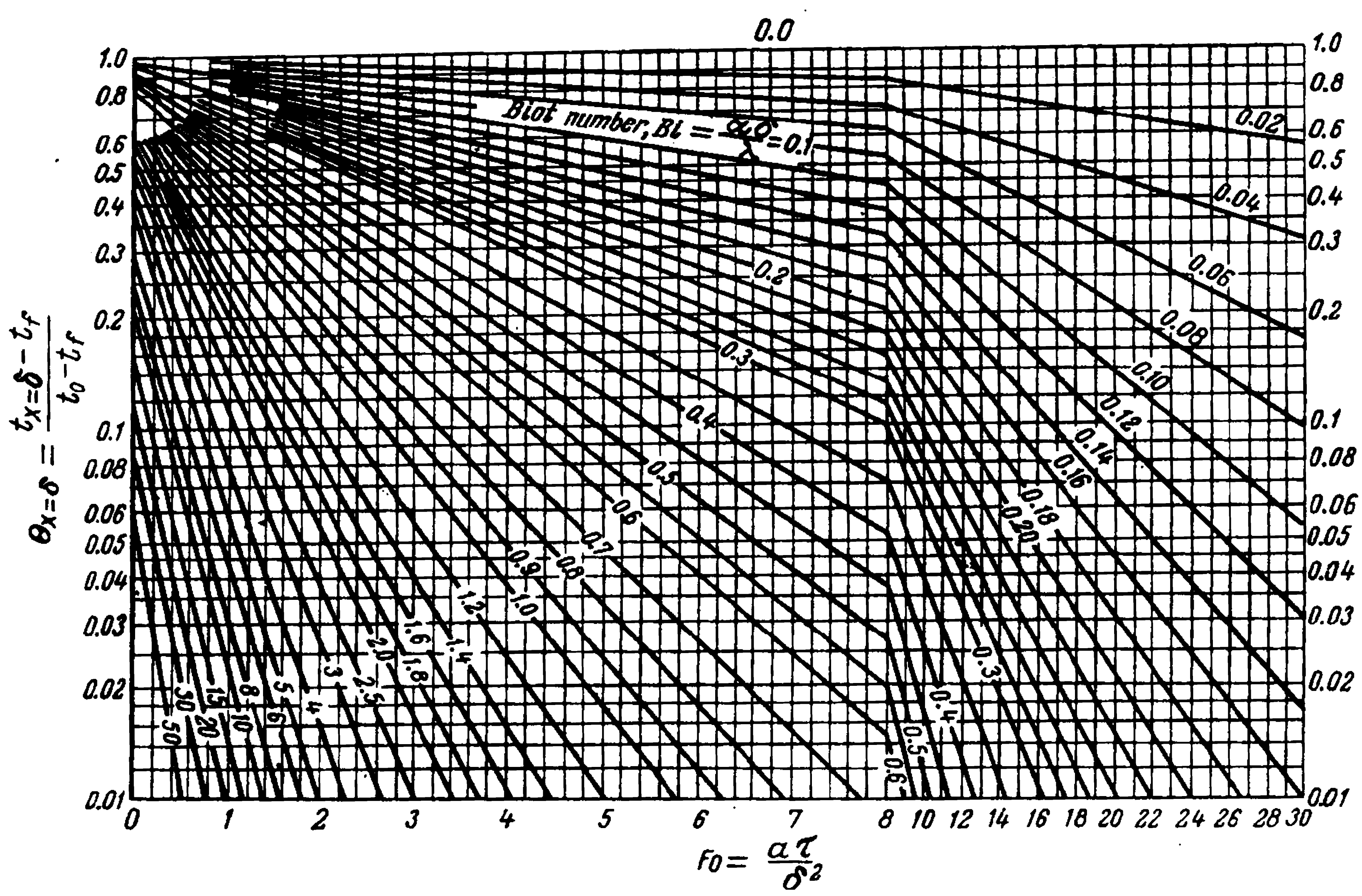

Fig. 2-2. $\Theta = f_2(Fo, Bi)$ for the surface of a thin plate [24]

Table 2-1

Calculation Coefficients for a Cooled (Heated) Plate 2δ Thick [24]

$Bi=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P	N	$Bi=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P	N	$Bi=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P	N
0.00	0.0000	0.0000	1.000	1.000	0.55	0.6800	0.462	0.836	1.076	6.0	1.3496	1.82	0.273	1.248
0.01	0.0998	0.0100	0.997	1.002	0.60	0.7051	0.497	0.823	1.081	7.0	1.3766	1.90	0.241	1.254
0.02	0.1410	0.0199	0.993	1.003	0.70	0.7506	0.564	0.798	1.092	8.0	1.3978	1.95	0.216	1.257
0.04	0.1987	0.0397	0.987	1.006	0.80	0.7910	0.626	0.774	1.102	9.0	1.4149	2.00	0.196	1.260
0.06	0.2425	0.0584	0.981	1.010	0.90	0.8274	0.684	0.751	1.111	10	1.4289	2.04	0.180	1.262
0.08	0.2791	0.0778	0.974	1.013	1.00	0.8603	0.740	0.729	1.119	12	1.4420	2.08	0.152	1.265
0.10	0.3111	0.0968	0.967	1.016	1.20	0.9171	0.841	0.689	1.134	14	1.4560	2.12	0.132	1.267
0.12	0.3397	0.1154	0.960	1.020	1.40	0.9649	0.931	0.653	1.148	16	1.4700	2.16	0.116	1.268
0.14	0.3656	0.1337	0.954	1.023	1.60	1.0008	1.016	0.619	1.159	18	1.4830	2.20	0.104	1.269
0.16	0.3896	0.1518	0.948	1.026	1.80	1.0440	1.090	0.587	1.169	20	1.4961	2.24	0.094	1.270
0.18	0.4119	0.1697	0.942	1.029	2.00	1.0769	1.160	0.559	1.179	25	1.5070	2.27	0.076	1.271
0.20	0.4328	0.1874	0.936	1.031	2.2	1.1054	1.222	0.535	1.186	30	1.5200	2.30	0.065	1.271
0.22	0.4525	0.2048	0.930	1.034	2.4	1.1300	1.277	0.510	1.193	35	1.5260	2.33	0.056	1.272
0.24	0.4712	0.2220	0.924	1.037	2.6	1.1541	1.332	0.488	1.200	40	1.5325	2.35	0.050	1.272
0.26	0.4889	0.2390	0.918	1.040	2.8	1.1747	1.380	0.468	1.205	50	1.5400	2.37	0.040	1.272
0.28	0.5058	0.2558	0.912	1.042	3.0	1.1925	1.420	0.448	1.210	60	1.5451	2.39	0.0333	1.273
0.30	0.5218	0.2723	0.906	1.045	3.5	1.2330	1.52	0.406	1.221	70	1.5490	2.40	0.0286	1.273
0.35	0.5590	0.3125	0.891	1.052	4.0	1.2646	1.59	0.370	1.229	80	1.5514	2.41	0.0250	1.273
0.40	0.5932	0.3516	0.877	1.058	4.5	1.2880	1.66	0.338	1.235	90	1.5520	2.41	0.0222	1.273
0.45	0.6240	0.3894	0.863	1.064	5.0	1.3138	1.73	0.314	1.240	100	1.5560	2.42	0.02001	1.273
0.50	0.6533	0.4264	0.849	1.070	5.5	1.3340	1.78	0.293	1.244	∞	1.5708	2.467	0.0000	1.273

In the case considered, at Bi = 3.73, we find from Table 2-1: $N = 1.224$, $P = 0.390$, $\varepsilon_1 = 1.248$, $\varepsilon_1^2 = 1.56$. Hence, with Fo = 1

$$\Theta_{x=\delta/2} = 1.224 \cos\left(\frac{1.248}{2}\right) \exp(-1.56) = 0.208;$$

$$t_{x=\delta/2} = t_f + \Theta_{x=\delta/2}(t_0 - t_f) =$$

$$= 15 + 0.208 \times 125 = 41°\mathrm{C};$$

$$\Theta_{x=0} = 1.224 \exp(-1.56) = 0.257;$$

$$\Theta_{x=\delta} = 0.390 \exp(-1.56) = 0.082.$$

2-3. Determine the interval of time after the expiration of which a steel plate heated to a temperature $t_0 = 500$ °C and placed into the open air at a temperature $t_f = 20$ °C will acquire a temperature, differing from the ambient temperature by not more than 1 per cent.

The plate thickness $2\delta = 20$ mm; the thermal conductivity of steel $\lambda = 45.5$ W/m·°C; the heat capacity of steel $c = 0.46$ kJ/kg·°C; the density of steel $\rho = 7900$ kg/m^3. The local coefficient of heat transfer from the surface of the plate to the surrounding air $\alpha = 35$ W/m^2·°C.

Hint. To evaluate the distribution of temperature over the cross section of the plate, first calculate the value of the Biot dimensionless:

$$\mathrm{Bi} = \frac{\alpha\delta}{\lambda} = \frac{35 \times 0.01}{45.5} = 0.0077 \ll 0.1.$$

Since Bi ≪ 0.1, the temperature can be assumed to be the same at all points over the cross section and make use of the formula

$$\Theta = \exp(-\mathrm{BiFo}).$$

Answer

2 h 15 min.

2-4 Determine the time τ required to heat a steel plate of thickness $2\delta = 24$ mm from the initial temperature $t_0 = 25$ °C to the final temperature $t = 450$ °C by placing the plate into a furnace in which the temperature $t_f = 600$ °C is maintained.

The thermal conductivity, heat capacity and the density of steel are respectively $\lambda = 45.4$ W/m·°C, $c = 0.502$ kJ/kg·°C, and $\rho = 7800$ kg/m^3; the local coefficient of heat transfer from the hot surroundings in the furnace to the surface of the steel plate $\alpha = 23.3$ W/m²·°C.

Answer

$$\tau = 45 \text{ min.}$$

2-5. A long steel shaft of a diameter $d = 2r_0 = 120$ mm which is at a temperature $t_0 = 20$ °C is placed into a furnace with a temperature $t_f = 820$ °C.

Determine the time τ required to heat the shaft, if the process of heating is considered terminated when the temperature at the axis of the shaft $t_{r=0} = 800$ °C. Also determine the surface temperature of the shaft $t_{r=r_0}$ at the end of heating.

The thermal conductivity and the thermal diffusivity of steel are respectively equal to $\lambda = 21$ W/m·°C and $a = 6.11 \times 10^{-6}$ m²/s. The local coefficient of heat transfer to the surface of the shaft in the furnace $\alpha = 140$ W/m²·°C.

Answer

$$\tau = 51 \text{ min}; \; t_{r=r_0} = 804 \text{ °C}.$$

Solution

The temperatures at the axis and surface of a long cylinder heated (cooled) in surroundings at a constant temperature can be determined with the aid of the graphs $\Theta_{r=0} = F$ (Bi, Fo), Fig. 2-3, and $\Theta_{r=r_0} = F_2$ (Bi, Fo), Fig. 2-4.

In the case considered

$$\text{Bi} = \frac{\alpha r_0}{\lambda} = \frac{140 \times 0.06}{21} = 0.4;$$

$$\Theta_{r=0} = \frac{t_f - t_{r=0}}{t_f - t_0} = \frac{820 - 800}{820 - 20} = 0.025$$

Using these values of Bi and $\Theta_{r=0}$, we find from the graph in Fig. 2-3 the dimensionless term Fo = 5.2. Hence, the time required to heat the shaft

$$\tau = \frac{r_0^2 \text{Fo}}{a} = \frac{(60 \times 10^{-3})^2 \times 5.2}{6.11 \times 10^{-6}} = 3060 \text{ s} = 51 \text{ min.}$$

With Bi = 0.4 and Fo = 5.2 the dimensionless surface temperature of the shaft is determined from the graph in Fig. 2-4:

$$\Theta_{r=r_0} = \frac{t_f - t_{r=r_0}}{t_f - t_0} = 0.02.$$

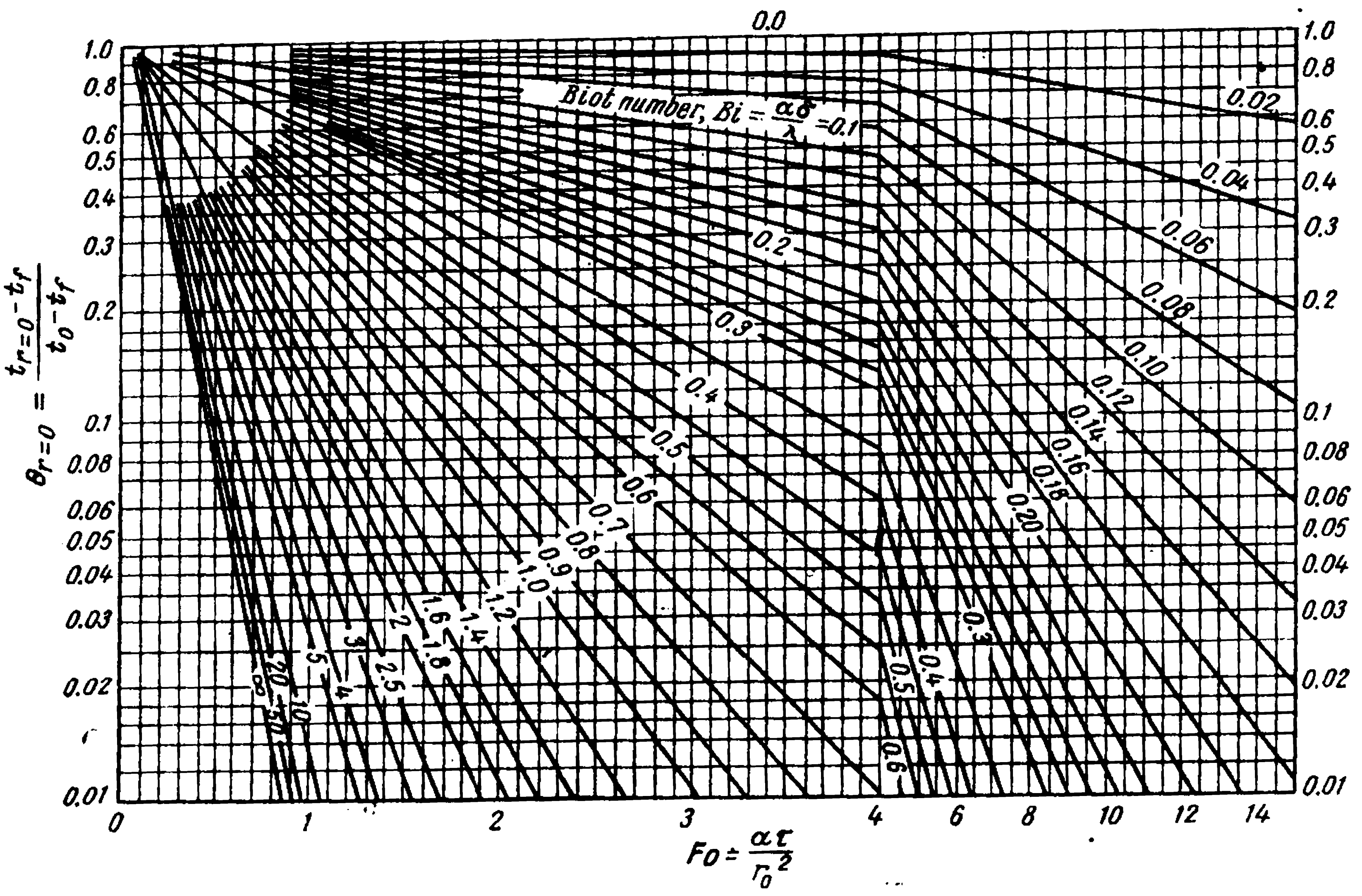

Fig. 2-3. $\Theta = F_1(\mathrm{Fo}, \mathrm{Bi})$ for the axis of a cylinder [24]

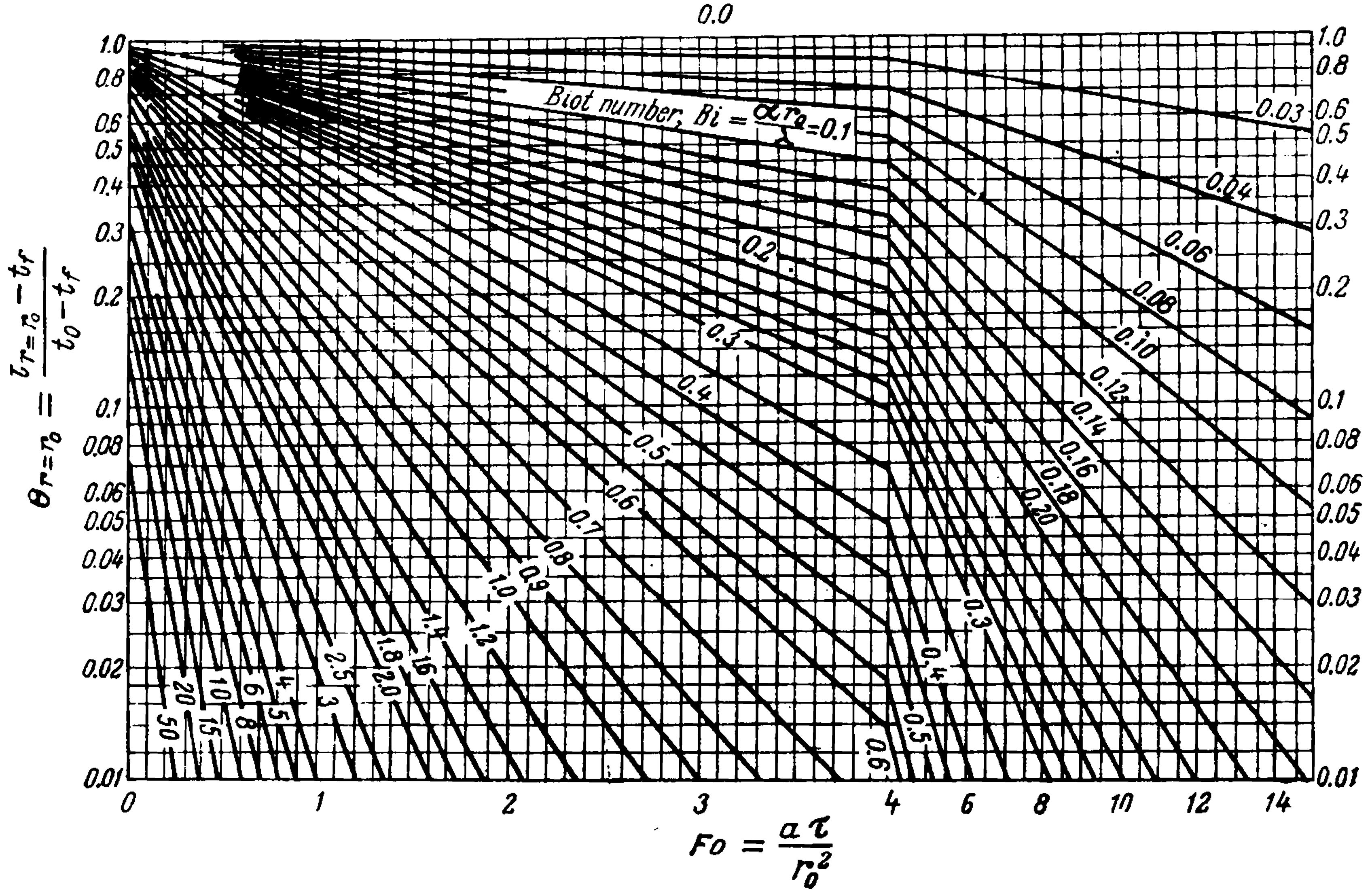

Fig. 2-4. $\Theta = F_2(\text{Fo}, \text{Bi})$ for the surface of a cylinder [24]

consequently,

$$t_{r=r_0} = t_f - 0.02\,(t_f - t_0) = 820 - 0.02 \times (820 - 20) = 804\ ^\circ\text{C}.$$

2-6. Under the conditions of problem 2-5 determine the temperatures at the axis and the surface of the shaft after the expiration of 20 and 40 min since the shaft has been placed into the furnace.

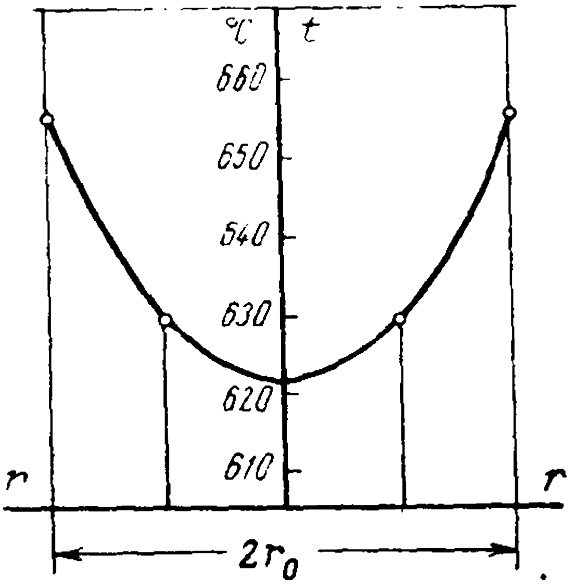

Fig. 2-5. Solution to problems 2-6 and 2-7

Answer

with $\tau = 20$ min: $t_{r=r_0} = 656\ ^\circ\text{C}$, $t_{r=0} = 620\ ^\circ\text{C}$; with $\tau = 40$ min: $t_{r=r_0} = 763\ ^\circ\text{C}$, $t_{r=0} = 755\ ^\circ\text{C}$.

2-7. Under the conditions of problem 2-5 determine the temperature at a distance $r = 0.5\,r_0$ from the shaft axis through the expiration of time $\tau = 20$ min after the beginning of heating.

Calculate the temperatures at the surface and axis of the shaft after the expiration of the same interval of time and compare calculation results with the answer to problem 2-6.

Answer

$$t_{r=0} = 621^\circ\text{C}, \quad t_{r=0.5r_0} = 630\ ^\circ\text{C}; \quad t_{r=r_0} = 656^\circ\text{C}.$$

The distribution of temperature in the shaft cross section is shown graphically in Fig. 2-5 ($\tau = 20$ min).

Solution

The dimensionless temperature of a long cylinder being heated (or cooled) in constant temperature surroundings is expressed by the following equation

$$\Theta = \sum_{n=1}^{\infty} \frac{2J_1(\varepsilon_n)}{\varepsilon_n [J_0^2(\varepsilon_n) + J_1^2(\varepsilon_n)]} J_0\left(\varepsilon_n \frac{r}{r_0}\right) \exp(-\varepsilon_n^2 \mathrm{Fo}), \qquad (2\text{-}2)$$

where $J_0(\varepsilon_n)$ and $J_1(\varepsilon_n)$ are respectively first-kind Bessel functions of the zero and first orders.

Since in the case considered the dimensionless term

$$\mathrm{Fo} = \frac{a\tau}{r_0^2} = \frac{6.11 \times 10^{-6} \times 1200}{(60 \times 10^{-3})^2} = 2.04 > 0.25,$$

we may use only the first term of the series; then

$$\Theta = N_0 J_0\left(\varepsilon_1 \frac{r}{r_0}\right) \exp(-\varepsilon_1^2 \mathrm{Fo})$$

and the dimensionless temperature at the cylinder axis is

$$\Theta_{r=0} = N_0 \exp(-\varepsilon_1^2 \mathrm{Fo}),$$

and the dimensionless surface temperature of the cylinder

$$\Theta_{r=r_0} = P_0 \exp(-\varepsilon_1^2 \mathrm{Fo}).$$

The values of N_0, P_0, ε_1 and ε_1^2 are given in Table 2-2 as a function of the dimensionless term Bi.

In the case considered, with Bi = 0.4, we find from the table: $N_0 = 1.093$, $P_0 = 0.903$, $\varepsilon_1 = 0.8516$, $\varepsilon_1^2 = 0.726$. Hence, with Fo = 2.04

$$\Theta_{r=0.5r_0} = 1.093 J_0\left(\frac{0.8516}{2}\right) \exp(-0.726 \times 2.04) =$$

$$= 1.093 \times 0.9548 \times 0.2276 = 0.238;$$

$$t_{r=0.5r_0} = t_f - \Theta_{r=0.5r_0}(t_f - t_0) = 820 - 0.238 \times 800 = 630\ ^\circ\mathrm{C};$$

$$\Theta_{r=0} = 1.093 \times 0.2276 = 0.2485;$$

$$t_{r=0} = 820 - 0.2485 \times 800 = 621\ ^\circ\mathrm{C};$$

$$\Theta_{r=r_0} = 0.903 \times 0.2276 = 0.205;$$

$$t_{r=r_0} = 820 - 0.205 \times 800 = 656\ ^\circ\mathrm{C}.$$

Calculation Coefficients for a Cooled (Heated) Cylinder with a Radius r_0 [24]

$\mathrm{Bi}=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P_0	N_0	$\mathrm{Bi}=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P_0	N_0	$\mathrm{Bi}=\frac{\alpha\delta}{\lambda}$	ε_1	ε_1^2	P_0	N_0
0.00	0.0000	0.0000	1.000	1.000	0.55	0.9808	0.962	0.869	1.124	6.0	2.0490	4.20	0.299	1.527
0.01	0.1412	1.0200	0.998	1.002	0.60	1.0184	1.036	0.858	1.134	7.0	2.0937	4.38	0.262	1.541
0.02	0.1995	0.0398	0.995	1.005	0.70	1.0873	1.184	0.836	1.154	8.0	2.1286	4.53	0.234	1.551
0.04	0.2814	0.0792	0.990	1.010	0.80	1.1490	1.322	0.815	1.172	9.0	2.1566	4.65	0.210	1.560
0.06	0.3438	0.1183	0.985	1.014	0.90	1.2048	1.453	0.795	1.190	10	2.1795	4.75	0.191	1.566
0.08	0.3960	0.1569	0.980	1.019	1.00	1.2558	1.580	0.774	1.208	12	2.2181	4.92	0.161	1.575
0.10	0.4417	0.1951	0.975	1.024	1.20	1.3450	1.81	0.738	1.239	14	2.2428	5.03	0.137	1.581
0.12	0.4726	0.2329	0.970	1.029	1.40	1.4250	2.03	0.704	1.268	16	2.2627	5.12	0.120	1.585
0.14	0.5200	0.2704	0.965	1.034	1.60	1.4900	2.22	0.671	1.295	18	2.2760	5.18	0.106	1.588
0.16	0.5545	0.3075	0.960	1.039	1.80	1.5460	2.39	0.639	1.319	20	2.2890	5.24	0.095	1.590
0.18	0.5868	0.3443	0.956	1.044	2.00	1.5994	2.55	0.610	1.340	25	2.3108	5.34	0.076	1.595
0.20	0.6170	0.3807	0.951	1.048	2.2	1.6432	2.70	0.584	1.357	30	2.3261	5.41	0.065	1.598
0.22	0.6455	0.4167	0.946	1.053	2.4	1.6852	2.84	0.558	1.375	35	2.3366	5.46	0.056	1.600
0.24	0.6726	0.4524	0.941	1.057	2.6	1.7234	2.97	0.534	1.392	40	2.3455	5.50	0.050	1.602
0.26	0.6983	0.4877	0.937	1.062	2.8	1.7578	3.09	0.513	1.406	50	2.3572	5.55	0.040	1.603
0.28	0.7229	0.5226	0.932	1.067	3.0	1.7887	3.20	0.492	1.420	60	2.3651	5.59	0.0333	1.604
0.30	0.7465	0.5572	0.927	1.071	3.5	1.8547	3.44	0.446	1.449	70	2.3707	5.62	0.0286	1.604
0.35	0.8012	0.642	0.915	1.082	4.0	1.9081	3.64	0.407	1.472	80	2.3750	5.64	0.0250	1.605
0.40	0.8516	0.726	0.903	1.093	4.5	1.9519	3.81	0.374	1.489	90	2.3791	5.66	0.0222	1.605
0.45	0.8978	0.806	0.891	1.103	5.0	1.9898	3.96	0.345	1.504	100	2.3809	5.67	0.0200	1.606
0.50	0.9408	0.888	0.880	1.114	5.5	2.0224	4.09	0.320	1.516	∞	2.4048	5.784	0.0000	1.606

2-8. A parallelepiped-shaped steel ingot, measuring 200 × × 400 × 500 mm (Fig. 2-6), with an initial temperature $t_0 = 20\,^\circ\mathrm{C}$ is placed into a furnace with a temperature $t_f = 1400\,^\circ\mathrm{C}$.

Determine the temperature t_c at the centre of the ingot in a time interval $\tau = 1.5$ h after the ingot was charged into the furnace.

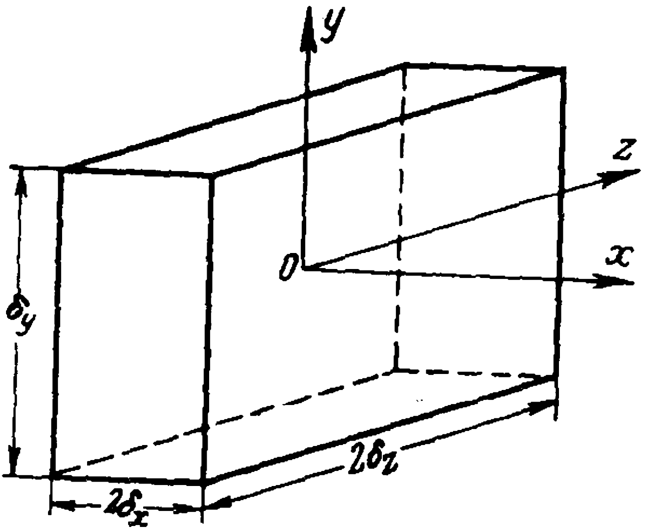

Fig. 2-6. Solution to problem 2-8

The thermal conductivity and the thermal diffusivity of steel are respectively $\lambda = 37.2$ W/m·°C and $a = 6.94 \times 10^{-6}$ m²/s, and the local coefficient of heat transfer at the surface of the ingot $\alpha = 186$ W/m²·°C.

Answer:

$$t_c = 1282\,^\circ\mathrm{C}.$$

Solution

The dimensionless temperature at any point in a parallelepiped is equal to the product of the dimensionless temperatures of three infinite plates which are mutually perpendicular and form the parallelepiped upon intersection. Hence, the temperature at the centre of the parallelepiped can be found from the equation

$$\frac{t_f-t_c}{t_f-t_0}=\frac{t_f-t_{x=0}}{t_f-t_0}\cdot\frac{t_f-t_{y=0}}{t_f-t_0}\cdot\frac{t_f-t_{z=0}}{t_f-t_0}.$$

The temperatures of the plates $t_{x=0}$, $t_{y=0}$, $t_{z=0}$ can be found from the graph on which the temperature at the centre of an infinite plate is plotted as a function of the dimensionless terms Bi and Fo (Fig. 2-1). For a plate of thickness $2\delta_x = 200$ mm we have:

$$\mathrm{Fo}_x = \frac{a\tau}{\delta_x^2} = \frac{6.94 \times 10^{-6} \times 5400}{0.1^2} = 3.75;$$

$$\mathrm{Bi}_x = \frac{\alpha\delta_x}{\lambda} = \frac{186 \times 0.1}{37.2} = 0.5.$$

From the graph we find that with $\mathrm{Fo}_x = 3.75$ and $\mathrm{Bi}_x = 0.5$

$$\frac{t_f - t_{x=0}}{t_f - t_0} = 0.22.$$

By analogy, for a plate of thickness $2\delta_y = 400$ mm we have:

$$\mathrm{Fo}_y = 0.937; \quad \mathrm{Bi}_y = 1.0.$$

From the graph we find that

$$\frac{t_f - t_{y=0}}{t_f - t_0} = 0.57.$$

For a plate of thickness $2\delta_z = 500$ mm

$$\mathrm{Fo}_z = 0.6; \quad \mathrm{Bi}_z = 1.25$$

and

$$\frac{t_f - t_{z=0}}{t_f - t_0} = 0.68.$$

Consequently,

$$\frac{t_f - t_c}{t_f - t_0} = 0.22 \times 0.57 \times 0.68 = 0.0852$$

and the temperature at the centre of the ingot

$$t_c = t_f - 0.0852\,(t_f - t_0) =$$
$$= 1400 - 0.0852\,(1400 - 200) = 1282\,^\circ\mathrm{C}.$$

2-9. An ingot is being heated under conditions considered in problem 2-8. Determine the temperature at the centre of the ingot sides, measuring 200 × 400 and 200 × 500 mm.

Answer

$$t_{x=0;\ y=0;\ z=\delta_z} = 1331\,^\circ\mathrm{C};$$

$$t_{x=0;\ y=\delta_y;\ z=0} = 1323\,^\circ\mathrm{C},$$

2-10. A cylindrical steel ingot with a diameter $d = 80$ mm and length $l = 160$ mm (Fig. 2-7) was first heated uniformly to a temperature $t_0 = 800$ °C. The billet is then cooled in the open air which is at a temperature $t_f = 30$ °C.

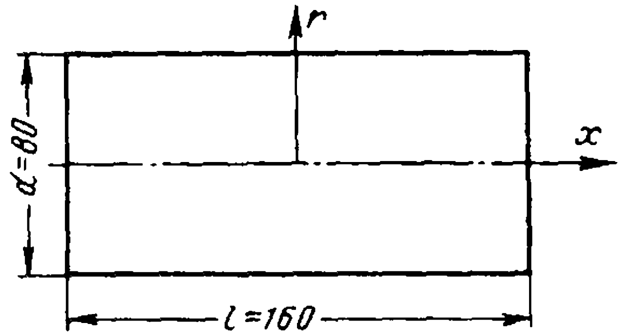

Fig. 2-7. Solution to problem 2-10

Determine the temperature at the centre of the ingot $t_{x=0;\ r=0}$, and at the centre of its side $t_{r=0;x=l/2}$ in a time interval $\tau = 30$ min after cooling is initiated.

The thermal conductivity and thermal diffusivity of steel are respectively $\lambda = 23.3$ W/m·°C and $a = 6.11 \times 10^{-6}$ m²/s.

The local coefficient of heat transfer from the surface of the billet to the cooling air $\alpha = 118$ W/m²·°C

Answer

$$t_{r=0;\ x=0} = 55\ ^\circ\text{C};\quad t_{r=0;\ x=l/2} = 50\ ^\circ\text{C}.$$

2-11. Under the conditions for cooling a steel billet assumed in problem 2-10 determine the temperature at the centre of the billet and at the middle of its side surface, if the dimensions of the billet are doubled, i.e., $d = 160$ mm and $l = 320$ mm, all other conditions remaining as in problem 2-10.

Answer

$$t_{r=0;\ x=0} = 211\ ^\circ\text{C};\quad t_{r=0;\ x=l/2} = 153\ ^\circ\text{C}.$$

2-12. A long rectangular steel beam, measuring 400 × 320 mm in cross section, is placed at a temperature $t_0 = 1000$ °C into surroundings at a temperature $t_f = 20$ °C for the purpose of cooling.

The thermal conductivity of steel $\lambda = 32$ W/m·°C and thermal diffusivity $a = 7 \times 10^{-6}$ m²/s; the local coefficient of heat transfer from the surface of the beam to surroundings remains constant throughout the process of cooling and is equal to 170 W/m²·°C.

Calculate the temperature at the axis of the beam for a time interval $\tau = 1, 2, 3$ and 4 h after cooling is initiated.

Answer

$$\Theta_1 = 0.403;\quad \Theta_2 = 0.130;\quad \Theta_3 = 0.0424;$$
$$\Theta_4 = 0.0137;$$
$$t_1 = 415^\circ\text{C};\quad t_2 = 148^\circ\text{C};\quad t_3 = 62^\circ\text{C};$$
$$t_4 = 23^\circ\text{C}.$$

2-13. A steel plate of thickness $2\delta = 400$ mm is heated in a furnace inside which a constant temperature $t_f = 800^\circ$C is maintained. When placed into the furnace the plate had a uniform temperature $t_0 = 30^\circ$C. The local coefficient of heat transfer to the surface of the heated plate remains constant and equal to $\alpha = 200$ W/m²·°C. The two other dimensions of the plate, width and length, are large compared with its thickness and the temperature field of the plate can be considered as unidimensional.

Determine the amount of heat transferred to 1 m² of the plate in 2 hours after heating is initiated. The thermal conductivity of steel $\lambda = 37.2$ W/m·°C and the thermal diffusivity $a = 7 \times 10^{-6}$ m²/s; the density of steel $\rho = 7800$ kg/m³.

Answer

$$\bar{\Theta} = 0.098;\quad Q = 1470 \times 10^3 \text{ kJ/m}^2.$$

Solution

The calculation of the amount of heat rejected from (or transferred to) the plate in the course of cooling (heating) during a time interval ranging from $\tau = 0$ to τ practically resolves into the finding the mean dimensional temperature at the moment of time τ, i.e. it can be calculated by the formula

$$Q = Q_t (1 - \bar{\Theta}), \tag{2-3}$$

where Q_t is the total amount of heat which can be rejected from or transferred to the plate during a time interval ranging from $\tau = 0$ to $\tau = \infty$:

$$Q_t = 2\delta f \rho c (t_0 - t_f), \ J, \qquad (2\text{-}4)$$

where f is the area of one side of the plate.

For a plate the mean dimensionless temperature at the moment of time τ can be calculated from the formula

$$\bar{\Theta} = \sum_{n=1}^{\infty} \frac{2\sin^2 \varepsilon_n}{\varepsilon_n^2 + \varepsilon_n \sin \varepsilon_n \cos \varepsilon_n} \ \mathrm{p}(-\varepsilon_n^2 \mathrm{Fo}). \qquad (2\text{-}5)$$

Substituting the corresponding values of the quantities given in formulating the problem into formula (2-4), we obtain:

$$Q_t = 0.4 \times 7800 \times 682\,(800-30) = 1630 \times 10^3 \ \mathrm{kJ/m^2},$$

where

$$c = \frac{\lambda}{\rho a} = \frac{37.2}{7800 \times 7 \times 10^{-6}} = 682 \ \mathrm{J/kg \cdot {}^\circ C}.$$

Calculate the dimensionless terms Fo and Bi:

$$\mathrm{Fo} = \frac{a\tau}{\delta^2} = \frac{7 \times 10^{-6} \times 7200}{0.2^2} = 1.26;$$

$$\mathrm{Bi} = \frac{\alpha\delta}{\lambda} = \frac{200 \times 0.2}{37.2} = 1.075.$$

The magnitude of the dimensionless term $\mathrm{Fo} > 0.3$ and for calculations with a sufficient accuracy we can use the first term of the sum (2-5). Using the value of Bi, we find the value of ε_1 from Table 2-1. Substituting this value of ε_1 into formula (2-5), we find:

$$\bar{\Theta} = 0.098.$$

Substituting the calculated values of Q_t and $\bar{\Theta}$ into equation (2-3), we find that

$$Q = 1630 \times 10^3 (1 - 0.098) = 1470 \times 10^3 \ \mathrm{kJ/m^2}.$$

2-14. A steel cylinder with a diameter $d = 500$ mm is cooled in a medium which is at a constant temperature $t_f = 15^\circ\mathrm{C}$. At the initial moment of time the temperature of the cylinder was the same at all points and equal to $t_0 = 450^\circ\mathrm{C}$. The local coefficient of heat transfer from all points on the surface of the cylinder in the process of cooling remained constant and equal to 160 W/m²·°C.

The thermal conductivity, thermal diffusivity and density of steel are respectively:

$$\lambda = 49\ \text{W/m}\cdot{}^\circ\text{C};\quad a = 1.4 \times 10^{-5}\ \text{m}^2/\text{s};\quad \rho = 7850\ \text{kg/m}^3.$$

Determine the amount of heat transferred from length of 1 m of the cylinder to surroundings in three hours after cooling is initiated.

Answer

$$Q = 297 \times 10^3\ \text{kJ/m}.$$

2-15. A rectangular billet, or slab, measuring 480 × 360 × 280 mm in size, is heated in a furnace at a constant temperature $t_f = 800$°C. Before the beginning of heating all points of the billet were at the same temperature $t_0 = 20$°C. The local coefficient of heat transfer from surroundings to the sides of the billet remains constant throughout the process of heating and equal to 200 W/m²·°C.

The thermal conductivity, thermal diffusivity and density of steel are respectively: $\lambda = 37.2$ W/m·°C; $a = 7 \times 10^{-6}$ m²/s; $\rho = 7800$ kg/m³.

Determine the amount of heat transferred to the billet in 2.5 hours after heating is initiated.

Answer

$$Q = 189 \times 10^3\ \text{kJ}.$$

2-16. A cylindrical steel billet of a diameter $d = 620$ mm and length $l = 700$ mm is cooled in surroundings with a constant temperature $t_f = 20$°C. Before the beginning of cooling the temperature of the billet was $t_0 = 600$°C. The local coefficient of heat transfer from the surface of the billet to surroundings remains constant in the course of cooling and equal to 160 W/m²·°C.

The thermal conductivity, thermal diffusivity and density of steel are respectively: $\lambda = 4.9$ W/m·°C, $a = 1.4 \times 10^{-5}$ m²/s, $\rho = 7850$ kg/m³.

Determine the amount of heat transferred from the billet to surroundings in 2.8 hours after cooling is initiated.

Answer

$$Q = 426 \times 10^3\ \text{kJ}.$$

2-17. Both surfaces of a brick wall of thickness $2\delta = 500$ mm are exposed to surroundings at a constant temperature 18°C. The thermal conductivity, thermal diffusivity and density of the material are respectively: $\lambda = 0.7$ W/m·°C, $a = 0.647 \times 10^{-6}$ m^2/s, $\rho = 1700$ kg/m^3.

How will the temperature change on the surface and at the middle of the brick wall in 1 h, if the temperature of surroundings drops suddenly to 8°C? The coefficient of heat transfer from the surface of the brick wall to surroundings remains constant and equal to $\alpha = 7$ W/m^2·°C.

Note. Under the conditions of the problem the dimensionless term $\mathrm{Fo} < 0.3$, therefore, to find the temperature, we should not confine ourselves to the first term of the series and have to calculate not less than three terms of the sum. (The values of the roots of equation 2-1 can be found in [24].)

Answer

The surface temperature of the brick wall $t_{x=\delta} = 14.3$ °C. The temperature at the middle of the brick wall $t_{x=0} = 18$ °C, hence, in a time interval of 1 h the temperature disturbance will practically fail to reach the middle of the wall.

2-18. In an experimental plant, used to determine the thermal diffusivity of solids by the method of the regular regime, the material tested is placed into a cylindrical calorimeter of a diameter $d = 50$ mm and length $l = 75$ mm. Upon preliminary heating the calorimeter is cooled in a water thermostat (Fig 2-8) in which the water temperature t_f is maintained constant and equal to 20 °C.

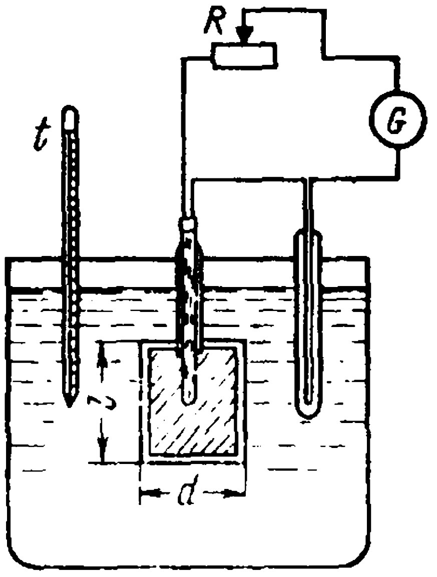

Fig. 2-8. Solution to problem 2-18

Calculate the thermal diffusivity of the material tested, if in the process of cooling, after the regular regime sets in, the temperature of the specimen at the spot where the thermocouple is introduced decreases from $t_1 = 30$ °C to $t_2 = 22$ °C in a time interval $\Delta\tau = 7$ min.

Answer

$$a = 3.47 \times 10^{-7} \text{ m}^2/\text{s},$$

2-19. In an experimental plant, used to determine the thermal conductivity of solids by the method of the regular regime, the material tested is placed into a spherical calorimeter with a radius $r_0 = 30$ mm. Upon preliminary heating the calorimeter is cooled in an air thermostat in which the temperature t_f is maintained constant and equal to 20 °C.

Preliminary investigations have made it possible to find that the local coefficient of heat transfer from the surface of the calorimeter to the surrounding air $\alpha = 7$ W/m²·°C and the thermal diffusivity $a = 3.47 \times 10^{-7}$ m²/s (see problem 2-18).

Calculate the thermal conductivity of the material tested, if in the process of cooling, after the regular regime sets in, the temperature at the centre of the calorimeter dropped from $t_1 = 27$ °C to $t_2 = 24$ °C in a time interval $\Delta\tau = 15$ min.

Answer

$$\lambda = 0.35 \text{ W/m}\cdot{}^\circ\text{C}.$$

2-20. Determine the rate of cooling of a solid which at the moment of time $\tau = 0$ had a uniform temperature $t_0 = 210$°C. For the purpose of cooling the solid was placed into surroundings at a constant temperature $t_f = 195$ °C. The results of measuring the excess temperature of the body at different moments of time after cooling is initiated, expressed in galvanometer scale divisions, are given below:

τ, min	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
Number of divisions	300	300	281	262	245	230	214	200	187	175	165	155	145	131.5	129

Answer

$$m = 2.2 \times 10^{-3} \text{ 1/s}.$$

2-21. Determine the local coefficient of heat transfer by free convection from the surface of a sphere to the air. The steel sphere is $d = 60$ mm in diameter and during the period of regular cooling the rate of cooling of the sphere $m = 16.7 \times 10^{-6}$ 1/s. Assume a factor of non-uniform temperature distribution $\psi = 1$, The density and heat capacity of steel are assumed to be: $\rho = 7900$ kg/m^3, $c = 460$ J/kg·°C.

Check whether it is right to assume $\psi = 1$ in this case.

Answer

$$\alpha = 6.06 \text{ W/m}^2\text{·°C}.$$

Chapter Three

TREATMENT OF TEST DATA BY THE METHOD OF THE THEORY OF SIMILARITY

3-1. It is necessary to determine by experiment the distribution of temperature in a long shaft $d = 400$ mm in a time interval $\tau = 2.5$ h after the shaft was charged into a furnace.

The thermal conductivity and thermal diffusivity of steel are respectively: $\lambda = 42$ W/m·°C, $a = 1.18 \times 10^{-5}$ m²/s. In the furnace the local coefficient of heat transfer to the shaft is equal to $\alpha = 116$ W/m²·°C.

It is decided to conduct the investigation in a small furnace using for a test specimen a geometrically similar model of the shaft fabricated from alloyed steel. For the model $\lambda_m = 16$ W/m·°C; $a_m = 0.53 \times 10^{-5}$ ·m²/s, $\alpha_m =$ $= 150$ W/m²·°C.

Determine the diameter d_m of the model and how soon after the model is charged into the furnace it is necessary to measure the distribution of temperature in the model.

Answer

$$d_m = 117.5 \text{ mm}; \quad \tau_m = 1735 \text{ s}.$$

Solution

The temperature fields of the shaft and its model will be similar when the dimensionless terms for the shaft and model will be the same:

$$\mathrm{Bi}_m = \mathrm{Bi} \text{ and } \mathrm{Fo}_m = \mathrm{Fo}.$$

For the shaft the dimensionless terms

$$\mathrm{Bi} = \frac{\alpha r}{\lambda} = \frac{116 \times 0.2}{42} = 0.552;$$

$$\mathrm{Fo} = \frac{a\tau}{r^2} = \frac{1.18 \times 10^{-5} \times 9 \times 10^3}{0.2^2} = 2.66.$$

From the condition $\mathrm{Bi}_m = \mathrm{Bi}$ we find the diameter for the shaft model

$$d_m = 2r_m = 2\frac{\lambda_m}{\alpha_m}\,\mathrm{Bi} = 2\,\frac{16}{150}\,0.552 = 0.1175 \text{ m}.$$

From the condition $\mathrm{Fo}_m = \mathrm{Fo}$ we find the unknown time interval:

$$\tau_m = \frac{r_m^2}{a_m}\,\mathrm{Fo} = \frac{(0.05875)^2}{0.53\times 10^{-5}}\,2.66 = 1735 \text{ s}.$$

3-2. Determine the diameter of the model of the shaft d_m and the required value of the local heat-transfer coefficient α_m at which, under the conditions assumed in problem 3.1, the temperature fields shall become similar in time $\tau_m = 15$ min after the model of the shaft is charged into the furnace.

Also determine the relations of the linear dimensions, time and temperatures for the shaft and model, if it is known that at the moment of charging their temperatures and the temperature inside the furnace were respectively equal to:

$$t_0 = 10°\mathrm{C},\quad t_{0m} = 20°\mathrm{C},\quad t_f = 1000°\mathrm{C},\quad t_{fm} = 200°\mathrm{C}.$$

Answer

$$d_m = 85 \text{ mm};\quad \alpha_m = 208 \text{ W/m}^2\cdot°\mathrm{C};$$

$$r/r_m = 4.7;\quad \tau/\tau_m = 10;\quad t = 5.5t_m - 100.$$

3-3. The orifice plate mounted on the superheated steam main having a diameter $d = 400$ mm must be calibrated, i.e. the problem is to find the dependence $\Delta p = f(G)$, where Δp is the drop in static pressure across the orifice, Pa, and G—the rate of steam flow, kg/s.

Since due to operating conditions it was impossible to calibrate the orifice directly on the steam pipeline, a scale model was constructed of 1/5 natural size.

The model was tested with water flowing through the orifice at a temperature $t_{fm} = 20°\mathrm{C}$ and values of the pressure drops across the orifice were obtained at different rates of water flow. The measurement results are given below:

Δp Pa .	477	1178	4520	18 050	72 200
G, kg/s . .	2.22	4.44	8.88	17.76	35.52

Find the dependence $\Delta p = f(G)$ for the actual pipeline with steam flowing in it in the self-similar region and indicate the limits for its application. The steam pressure $p = 98$ kPa. The steam temperature $t_f = 250°C$.

Answer

$$\Delta p = 222G^2 \text{ at } \mathrm{Re} > 1.42 \times 10^5.$$

Solution

Let us treat the test results, expressed in dimensionless terms or criteria of similarity, and plot the dependence $\mathrm{Eu} = f(\mathrm{Re})$ This dependence will also be true for steam. Therefore, the dependence $\mathrm{Eu} = f(\mathrm{Re})$ can be used to find the dependence $\Delta p = f(G)$ for steam flowing in the actual pipeline.

To determine the dependence $\mathrm{Eu} = f(\mathrm{Re})$, calculate the values of dimensionless terms for the test data obtained in calibrating the orifice on a model.

The Euler number

$$\mathrm{Eu} = \frac{\Delta p_m}{\rho_m w_m^2}.$$

Taking into account that the velocity

$$w_m = \frac{4G_m}{\rho_m \pi d_m^2},$$

we obtain

$$\mathrm{Eu} = \frac{\Delta p_m}{\rho_m} \left(\frac{\rho_m \pi d_m^2}{4G_m} \right)^2.$$

With $t_{f \cdot m} = 20°C$ the density of water $\rho_m = 998$ kg/m³. $\nu_m = 1 \times 10^{-6}$ m²/s*.

Substituting the known values we find that

$$\mathrm{Eu} = 998 \left(\frac{3.14 \times 0.08^2}{4} \right)^2 \frac{\Delta p_m}{G_m^2} = 2.51 \times 10^{-2} \frac{\Delta p_m}{G_m^2}.$$

The Reynolds number

$$\mathrm{Re} = \frac{w_m d_m}{\nu_m} = \frac{4G_m}{\rho_m \nu_m \pi d_m} = 15\,930 G_m$$

---

* Here and below, in solving problems, the physical properties of fluids and other needed reference materials can be taken from tables (see Appendices, p. 356).

where

$$d_m = \frac{d}{5} = \frac{0.4}{5} = 0.08 \text{ m}.$$

Substituting the values of G_m and Δp_m, obtained when calibrating the orifice, we calculate the values of dimensionless terms. Calculation results are given in the following table:

Δp_m, Pa	G_m, kg/s	W_m, m/s	Eu	Re
477	2.22	0.443	2.44	35 400
1 178	4.44	0.886	1.505	70 800
4 520	8.88	1.772	1.44	141 600
18 050	17.76	3.544	1.44	283 200
72 200	35.52	7.088	1.44	566 400

These data are used to plot the curve Eu = f (Re) shown in Fig. 3-1.

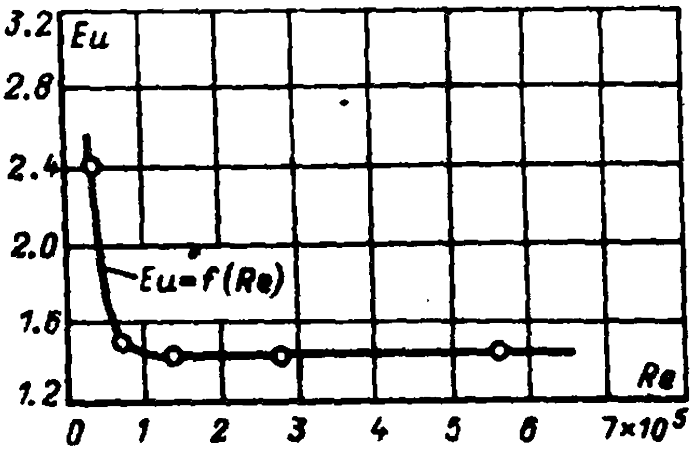

Fig. 3-1. Solution to problem 3-3

From the table and graph it is clear that at Re > 1.42×10^5 the Euler number Eu = const = 1.44 (the self-similarity region). Consequently, when steam flows through the full-size pipeline at Re > 1.42×10^5 the Euler term Eu = 1.44. Let us make use of this relationship to find the unknown relation. For the full-size pipeline, in the event of steam flow,

$$\Delta p = \mathrm{Eu}\rho w^2 = \frac{1.44}{v} w^2 = \frac{1.44}{2.452} w^2 = 0.587 w^2,$$

where at $p = 98$ kPa and $t_f = 250$°C the specific volume $v = 2.452$ m^3/kg.

Replacing velocity with the rate of flow

$$w = \frac{Gv}{0.785 d^2},$$

where G is the rate of flow, kg/s, we obtain:

$$\Delta p = 0.587 \left(\frac{2.452}{0.785 \times 0.4^2}\right)^2 G^2,$$

whence

$$\Delta p = 222 G^2 \text{ at } \mathrm{Re} > 1.42 \times 10^6.$$

3-4. With liquid in laminar flow in a straight round tube of constant cross-sectional area the drop in pressure, Pa, at a considerable distance from the tube inlet over section l long will be determined by the equation

$$\Delta p = \frac{128}{\pi} \frac{\mu V l}{d^4},$$

where μ = dynamic viscosity, Pa·s;
V = volumetric rate of flow, m^3/s;
d = tube diameter, m.

Represent this equation in the form of a dependence between dimensionless terms and also in the form of the dependence for the friction factor:

$$\mathrm{Eu} = f(\mathrm{Re}) \text{ and } \xi = f_1(\mathrm{Re}),$$

where

$$\xi = 2 \frac{\Delta p}{\rho w^2} \frac{d}{l}.$$

Answer

$$\mathrm{Eu} = \frac{32}{\mathrm{Re}} \frac{l}{d}; \quad \xi = \frac{64}{\mathrm{Re}}.$$

3-5. The hot water flowing through a tube having a diameter $d = 16$ mm and length $l = 2.1$ m transfers heat through the wall of the tube to the medium to which the outside surface of the tube is exposed.

The rate of water flow through the tube $G = 0.0091$ kg/s; the water inlet temperature $t_{f1} = 87.2$°C, the water outside temperature $t_{f2} = 29$°C; the mean temperature of the wall of the tube $t_w = 15.3$°C.

Calculate the magnitudes of the dimensionless terms Nu, Re and Pe, assuming as the reference temperature the arithmetic mean temperature of the water. Relate the heat-transfer coefficient to the arithmetic mean difference between the temperatures of the water and wall.

Answer

$$\mathrm{Nu} = 11.9; \quad \mathrm{Re} = 1485; \quad \mathrm{Pe} = 4600.$$

3-6. Calculate the local heat-transfer coefficient and the Nusselt number for the conditions assumed in the problem 3-5, reducing the local heat-transfer coefficient to the logarithmic mean difference between the temperatures of the water and wall. Compare the obtained values with the results of the problem 3-5.

Answer

$$\alpha_{\log} = 597\ \mathrm{W/m^2 \cdot {}^\circ C}; \quad \mathrm{Nu}_{\log} = 14.5.$$

3-7. Under the conditions assumed in the problem 3-5 calculate the Euler number and the friction resistance coefficient, or friction factor, if the pressure drop across the tube $\Delta p = 5.88$ Pa.

Answer

$$\mathrm{Eu} = 2.82; \quad \xi = 0.0431.$$

3-8. A 1/8 scale model of a steam boiler was used to investigate heat transfer by convection, using air as a working medium instead of flue gases. At different air velocities, the following values of the heat-transfer coefficient were obtained for the first pass of the 1/8 scale model:

W_m, m/s	2.0	3.14	4.65	8.8
α_m, W/m$^2 \cdot$°C	50.4	68.6	90.[illegible]	141

The mean temperature of the air flowing through the boiler $t_{f \cdot m} = 20$°C. The diameter of the tubes used in the model $d_m = 12.5$ mm. When treating the test results, the local heat-transfer coefficient α_m was related to the arithmetic mean difference between the temperatures of the air and tube wall.

Based on the data obtained in testing the model, find the calculation formula for heat transfer by convection in the 1st boiler pass in the form of the dependence Nu = f(Re).

Answer

$$\mathrm{Nu} = 0.15\mathrm{Re}^{0.685}.$$

Solution

Using the data obtained in testing the model, we seek the dependence for heat transfer in the form $\mathrm{Nu} = C\mathrm{Re}^n$.

The Nusselt number $\mathrm{Nu}_m = \frac{\alpha_m d_m}{\lambda_f}$ and the Reynolds number $\mathrm{Re} = \frac{w_m d_m}{\nu_f}$, where at $t_{f,m} = 20°\mathrm{C}$ the thermal conductivity of air $\lambda_f = 0.026$ W/m·°C and kinematic viscosity $\nu_f = 15.06 \times 10^{-6}$ m²/s.

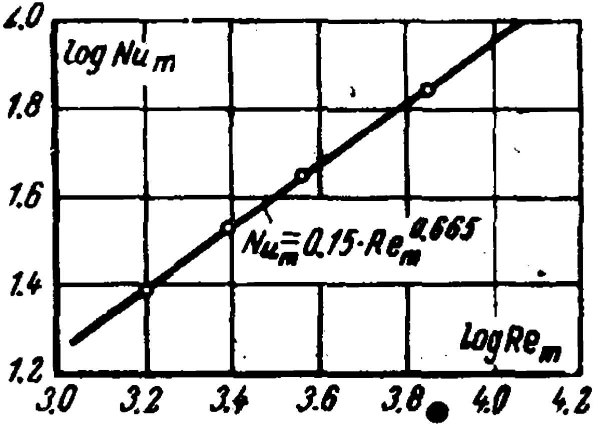

Fig. 3-2. Solution to problem 3-8

Substituting the corresponding values into the expressions for the Nusselt and Reynolds numbers, we get:

$$\mathrm{Nu}_m = 0.481\alpha_m; \quad \mathrm{Re}_m = 830w_m.$$

The results obtained in calculating the dimensionless terms for corresponding values of α_m and w_m are tabulated below:

w_m, m/s	α_m, W/m²·°C	Re_m	Nu_m
2.0	50.4	1660	24.2
3.14	68.6	2600	33.0
4.65	90.6	3860	43.6
8.8	141	7300	68.0

Using the data given in the table, we plot the dependence $\mathrm{Nu}_m = f(\mathrm{Re}_m)$ in logarithmic coordinates (Fig. 3-2). The slope of the curve in respect to the abscissa axis is used to determine the exponent n, then the constant $C = \mathrm{Nu}_m/\mathrm{Re}_m^n$ is found. The obtained calculation formula $\mathrm{Nu} = 0.15\mathrm{Re}^{0.685}$ is true within the limits $1600 \leqslant \mathrm{Re} \leqslant 7300$.

3-9. Determine the amount of heat transferred from the gases to the walls of the tubes in the 1st boiler pass (the results obtained in investigating this boiler pass on a model were given in problem 3-8), if the following data are known: the mean gas velocity $w = 6$ m/s; the flue gas temperature at the beginning and end of the 1st pass $t_{f1} = 900$°C and $t_{f2} = 700$°C respectively; the tube wall temperature $t_w = 250$°C; the heating surface of the pass $F = 500$ m^2.

As the reference temperature assume the arithmetic mean temperature $t_f = 0.5\ (t_{f1} + t_{f2})$. The composition of the flue gases is as follows: $\bar{p}_{CO_2} = 0.13$, $\bar{p}_{H_2O} = 0.11$, $\bar{p}_{N_2} = 0.76$.

Answer

$$\alpha = 44.4\ \mathrm{W/m^2 \cdot {}^\circ C}; \quad Q = 12.2 \times 10^3\ \mathrm{kW}.$$

3-10. An experimental outfit was used to determine heat transfer for a single cylinder in cross flow of air. The tests enabled to obtain the values of the local heat-transfer coefficients α_1 and α_2, W/m^2·°C, for two cylinders of diameters $d_1 = 10$ mm and $d_2 = 20$ mm respectively at constant temperature $t_f = 20$ °C and different velocities of the main-stream flow, w, m/s. The test data are tabulated below:

w, m/s	2.0	5.0	10	20
α_1, W/m^2·°C	39.5	71.2	106.5	165.3
α_2, W/m^2·°C	31.2	55.6	83.4	128

Find the dependence between the dimensionless terms for the process of heat transfer, $\mathrm{Nu}_f = C\mathrm{Re}_f^n$. Compare the graphs $\alpha_1 = f_1(w)$ and $\alpha_2 = f_2(w)$.

Answer

$$\mathrm{Nu}_f = 0.18\mathrm{Re}_f^{0.62}.$$

A comparison of the graphs shows that at the same velocity the local heat-transfer coefficients differ from each other by about 30 per cent. When treated in the dimensionless terms, the dependence $\mathrm{Nu} = f(\mathrm{Re})$ happens to be unambiguous for the two cylinders.

3-11. Under the conditions of free convection the heat losses from the surface of horizontal steam pipelines were investigated on a laboratory outfit, where measurements were taken on a horizontal tube of a diameter $d = 30$ mm.

The tests were conducted with different tube wall temperatures; the following values for the local heat-transfer coefficient were obtained:

α, W/m²·°C . . .	11.75	12.34	12.87	13.34	13.75
t_w, °C	210	250	290	330	370

At a distance from the tube the ambient air temperature t_f remained constant and equal to 30°C.

Using the obtained experimental data on the local heat-transfer coefficients, find the generalized calculation relationship for heat transfer by free convection. Taking into account the fact that for air the dimensionless term Pr remains practically constant in a wide range of temperatures, try to derive a relationship of the kind $\mathrm{Nu} = f(\mathrm{Gr})$.

When treating the test data, use as a reference temperature the temperature of air at a distance from the tube.

Answer

$$\mathrm{Nu} = 0.47\mathrm{Gr}^{0.25} \text{ at } 6 \times 10^5 < \mathrm{Gr} < 1.2 \times 10^6.$$

Chapter Four

HEAT TRANSFER FOR A PLATE IN FORCED LONGITUDINAL FLOW

4-1. A thin plate of length $l_0 = 2$ m and width $a = 1.5$ m is exposed to a flow of air parallel to its surface (Fig. 4-1). The velocity and temperature of the free-stream flow of air are respectively $w_0 = 3$ m/s and $t_0 = 20°C$. The temperature at the surface of the plate $t_w = 90°C$.

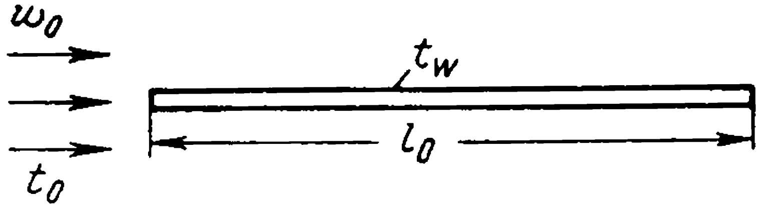

Fig. 4-1. Solution to problem 4-1

Determine the lengthwise mean local coefficient of heat transfer from the plate to the air and the amount of heat transferred.

Answer

$$\alpha = 4.87\ \text{W/m}^2\cdot°\text{C};\quad Q = 2050\ \text{W}.$$

Solution

For the air at a temperature $t_0 = 20°C$ $\nu = 15.06 \times 10^{-6}$ m²/s $\lambda = 2.59 \times 10^{-2}$ W/m·°C, Pr = 0.703. The Reynolds number

$$\text{Re} = \frac{w_0 l_0}{\nu} = \frac{3 \times 2}{15.06 \times 10^{-6}} = 3.98 \times 10^5 < 5 \times 10^5\,^*,$$

consequently in the boundary layer the pattern of flow is laminar. Under these conditions the lengthwise mean local

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[*] For practical calculation the critical Reynolds number, Re_{cr}, is often assumed equal to 1×10^6.

coefficient of heat transfer from the plate to the air can be calculated from formula [4]

$$\mathrm{Nu} = 0.67\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}, \qquad (4\text{-}1)$$

where

$$\mathrm{Nu} = \frac{\alpha l_0}{\lambda}; \quad \mathrm{Re} = \frac{w_0 l_0}{\nu},$$

and the physical properties are selected for the temperature of the free-stream flow, t_0.

In the case considered

$$\mathrm{Nu} = 0.67(3.98 \times 10^5)^{1/2}(0.703)^{1/3} = 375$$

and the local heat-transfer coefficient

$$\alpha = \mathrm{Nu}\frac{\lambda}{l_0} = 375 \cdot \frac{2.59 \times 10^{-2}}{2} = 4.87 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The amount of heat transferred from the both sides of the plate

$$Q = \alpha(t_w - t_0)F = 4.87\,(90-20) \times 2 \times 2 \times 1.5 = 2050 \text{ W}.$$

4-2. For the conditions of problem 4-1 calculate the thickness of the hydrodynamic boundary layer and the values of the local heat-transfer coefficients at various distances from the front edge of the plate: $x = 0.1l_0$, $0.2l_0$, and $1.0l_0$. Plot the graph showing the dependence of the thickness of the hydrodynamic boundary layer δ_l and local heat-transfer coefficient on the relative distance x/l_0.

Answer

x/l_0	0.1	0.2	0.5	1.0
δ_l, mm	4.66	6.58	10.4	14.7
α_x, W/m$^2\cdot{}^\circ$C	7.73	5.65	3.45	2.44

Solution

In accordance with the conditions of problem 4-1, the transfer of heat takes place in the laminar boundary layer. The thickness of the laminar boundary layer and the local heat-transfer coefficient at a distance x from the front edge of the plate are determined from formula [4, 12]:

$$\delta_l = \frac{4.64x}{\sqrt{\mathrm{Re}_x}} \tag{4-2}$$

and

$$\mathrm{Nu}_x = 0.335\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}, \tag{4-3}$$

where

$$\mathrm{Nu}_x = \frac{\alpha_x x}{\lambda} \quad \text{and} \quad \mathrm{Re}_x = \frac{w_0 x}{\nu}.$$

At a distance from the front edge $x = 0.1\ l_0$

$$\mathrm{Re}_x = \frac{w_0(0.1l_0)}{\nu} = \frac{3\times0.2}{15.06\times10^{-6}} = 3.98\times10^4;$$

$$\delta_l = \frac{4.64\times0.2}{\sqrt{3.98\times10^4}} = 4.66\times10^{-3}\ \mathrm{m};$$

$$\mathrm{Nu}_x = 0.335\,(3.98\times10^4)^{1/2}\,(0.703)^{1/3} = 59.5$$

$$\alpha_x = \mathrm{Nu}_x\frac{\lambda}{x} = 59.5\,\frac{2.59\times10^{-2}}{0.2} = 7.73\ \mathrm{W/m^2\cdot{}^\circ C}.$$

In a similar way, we can calculate the unknown quantities for other values of the relative distance x/l_0. The calculation

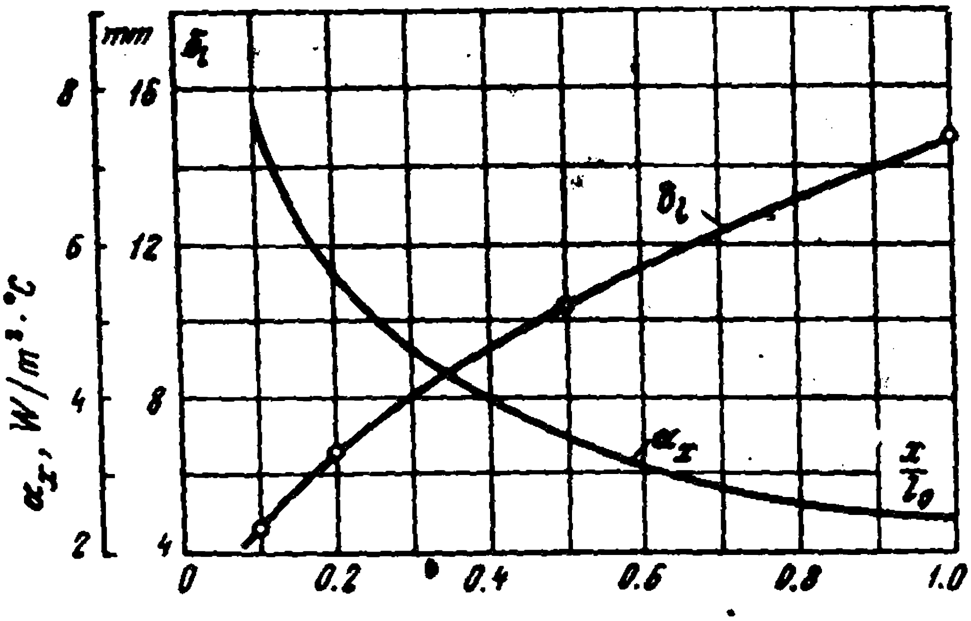

Fig. 4-2. Solution to problem 4-2

results are compiled in the table given in the answer to the problem, and are presented graphically in Fig. 4-2.

4-3. A thin plate of length $l_0 = 125$ mm is placed in a longitudinal flow of fluid. The temperature of the free-stream flow $t_0 = 20°C$.

Calculate the critical length x_{cr}; the maximum thickness of the laminar boundary layer $\delta_{l\,cr}$; the values of the local heat-transfer coefficients, and the thickness of the laminar boundary layer at distances $x = 0.1l_0$, $0.2l_0$, $0.5l_0$, and $1.0l_0$ from the front edge of the plate.

Calculations should be carried out for two cases:

(a) the plate is exposed to air with a free-stream flow velocity $w_0 = 10$ m/s;

(b) the plate is exposed to a longitudinal water flow with $w_0 = 2$m/s.

Assume a critical Reynolds number $\mathrm{Re}_{x\,cr} = 5 \times 10^5$.

Answer

When exposed to airflow $x_{cr} = 0.75$ m, $\delta_{l,cr} = 4.93$ mm; when exposed to water flow $x_{cr} = 0.25$ m, $\delta_{l\,cr} = 1.65$ mm.

x/l_0	0.1	0.2	0.5	1.0
α_x, W/m$^2\cdot$°C:				
air	56.4	39.9	25.1	17.8
water	4820	3420	2150	1520
δ_l, mm:				
air	0.635	0.895	1.42	2.0
water	0.366	0.516	0.822	1.15

4-4. Under the conditions of problem 4-3 calculate the values of the mean local heat-transfer coefficient and of the rate of heat flow from length of 1 m of the plate, q_l, for air and water, if the surface temperature of the plate $t_w = 50°C$.

Answer

When the plate is exposed to air flow $\alpha = 35.7$ W/m$^2\cdot$°C, $q_l = 279$ W/m; in the event of water flow $\alpha = 3050$ W/m$^2\cdot$°C, $q_l = 2.3 \times 10^4$ W/m.

4-5. A thin constantan band, measuring 0.1×5 mm in cross section is heated by electric current of intensity $I = 20$ A. The specific resistance of the band $R_l = 1.0$ Ohm/m.

The band is exposed to water flow parallel to its surface. The velocity and temperature of the free-stream flow $w_0 = 0.5$ m/s, $t_0 = 10°C$.

Determine the temperature of the band at a distance of 25 and 200 mm from its front edge.

Answer

At $x = 25$ mm $t_w = 35.2$°C;
at $x = 200$ mm $t_w = 81.6$°C.

4-6. A flat plate of length $l = 1$ m is exposed to air flow parallel to its surface. The velocity and temperature of the free-stream air flow $w_0 = 80$ m/s and $t_0 = 10$°C. A turbulizing grid is placed upstream of the plate, resulting in that the fluid is in turbulent flow in the boundary layer over the whole length of the plate.

Calculate the mean local coefficient of heat transfer from the surface of the plate and the value of the local heat-transfer coefficient on the back edge. Also calculate the hydrodynamic boundary layer on the back edge of the plate.

Answer

The mean local heat-transfer coefficient $\alpha = 202$ W/m²·°C. The local heat-transfer coefficient at a distance $x = l_0$ $\alpha_{x=0}' = 157.5$ W/m²·°C; the thickness of the hydrodynamic boundary layer at $x = l_0$ $\delta_t = 16.5$ mm.

Solution

With a temperature of the free-stream flow $t_0 = 10$°C the physical properties of air are: $\nu = 14.16 \times 10^{-6}$ m²/s, $\lambda = 2.51 \times 10^{-2}$ W/m·°C.

The Reynolds number

$$\mathrm{Re} = \frac{w_0 l_0}{\nu} = \frac{80 \times 1.0}{14.6 \times 10^{-6}} = 5.65 \times 10^6 > 5 \times 10^5.$$

The pattern of flow in the boundary layer on the plate is turbulent.

With the plate exposed to air flow the mean heat-transfer coefficient for the turbulent boundary layer can be calculated from the following formula [17]:

$$\mathrm{Nu} = 0.032\mathrm{Re}^{0.8}. \tag{4-4}$$

Substituting the obtained value of the Reynolds number into formula 4-4, we get:

$$\mathrm{Nu} = 0.032(5.65 \times 10^6)^{0.8} = 8050 \text{ and}$$

$$\alpha = \mathrm{Nu}\frac{\lambda}{l_0} = 8050\,\frac{2.51 \times 10^{-2}}{1.0} = 202 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

For a plate in a longitudinal air flow with a turbulent boundary layer the local heat-transfer coefficient can be calculated from the following formula [17]:

$$\mathrm{Nu}_x = 0.0255\mathrm{Re}_x^{0.8}, \qquad (4\text{-}5)$$

where $\mathrm{Nu}_x = \alpha_x x/\lambda$ and $\mathrm{Re}_x = w_0 x/\nu$.

The value of the local coefficient of heat transfer on the back edge of the plate can be found, assuming $x = l_0$; then $\mathrm{Re}_x = 5.65 \times 10^6$, $\mathrm{Nu}_x = 0.255(5.65 \times 10^6)^{0.8} = 6280$ and

$$\alpha_{x=l_0} = \mathrm{Nu}_{x=l_0}\frac{\lambda}{l_0} = 6280\,\frac{2.51 \times 10^{-2}}{1.0} = 157.5\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The local thickness of the turbulent hydrodynamic boundary layer can be calculated from the formula [27]:

$$\delta_t = \frac{0.37x}{\sqrt[5]{\mathrm{Re}_x}}. \qquad (4\text{-}6)$$

Substituting the values of the known quantities, we obtain at $x = l_0$:

$$\delta_t = \frac{0.37 \times 1.0}{\sqrt[5]{5.65 \times 10^6}} = 0.0165\ \mathrm{m}.$$

4-7. Under the conditions of problem 4-6 calculate the thickness of the hydrodynamic boundary layer and the values of the local heat-transfer coefficients at distances $x = 0.1l_0$, $0.2l_0$, $0.5l_0$, and $0.8l_0$ from the front edge of the plate. Plot the graph showing the variation of the thickness of the hydrodynamic boundary layer and of the local values of the heat-transfer coefficients over the length of the plate.

Answer

Calculation results are represented graphically in Fig. 4-3 and tabulated below:

x, m	0.1	0.2	0.5	0.8
α_x, W/m²·°C	256	219	185	165
δ_t, mm	2.62	4.55	9.49	13.83

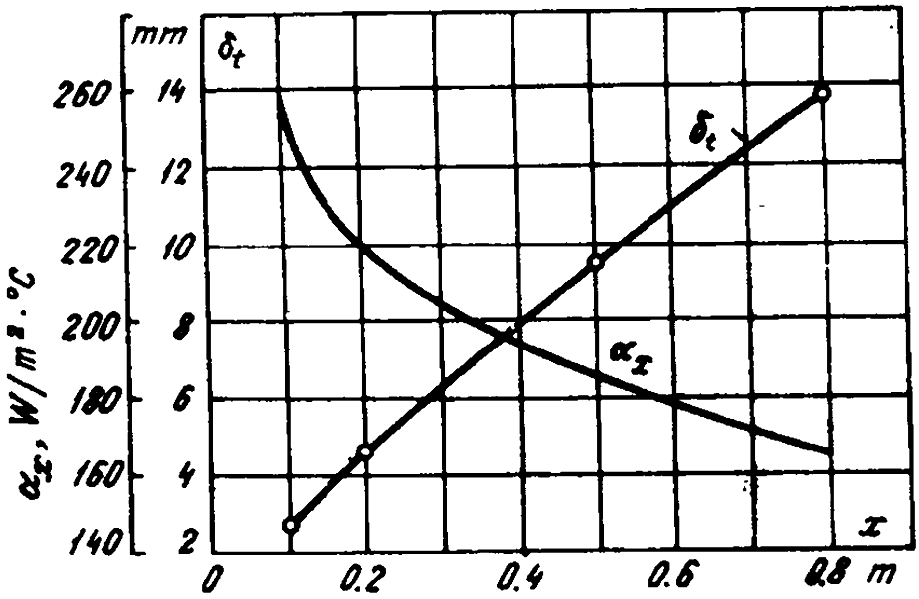

Fig. 4-3. Solution to problem 4-7

4-8. A flat plate is exposed to air flow parallel to its surface. The velocity and temperature of the free-stream air flow are respectively $w_0 = 6$ m/s and $t_0 = 20°C$.

Calculate the quantity of heat transferred from the plate to the air, if the surface temperature of the plate $t_w = 80°C$, the length of the plate in the direction of flow $l = 1$ m, and the dimension of the plate across flow $b = 0.9$ m*.

Answer

$$Q = 1.3 \text{ kW}.$$

4-9. A thin plate of length $l = 0.2$ m is exposed to a longitudinal air flow. The velocity and temperature of the free-stream flow respectively $w_0 = 150$ m/s and $t_0 = 20°C$.

Determine the mean heat-transfer coefficient and the rate of heat flow from the surface of the plate, if the surface temperature of the plate $t_w = 50°C$. Calculate, assuming the pattern of flow to be turbulent in the boundary layer over the entire length of the plate.

Answer

$$\alpha = 454 \text{ W/m}^2\cdot°\text{C}; \quad q = 9080 \text{ W/m}^2.$$

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* Here and below (Chapters 4 through 7) thermal radiation heat transfer is ignored.

Solution

At the temperature of the free-stream flow $t_0 = 20°C$ the physical properties of air are:

$$\nu = 15.06 \times 10^{-6} \text{ m}^2/\text{s}; \quad \lambda = 2.59 \times 10^{-2} \text{ W/m}\cdot°\text{C}; \quad c_p = 1.0 \text{ kJ/kg}\cdot°\text{C}.$$

The Reynolds number

$$\text{Re} = \frac{w_0 l}{\nu} = \frac{150 \times 0.2}{15.06 \times 10^{-6}} = 1.99 \times 10^6.$$

The Mach number

$$M = \frac{w}{a} = \frac{150}{344} = 0.436,$$

where the velocity of the sound in air

$$a = 20.1\sqrt{T_0} = 20.1\sqrt{293} \doteq 344 \text{ m/s}.$$

The formula (4-4) is valid for calculating heat transfer from a plate exposed to air flow at a high subsonic velocity at $10^5 < \text{Re} < 2 \times 10^6$ and $0.25 < M < 0.8$, provided the heat-transfer coefficient is related to the difference between the temperature of the wall and adiabatic wall temperature $t_{a.w}$ [17]:

$$t_{a.w} = t_0 + r\frac{w_0^2}{2c_p},$$

where the recovery factor for a plate in a longitudinal flow with a boundary turbulent layer can be assumed equal to $r = \sqrt[3]{\text{Pr}}$.

For air at $t_0 = 20°C$ $r = \sqrt[3]{0.703} = 0.89$.

In the case considered

$$\text{Nu} = 0.032\text{Re}^{0.8} = 0.032(1.99 \times 10^6)^{0.8} = 3500$$

and

$$\alpha = \text{Nu}\frac{\lambda}{l} = 3500\frac{2.59 \times 10^{-2}}{0.2} = 454 \text{ W/m}^2\cdot°\text{C}.$$

The adiabatic wall temperature

$$t_{a.w} = 20 + 0.89\frac{150^2}{2 \times 1 \times 10^3} = 30°\text{C}$$

and the rate of heat flow

$$q = \alpha(t_w - t_{a.w}) = 454(50-30) = 9080 \text{ W/m}^2.$$

4-10. Calculate the mean local heat-transfer coefficient and the amount of heat transferred from the surface of the plate placed in a longitudinal air flow parallel to its surface.

The velocity and temperature of the free-stream flow are respectively: $w_0 = 200$ m/s and $t_0 = 30°\text{C}$. The surface temperature of the plate $t_w = 90°\text{C}$. The plate is 120 mm long (l) and 200 mm wide (b).

Solve the problem assuming the boundary layer to be turbulent over the entire length of the plate.

Answer

$$\alpha = 640\ \text{W/m}^2\cdot°\text{C};\quad Q = 1.3\ \text{kW}.$$

Chapter Five

HEAT TRANSFER AND HYDRAULIC RESISTANCE WITH FORCED FLOW OF FLUID IN TUBES

5-1. Calculate the mean heat-transfer coefficient for transformer oil flowing in a tube with a diameter $d = 8$ mm and length $l = 1$ m, if the lengthwise mean oil temperature

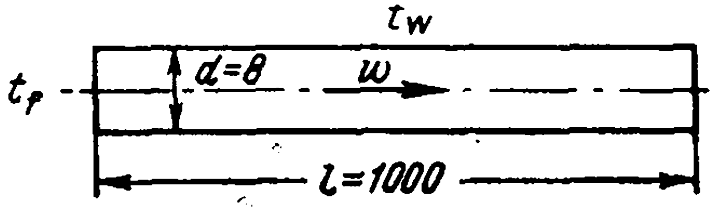

Fig. 5-1. Solution to problem 5-1

$t_f = 80$°C; the mean temperature of the wall of the tube $t_w = 20$°C, and the velocity of the oil $w = 0.6$ m/s (Fig. 5-1).

Answer

$$\alpha = 138 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Solution

To determine the pattern of oil flow, we calculate the value of the Reynolds number.

At $t_f = 80$°C the kinematic viscosity of oil $\nu_f = 3.66 \times 10^{-6}$ m²/s and the Reynolds number

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{0.6\times 8\times 10^{-3}}{3.66\times 10^{-6}} = 1310.$$

Inasmuch as $\mathrm{Re}_f < 2300$ the pattern of flow is laminar.

In order to determine the effect of free convection on heat transfer, we must calculate the value of the product $(\mathrm{GrPr})_h$, where as the reference temperature is used

$$t_h = 0.5(t_f + t_w), \text{ and } t_f = 0.5(t_{f1} + t_{f2}).$$

In the case considered

$$t_h = 0.5(80 + 20) = 50\,^\circ\mathrm{C}.$$

At this temperature

$$\nu_h = 7.58 \times 10^{-6}\ \mathrm{m^2/s};\quad \beta_h = 7.05 \times 10^{-4}\ \mathrm{K^{-1}};\quad \mathrm{Pr}_h = 111$$

$$\text{and } (\mathrm{GrPr})_h = g\beta_h \frac{(t_f - t_w)\,d^3}{\nu_h^2}\,\mathrm{Pr}_h =$$

$$= 9.81 \times 7.05 \times 10^{-4}\,\frac{(80-20)\,(8\times 10^{-3})^3}{(7.58\times 10^{-6})^2}\,111 = 36 \times 10^5.$$

Since $(\mathrm{GrPr})_h < 8 \times 10^5$, the free convection does not affect materially the intensity of heat transfer and the pattern of flow is viscous.

With fluid in viscous flow in tubes at a constant wall temperature (t_w = const) the mean rate of heat transfer can be calculated from formula [15]:

$$\mathrm{Nu}_h = 1.55\left(\mathrm{Pe}_h \frac{d}{l}\right)^{1/3}\left(\frac{\mu_f}{\mu_w}\right)^{0.14}\varepsilon, \qquad (5\text{-}1)$$

where

$$\mathrm{Nu}_h = \frac{\alpha d}{\lambda_h};\quad \mathrm{Pe}_h \frac{d}{l} = \frac{4Gc_{ph}}{\pi l \lambda_h};\quad \alpha = \frac{q}{t_f - t_w};$$

the subscripts "w" and "h" indicate that the physical properties of the fluid (gas or liquid) are selected respectively at the wall temperature t_w and temperature $t_h = 0.5(t_f + t_w)$;

$$\varepsilon = 0.6\left(\frac{1}{\mathrm{Re}_f}\frac{l}{d}\right)^{-1/7}\left(1 + 2.5\frac{1}{\mathrm{Re}_f}\frac{l}{d}\right)$$

is the correction for the section of hydrodynamic stabilization. This correction is introduced when there is no section of hydrodynamic stabilization in front of the heated section of the tube and

$$\frac{1}{\mathrm{Re}_f}\frac{l}{d} < 0.1.$$

Formula (5-1) is valid at $\mathrm{Re}_f < 2300$;

$$\frac{1}{\mathrm{Pe}_h}\frac{l}{d} \leqslant 0.05;\quad (\mathrm{GrPr})_h \leqslant 8 \times 10^5;\quad 0.07 \leqslant \frac{\mu_w}{\mu_f} \leqslant 500\,^*.$$

---

* The correction for the effect of variable properties, found in formula (5-1), $(\mu_f/\mu_w)^{0.14}$, is not valid for gases.

In the case considered $t_f = 80°C$, $t_w = 20°C$, and $t_h = 50°C$.

The physical properties of oil:

$\rho_f = 844\ kg/m^3$; $\mu_f = 30.8 \times 10^4\ Pa \cdot s$; $\lambda_h = 0.108\ W/m \cdot °C$;

$$c_{ph} = 1.846\ kJ/kg \cdot °C;\quad \mu_w = 198.2 \times 10^{-4}\ Pa \cdot s.$$

The rate of oil flow

$$G = \rho_f w \frac{\pi d^2}{4} = 844 \times 0.6 \frac{\pi (8 \times 10^{-3})^2}{4} = 2.53 \times 10^{-2}\ kg/s.$$

The Peclet number at the reference temperature

$$Pe_h \frac{d}{l} = \frac{4G}{\pi l} \frac{c_{ph}}{\lambda_h} = \frac{4 \times 2.53 \times 10^{-2}}{3.14 \times 1.0} \frac{1.846 \times 10^3}{0.108} = 550;$$

$\frac{1}{Pe_h} \frac{l}{d} < 0.05$ and, consequently, formula (5-1) is applicable.

The correction for the initial hydrodynamic section

$$\frac{1}{Re_f} \frac{l}{d} = \frac{1}{1310} \frac{1}{8 \times 10^{-3}} = 0.0955 < 0.1 \text{ and}$$

$$\varepsilon = 0.6\,(0.0955)^{-1/7}(1 + 2.5 \times 0.0955) = 1.05.$$

The Nusselt number at the reference temperature

$$Nu_h = 1.55\,(550)^{1/3} \left(\frac{30.8}{198.2}\right)^{0.14} 1.05 = 10.2.$$

The heat-transfer coefficient

$$\alpha = Nu_h \frac{\lambda_h}{d} = 10.2 \frac{0.108}{8 \times 10^{-3}} = 138\ W/m^2 \cdot °C.$$

5-2. Determine the temperature of oil at the tube inlet and outlet and the pressure drop along the tube under the conditions assumed in problem (5-1).

Answer

$$t_{f1} = 82°C;\quad t_{f2} = 78°C;\quad \Delta p = 1640\ Pa.$$

Solution

The solution of problem (5-1) gives:

$\alpha = 138\ W/m^2 \cdot °C$; $t_f = 80°C$; $t_w = 20°C$; $G = 2.53 \times 10^{-2}\ kg/s$.

The quantity of the heat transferred

$$Q = \alpha(t_f - t_w)\pi dl =$$
$$= 138(80-20)3.14 \times 8 \times 10^{-3} \times 1.0 = 207 \text{ W}.$$

At the temperature $t_f = 80°\text{C}$ the heat capacity of oil $c_{pf} = 2.03$ kJ/kg·°C and the variation of oil temperature along the tube

$$t_{f1} - t_{f2} = \frac{Q}{Gc_{pf}} = \frac{207}{2.53 \times 10^{-2} \times 2.03 \times 10^3} = 4\text{ °C},$$

and the arithmetic mean oil temperature $t_f = 0.5(t_{f1} + t_{f2}) = 80°\text{C}$, whence $t_{f1} = 82°\text{C}$ and $t_{f2} = 78°\text{C}$.

With liquid (fluid) in viscous non-isothermal flow in tubes the friction factor can be determined from the following formula [19]:

$$\xi = \xi_i \left(\frac{\mu_w}{\mu_{f1}}\right)^n, \tag{5-2}$$

where ξ_i is the friction factor for isothermal flow,

$$\xi_i = \frac{64}{\text{Re}}; \quad n = C\left(\text{Pe}_1 \frac{d}{l}\right)^m \left(\frac{\mu_w}{\mu_{f1}}\right)^{-0.062};$$

with $\text{Pe}_1 d/l \leqslant 1500$ $C = 2.3$, $m = -0.3$;
with $\text{Pe}_1 d/l > 1500$ $C = 0.535$, $m = 0.1$.

In the case considered the inlet temperature of oil $t_{f1} = 82°\text{C}$ and at this temperature the heat capacity of oil $c_{pf1} = 2.04$ kJ/kg·°C; $\lambda_{f1} = 0.105$ W/m·°C; $\mu_{f1} = 29.7$ Pa·s. From the solution of problem (5-1) we have: $\text{Re}_f = 1310$ and $\mu_w = 198.2$ Pa·s, $\rho_f = 844$ kg/m³, $w = 0.6$ m/s. Then

$$\text{Pe}_1 \frac{d}{l} = \frac{4G}{\pi l} \frac{c_{pf1}}{\lambda_{f1}} = \frac{4 \times 2.53 \times 10^{-2}}{3.14 \times 1.0} \frac{2.04 \times 10^3}{0.105} = 625,$$

since $\text{Pe}_1 \frac{d}{l} < 1500$, then $C = 2.3$ and $m = -0.3$. The exponent n in formula (5-2)

$$n = 2.3(625)^{-0.3}\left(\frac{198.2}{29.7}\right)^{-0.062} = 0.3.$$

The friction factor

$$\xi = \frac{64}{\text{Re}_f}\left(\frac{\mu_w}{\mu_{f1}}\right)^n = \frac{64}{1310}\left(\frac{198.2}{29.7}\right)^{0.3} = 0.0865.$$

The drop in oil pressure across the tube

$$\Delta p = \xi \frac{\rho_f w^2}{2} \frac{l}{d} = 0.0865 \frac{844 \times 0.6^2}{2} \frac{1}{8 \times 10^{-3}} = 1640 \text{ Pa.}$$

5-3. How will the mean heat-transfer coefficient determined for the conditions assumed in problem 5-1 change, if the tube is made shorter five times ($l/d = 25$, instead of $l/d = 125$), with all other conditions of the problem remaining unchanged? Compare calculation results with the answer to problem 5-1.

Answer

$\alpha' = 262$ W/m²·°C. The lengthwise mean heat-transfer coefficient will increase 1.9 times.

5-4. Determine the hydraulic resistance offered to oil flow, or the pressure drop, assuming the conditions given in problem 5.3. Compare the answer with the calculation result for problem 5-2.

Answer

$\Delta p' = 276$ Pa. The resistance to flow will diminish 5.8 times.

5-5. With fluid in viscous flow in the tube how will change the local heat-transfer coefficient, if fluid velocity rises 2 and 4 times, while the diameter of the tube, mean fluid temperature and wall temperature remain unchanged.

The variation in the correction for the stabilization section ε should be ignored.

Answer

The heat-transfer coefficient will increase respectively $2^{1/3} \approx 1.26$ and $4^{1/3} \approx 1.59$ times.

5-6. With fluid in viscous flow in the tube how will change the Nusselt number and the heat-transfer coefficient, if the diameter of the tube is increased 2 and 4 times maintaining constant the mean temperature of the fluid and tube wall temperature: (a) at a constant fluid velocity and (b) at a constant rate of fluid flow?

The variation in the correction for the stabilization section ε should be ignored.

Answer

(a) At a constant velocity the Nusselt number Nu_h will increase respectively $2^{2/3} \approx 1.59$ and $4^{2/3} \approx 2.52$ times.

The coefficient of heat transfer from the fluid to the wall of the tube will diminish respectively 1.26 and 1.59 times.

(b) At a constant rate of fluid flow the Nu_h number is independent of the tube diameter. The heat-transfer coefficient will decrease respectively 2 and 4 times.

5-7. Water flows along a tube of diameter $d = 6$ mm with a velocity $w = 0.4$ m/s. The temperature of the tube wall $t_w = 50°\mathrm{C}$. What should the length of the tube be to ensure a water outlet temperature $t_{f_2} = 20°\mathrm{C}$ at a water inlet temperature $t_{w1} = 10°\mathrm{C}$?

Answer

$$l = 0.76 \text{ m}.$$

Solution

The lengthwise mean temperature of water

$$t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(10 + 20) = 15°\mathrm{C}$$

the kinematic viscosity of water $\nu_f = 1.16 \times 10^{-6}\ \mathrm{m^2/s}$ and the Reynolds number

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{0.4 \times 6 \times 10^{-3}}{1.16 \times 10^{-6}} = 2065.$$

The pattern of water flow is laminar. At a reference temperature

$$t_h = 0.5\,(t_f + t_w) = 0.5(15 + 50) = 32.5°\mathrm{C};$$

$$\nu_h = 0.769 \times 10^{-6}\ \mathrm{m^2/s};\quad \beta_h = 3.37 \times 10^{-4}\ \mathrm{K^{-1}};$$

$$\mathrm{Pr}_h = 5.14;$$

$$(\mathrm{GrPr})_h = g\beta_h \frac{(t_w - t_f)\,d^3}{\nu_h^2}\,\mathrm{Pr}_h = 9.81 \times 3.37 \times$$

$$\times 10^{-4}\,\frac{(50-15)\,(6 \times 10^{-3})^3}{(0.769 \times 10^{-6})^2}\,5.14 = 2.17 \times 10^5 < 8 \times 10^5.$$

Consequently, the water is in viscous flow and to determine the local coefficient of heat transfer let us use formula (5-1). Since the relative length of the tube is not known, the problem must be solved by the method of successive approximations.

Let us assume a relative length of the tube $l/d = 100$ and, consequently, $l = 100 \times 6 \times 10^{-3} = 0.6$ m.

The physical properties of water are as follows:

at $t_f = 15°\mathrm{C}$ $\mu_f = 1155 \times 10^{-6}$ Pa·s, $\rho_f = 999\ \mathrm{kg/m^3}$;

at $t_h = 32.5°C$ $\lambda_h = 0.631$ W/m·°C, $c_{p_r} 4.174$ kJ/kg·°C; at $t_w = 50°C$ $\mu_w = 549.4 \times 10^{-6}$ Pa·s.

The rate of water flow

$$G = \rho_f w \frac{\pi d^2}{4} = 999 \times 0.4 \frac{3.14 (6 \times 10^{-3})^2}{4} = 0.0113 \text{ kg/s}.$$

The Peclet number at the reference temperature

$$\mathrm{Pe}_h \frac{d}{l} = \frac{4G}{\pi l} \frac{c_{ph}}{\lambda_h} = \frac{4.0 \times 0.0113}{3.14 \times 0.6} \frac{4174}{0.631} = 159.$$

The correction for the section of hydrodynamic stabilization

$$\varepsilon = 0.6 \left(\frac{1}{\mathrm{Re}_f} \frac{l}{d}\right)^{-1/7} \left(1 + 2.5 \frac{1}{\mathrm{Re}_f} \frac{l}{d}\right) =$$

$$= 0.6 \left(\frac{100}{2065}\right)^{-1/7} \left(1 + 2.5 \frac{100}{2065}\right) \approx 1.04.$$

The Nusselt number at the reference temperature

$$\mathrm{Nu}_h = 1.55 \left(\mathrm{Pe}_h \frac{d}{l}\right)^{1/3} \left(\frac{\mu_f}{\mu_w}\right)^{0.14} \varepsilon =$$

$$= 1.55 (159)^{1/3} \left(\frac{1155}{549.4}\right)^{0.14} \times 1.04 = 9.7.$$

The heat-transfer coefficient

$$\alpha = \mathrm{Nu}_h \frac{\lambda_h}{d} = 9.7 \frac{0.631}{6 \times 10^{-3}} = 1020 \text{ W/m}^2\text{·°C}.$$

The amount of heat transferred through the wall

$$Q = G c_{pf} (t_{f2} - t_{f1}) = 0.0113 \times 4187 \times 10 = 473 \text{ W},$$

where c_{pf} is taken based on the mean temperature of the fluid $t_f = 15°C$. On the other hand, the amount of heat transferred

$$Q = \alpha (t_w - t_f) \pi d l.$$

Thus, after the first approximation we find

$$l = \frac{Q}{\alpha (t_w - t_f) \pi d} = \frac{473}{1020 (50 - 15) 3.14 \times 6 \times 10^{-3}} = 0.705 \text{ m}.$$

For the second approximation we select $l = 0.75$ m, repeat the calculation and obtain: $\mathrm{Pe}_h \frac{d}{l} = 183$, $\varepsilon = 1.03$, $\mathrm{Nu}_h = 8.94$, $\alpha = 940$ W/m²·°C. After the second approximation we get:

$$l = \frac{473}{940 \times 35 \times 3.14 \times 6 \times 10^{-3}} = 0.765 \text{ m}.$$

Since the assumed length of the tube coincides with a sufficient accuracy with the result obtained after the second approximation, there is no need in the third approximation, and we may assume a length of the tube $l = 0.76$ m.

5-8. Water flows with a velocity $w = 0.2$ m/s in a tube of diameter $d = 4$ mm and length $l = 200$ mm. The temperature of the tube wall $t_w = 70°\text{C}$. What will the temperature at the tube outlet be, if water inlet temperature $t_{f1} = 10°\text{C}$?

Answer:

$$t_{f2} = 27\ °\text{C}.$$

5-9. Oil, of the MK grade, flows in a tube with a diameter $d = 10$ mm. Oil temperature at the tube inlet $t_{f1} = 80°\text{C}$; the rate of oil flow $G = 120$ kg/h. What should the length of the tube be, if it is desired that at a tube wall temperature $t_w = 30°\text{C}$ the temperature of oil at the tube outlet t_{f2} be equal to 76°C?

Answer

$$l = 1.66 \text{ m}.$$

5-10. Determine the pressure drop across a tube in which oil flows under conditions assumed for problem 5-9. Compare the calculated pressure drop with that, taking place when oil is in isothermal flow at the same tube inlet temperature.

Answer

The pressure drop across the tube $\Delta p = 2.53 \times 10^4$ Pa. In the event of isothermal flow $\Delta p_i = 1.05 \times 10^4$ Pa which is about 2.5 times smaller.

5-11. MC-20 Grade oil flows through the pipes of an oil cooler of diameter $d = 5$ mm and length $l = 0.4$ m, as illustrated in Fig. 5-2. The temperature of tube walls $t_w = 30°\text{C}$.

The lengthwise mean temperature of oil in the tubes of the oil cooler $t_f = 70°\text{C}$.

Determine the total amount of heat transferred, if the oil cooler is built-up of $n = 120$ tubes, connected in parallel, and the total rate of oil flow through the oil cooler $G = 2.5$ kg/s.

Answer

$$Q = 9.1 \text{ kW}.$$

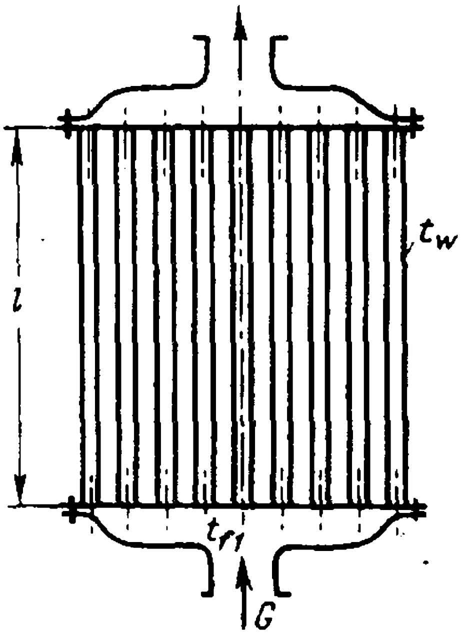

Fig. 5-2. Solution to problem 5-11

5-12. Determine the pressure drop involved and the power (without allowance for pump efficiency) consumption required to pump the oil through the oil cooler, assuming conditions as in problem 5-11. Assume an oil inlet temperature $t_{f1} = 70°\text{C}$; the local resistances to oil flow can be ignored.

Answer

$$\Delta p = 6.85 \times 10^4 \text{ Pa}; \quad N = 0.2 \text{ kW}.$$

5-13. How will change the heat-transfer coefficient, the quantity of the heat transferred and the pressure drop under the conditions of problems 5-11 and 5-12, if one oil cooler with tubes of length $l = 400$ mm is replaced with two coolers connected for parallel operation and built-up of tubes of length $l' = 200$ mm, preserving all other conditions the same as in problem 5-11 (Fig. 5-3)?

Answer

$$\alpha' \approx \alpha; \quad Q' \approx Q; \quad \Delta p' = \frac{1}{4}\Delta p.$$

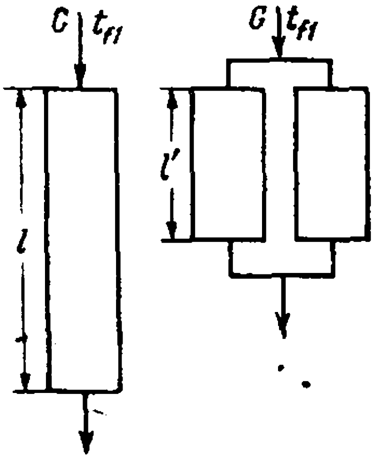

Fig. 5-3. Solution to problem 5-13

5-14. Water flows in a tube of diameter $d = 8$ mm. The tube is heated in a way that the rate of heat flow through the wall of the tube is constant over the perimeter and length and amounts to $q_w = 4 \times 10^4$ W/m^2.

Determine the value of the local heat-transfer coefficient and the tube wall temperature at a distance $x = 20d$

from the inlet into the heated section of the tube. The water inlet temperature $t_{f1} = 10°C$. The mean water flow velocity $w = 0.15$ m/s. Upstream of the heated section of the tube there is a section of hydrodynamic stabilization (Fig. 5-4).

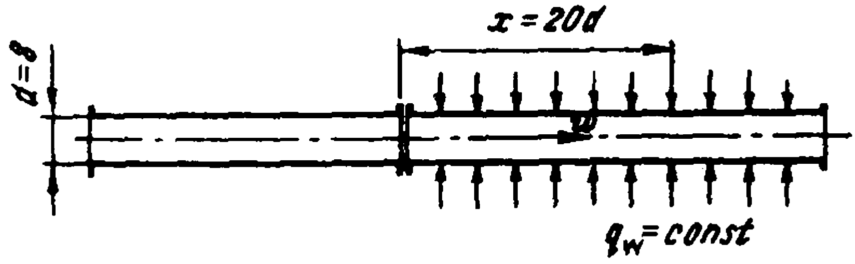

Fig. 5-4. Solution to problem 5-14

Answer

$$\alpha = 885 \text{ W/m}^2\cdot°\text{C}; \quad t_w = 60°\text{C}.$$

Solution

To determine the pattern of water flow, we calculate the Reynolds number. The rate of water flow

$$G = \rho w \frac{\pi d^2}{4} = 999 \times 0.15 \frac{\pi (8 \times 10^{-3})^2}{4} = 7.54 \times 10^{-3} \text{ kg/s}.$$

The bulk mean temperature of water in the section $x = 20d = 20 \times 8 \times 10^{-3} = 0.16$ m long

$$t_f = t_{f1} + \frac{q_w \pi d}{G c_p} x = 10 + \frac{4 \times 10^4 \times 3.14 \times 8 \times 10^{-3}}{7.54 \times 10^{-3} \times 4187} 0.16 =$$

$$= 10 + 5.1 = 15.1°\text{C},$$

where the heat capacity and density of the water are selected at a temperature of 15°C:

$$\rho = 999 \text{ kg/m}^3 \text{ and } c_{pf} = 4187 \text{ J/kg}\cdot°\text{C}.$$

At $t_f = 15°C$ the kinematic viscosity of water $\nu_f = 1.153 \times 10^{-6}$ m²/s and the Reynolds number $\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{0.15 \times 8 \times 10^{-3}}{1.153 \times 10^{-6}} = 1040$.

Since the Reynolds number $\mathrm{Re}_f < 2300$, water is in a laminar flow. In order to determine whether or not heat transfer is affected by free convection, it is necessary to calculate the magnitude of the product $(\mathrm{GrPr})_h$, but for this purpose the wall temperature should be known. Therefore, this check will be carried out at the end of the calculations, after the wall temperature t_w is determined. Calculations are done, assuming that natural, or free, convection does not affect heat transfer.

To calculate the local heat transfer when fluid is in viscous flow in tubes at a constant rate of heat flow, $q_w =$ const, use can be made of the following formula [15]:

$$\mathrm{Nu}_h = 1.31\varepsilon \left(\frac{1}{\mathrm{Pe}_h}\frac{x}{d}\right)^{-1/3}\left(1+2\frac{1}{\mathrm{Pe}_h}\frac{x}{d}\right)\left(\frac{\mu_w}{\mu_f}\right)^{-1/6}, \quad (5\text{-}3)$$

where

$$\mathrm{Nu}_h = \frac{\alpha d}{\lambda_h}; \quad \frac{1}{\mathrm{Pe}_h}\frac{x}{d} = \frac{\pi x \lambda_h}{4Gc_{ph}}; \quad \alpha = \frac{q_w}{t_w - t_f};$$

the subscripts "w" and "h" indicate that the physical properties of the fluid are selected at the wall temperature t_w and a reference temperature $t_h = 0.5\,(t_f + t_w)$ respectively.

The correction for the section of hydrodynamic stabilization

$$\varepsilon = 0.35\left(\frac{1}{\mathrm{Re}_f}\frac{x}{d}\right)^{-1/6}\left[1 + 2.85\left(\frac{1}{\mathrm{Re}_f}\frac{x}{d}\right)^{0.42}\right].$$

This correction is introduced when there is no section of hydrodynamic stabilization upstream of the heated section of the tube and $\frac{1}{Re_f}\frac{x}{d} \leqslant 0.064$.

Formula (5-3) is valid at $\mathrm{Re}_f < 2300$;

$$\frac{1}{\mathrm{Pe}_h}\frac{x}{d} \leqslant 0.04 \text{ and } 0.04 \leqslant \frac{\mu_w}{\mu_f} \leqslant 1.$$

In order to calculate with the aid of formula (5-3), the temperature of the wall, t_w, must be known. The calculation is, therefore, carried out by the method of successive approximations.

Evaluating the local coefficient of heat transfer from the wall of the tube to the flowing water to be $\alpha = 1000$ W/m$^2\cdot$°C, we get:

$$t_w = t_f + \frac{q_w}{\alpha} \approx 15.1 + \frac{4\times 10^4}{1\times 10^3} \approx 55^\circ\mathrm{C},$$

then, the reference temperature $t_h = 0.5\,(t_f + t_w) = 0.5\,(15.1 + 55) \approx 35$°C.

At this temperature

$$c_{ph} = 4174\ \text{J/kg}\cdot{}^\circ\text{C};\quad \lambda_h' = 0.624\ \text{W/m}\cdot{}^\circ\text{C};$$

with $t_f = 15.1$°C $\mu_f = 1152 \times 10^{-6}$ Pa·s;
with $t_w = 55$°C $\mu_w = 509.6 \times 10^{-6}$ Pa·s
and

$$\frac{1}{\text{Pe}_h}\frac{x}{d} = \frac{\pi x \lambda_h}{4Gc_{ph}} = \frac{3.14 \times 0.16 \times 0.624}{4 \times 7.54 \times 10^{-3} \times 4174} = 2.5 \times 10^{-3};$$

$$\frac{1}{\text{Pe}_h}\frac{x}{d} < 0.04;\quad \frac{\mu_w}{\mu_f} = \frac{509.6}{1152} = 0.442 > 0.04;$$

consequently, formula (5-3) is applicable.

In accordance with conditions assumed, $\varepsilon = 1$ and the number

$$\text{Nu}_h = 1.31\,(2.5 \times 10^{-3})^{-1/3}\,(1 + 2 \times 2.5 \times 10^{-3})\,(0.442)^{-1/6} = 11.1$$

The local heat-transfer coefficient

$$\alpha = \text{Nu}_h \frac{\lambda_h}{d} = 11.1\,\frac{0.624}{8 \times 10^{-3}} = 866\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

As the second approximation, the temperature of the wall

$$t_w = 15.1 \times \frac{4 \times 10^4}{866} = 15.1 + 46.2 = 61.3\ {}^\circ\text{C}.$$

Assuming the wall temperature $t_w \underset{\cdot}{=} 60$°C, we repeat the calculation and obtain:

$$t_h = 37.5\ {}^\circ\text{C};\quad \frac{1}{\text{Pe}_h}\frac{x}{d} = 2.54 \times 10^{-3};$$

$$\frac{\mu_w}{\mu_f} = 0.4;\quad \text{Nu}_h = 11.2;\quad \alpha = 885\ \text{W/m}^2\cdot{}^\circ\text{C};$$

as the third approximation, the temperature of the wall

$$t_w = 15.1 + \frac{4 \times 10^4}{885} = 15.1 + 45.2 = 60.3\ {}^\circ\text{C}.$$

Since the calculated wall temperature coincides with sufficient accuracy with the temperature assumed, the local heat-transfer coefficient can be assumed equal to $\alpha = 885$ W/m²·°C and $t_w = 60$°C.

Having determined the wall temperature t_w, let us check now whether heat transfer is affected by free convection.

At a reference temperature $t_h = 37.5°\text{C}$ $\nu_h = 0.695 \times 10^{-6}$ m²/s, $\beta_h = 3.71 \times 10^{-4}$ K⁻¹, Pr = 4.59, then

$$(\text{GrPr})_h = g\beta_h \frac{(t_w - t_f)\, d^3}{\nu_h^2} \text{Pr}_h =$$

$$= 9.81 \times 3.71 \times 10^{-4} \frac{(60 - 15.1)\,(8 \times 10^{-3})^3}{(0.695 \times 10^{-6})^2} 4.59 =$$

$$= 7.9 \times 10^5 < 8 \times 10^5$$

and it may be assumed that free convection will not affect materially the process of heat transfer.

5-15. Compare the local Nusselt numbers for fluid in a laminar flow in a circular tube on conditions of a constant heat flux on the wall, without a preceding section of hydrodynamic stabilization (Nu_h), and in the presence of such a section ($\text{Nu}_{h,st}$). Carry out the comparison for relative distances from the heated section amounting to x/d = 1, 2, 5, 10, 15 and 20. Assume the Reynolds number Re_f = 1800.

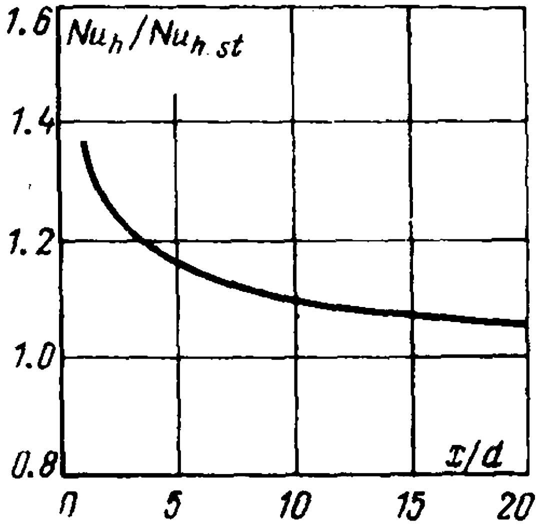

Fig. 5-5. Solution to problem 5-15

Answer

x/d	1	2	5	10	15	20
$\text{Nu}_h/\text{Nu}_{h\cdot st}$	1.37	1.26	1.16	1.10	1.07	1.06

Solution

In accordance with formula (5-3), the correction for the section of hydrodynamic stabilization of flow $\varepsilon = \text{Nu}_h/\text{Nu}_{h.st}$

can be calculated with the aid of the formula

$$\varepsilon = 0.35 \left(\frac{1}{\mathrm{Re}_f}\frac{x}{d}\right)^{-1/6} \left[1 + 2.85 \left(\frac{1}{\mathrm{Re}_f}\frac{x}{d}\right)^{0.42}\right].$$

In the case considered $\mathrm{Re}_f = 1800$ and for $x/d = 2$ we have:

$$\varepsilon = 0.35 \left(\frac{2}{1800}\right)^{-1/6} \left[1 + 2.85 \left(\frac{2}{1800}\right)^{0.42}\right] = 1.26.$$

The calculation results for other values of the relative length are given in the answer to problem 5-5 and in Fig. 5.5.

5-16. Determine the local heat-transfer coefficients and the temperatures of the inner surface of a tube, having a diameter $d = 10$ mm, at distances $x = 0.5$ m and $x = 1.0$ m from the inlet into the heated section of the tube. The tube is being heated maintaining a constant rate of heat transfer through the wall, $q_w = 1 \times 10^4$ W/m². The heat is transferred to and removed by transformer oil, having an inlet temperature $t_{f1} = 30$°C and flowing along the tube with a mean velocity $w = 2.5$ m/s.

Answer

with $x = 50d$ $\alpha = 326$ W/m²·°C, $t_w \approx 61$°C;
with $x = 100d$ $\alpha = 265$ W/m²·°C, $t_w \approx 69$°C.

5-17. A tube having a diameter $d = 6$ mm and length $l = 1600$ mm carries water at a mass rate of flow $G = 15$ kg/h. The tube is heated so that the rate of heat transfer on its inside surface may be assumed to be constant (q_w = const). At the tube inlet the water temperature $t_{f1} = 20$°C.

To what value can the rate of heat transfer q_w, W/m² be raised, if the temperature at the inside surface of the tube should not exceed $t_w \leqslant 100$°C? What will then the outlet bulk temperature of the water be?

Answer

$$q_w = 3.65 \times 10^4 \text{ W/m}^2; \quad t_{f2} = 43.7^\circ\text{C}.$$

5-18. Under the conditions of problem 5-17 how will change the admissible rate of heat transfer and the temperature of water at the tube outlet, if the rate of water flow is decreased two times, i.e. with $G = 7.5$ kg/h?

Answer

$q_w = 2.5 \times 10^4$ W/m^2, i.e., will decrease about 1.5 times; $t_{f2} = 52.5$°C.

5-19. Determine the relative length of the section of thermal stabilization, $l_{in.t}/d$, for a tube having a diameter $d = 14$ mm and carrying water in a laminar flow with a wall temperature maintained constant along the tube ($t_w =$ = const), if the mean temperature of water $t_f = 50$°C and $\mathrm{Re}_f = 1500$. Also calculate the local heat-transfer coefficient on the section of the tube where $l > l_{in.t}$.

Answer

$$l_{in.t}/d = 266; \quad \alpha = 170 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Solution

With water in a laminar flow and provided t_w = const, the relative length of the section of thermal stabilization may be assumed:

$$\frac{l_{in.t}}{d} \approx 0.5\mathrm{Pe}_f.$$

In the case considered at $t_f = 50$°C $\mathrm{Pr}_f = 3.55$ and $\mathrm{Pe}_f = \mathrm{Re}_f\mathrm{Pr}_f = 1500 \times 3.55 = 5320$, consequently, $l_{in.t}/d = 0.05 \times 5320 = 266$.

With $l > l_{in.t}$ the maximum value of the Nusselt number $\mathrm{Nu}_\infty = 3.66$, consequently,

$$\alpha = \mathrm{Nu}_\infty \frac{\lambda_f}{d} = 3.66 \frac{0.648}{14 \times 10^{-3}} = 170 \text{ W/m}^2\cdot{}^\circ\text{C},$$

where at $t_f = 50$°C $\lambda_f = 0.648$ W/m·°C.

5-20. Solve problem 5-19, if heat transfer takes place at a constant rate of heat transfer along the tube (q_w = const).

Answer

$$l_{in.t}/d = 372; \quad \alpha = 203 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Solution

With water in a laminar flow under the condition $q_w =$ = const the relative length of the section of thermal stabilization may be assumed equal to $l_{in.t}/d \approx 0.07\ \mathrm{Re}_f$, and $\mathrm{Nu}_\infty = 4.36$.

In the case considered (see problem 5-19)

$$\frac{l_{in.t}}{d}=0.07\times 5320\approx 372;$$

$$\alpha=4.36\frac{0.648}{14\times 10^{-3}}=203\ \mathrm{W/m^2\cdot {}^\circ C}.$$

5-21. Calculate the length of the section of thermal stabilization in a tube having a diameter $d = 10$ mm under the condition of a constant rate of heat transfer along the tube (q_w = const) and the Reynolds number $\mathrm{Re}_f = 1000$, if the tube carries the following liquids: transformer oil at a temperature $t_f = 100$°C; water at a temperature $t_f = 230$°C; mercury at a temperature $t_f = 120$°C, bismuth at a temperature $t_f = 400$°C; and sodium at a temperature $t_f = 400$°C.

Also determine the local heat-transfer coefficients for these liquids on the section of the tube where $l > l_{in.t}$.

In the calculations the effect of longitudinal heat transfer by conduction should be ignored.

Answer

Calculation results are tabulated below:

Liquid	$l_{in.t}$, m	α, W/m²·°C
Transformer oil	30.7	45.2
Water	0.616	278
Mercury	0.0126*	4 030
Bismuth	0.0102*	6 270
Sodium	0.00392*	27 900

5-22. Determine the local coefficient of heat transfer and the amount of heat transferred when water flow in a horizontal tube having a diameter $d = 10$ mm and length $l = 1.2$ m, if at midlength of the tube the temperatures of the water and tube wall are respectively $t_f = 30$°C and $t_w = 60$°C; the rate of water flow in the tube $G = 7\times$ $\times 10^{-3}$ kg/s.

---

* Actually, $l_{in.t}$ will be somewhat greater than the value obtained due to the presence of heat flow in the longitudinal direction.

Answer

$$\alpha = 1065\ \text{W/m}^2\cdot{}^\circ\text{C};\quad Q = 1200\ \text{W}.$$

Solution

To determine the pattern of water flow, we calculate the Reynolds number Re. At $t_f = 30^\circ\text{C}$ $\mu_f = 801\times10^{-6}$ Pa·s and

$$\text{Re}_f = \frac{4G}{\pi d\mu_f} = \frac{4.7\times10^{-3}}{3.14\times1\times10^{-2}\times801\times10^{-6}} = 1100 < 2300.$$

In accordance with the value of the Reynolds number, the water is in laminar flow.

In order to determine whether heat transfer is affected by free convection, we calculate the product $(\text{GrPr})_h$ where taken as the reference temperature is $t_h = 0.5\,(t_w + t_f)$ and $t_f = 0.5\,(t_{f1} + t_{f2})$. Hence, $t_h = 0.5\,(30 + 60) = 45^\circ\text{C}$.

At this temperature $\nu_h = 0.607\times10^{-6}$ m²/s, $\beta_h = 4.18\times10^{-4}\ \text{K}^{-1}$, and $\text{Pr}_h = 3.92$.

$$(\text{GrPr})_h = g\beta_h\frac{(t_w - t_f)\,d^3}{\nu_h^2}\,\text{Pr}_h =$$

$$= 9.81\times4.18\times10^{-4}\frac{(60-30)\,(1\times10^{-2})^3}{(0.607\times10^{-6})^2}\,3.92 =$$

$$= 1.31\times10^6 > 8\times10^5.$$

Consequently, free convection affects the process of heat transfer; the pattern of flow is of the viscous-gravitational kind.

For fluids in viscous-gravitational flow in horizontal tubes the mean rate of heat transfer can be calculated with the aid of the following formula [15]:

$$\text{Nu}_h = 0.8\left(\text{Pe}_h\frac{d}{l}\right)^{0.4}(\text{GrPr})_h^{0.1}\left(\frac{\mu_w}{\mu_f}\right)^{-0.14}, \qquad (5\text{-}4)$$

where

$$\alpha = \frac{q}{t_w - t_f};\quad \text{Gr}_h = g\beta_h\frac{(t_w - t_f)\,d^3}{\nu_h^2};\quad \text{Pe}_h = \frac{wd}{a_h};$$

$$t_f = 0.5\,(t_{f1} + t_{f2}).$$

The subscripts "w" and "h" indicate that corresponding physical properties are selected by the wall temperature t_w and reference temperature $t_h = 0.5\,(t_w + t_f)$.

Formula (5-4) is valid at

$$\mathrm{Re}_f = 3000;\quad 20 \leqslant \mathrm{Pe}_h d/l \leqslant 120;$$

$$10^6 \leqslant (\mathrm{GrPr})_h \leqslant 13 \times 10^6;\quad 2 \leqslant \mathrm{Pr}_h \leqslant 10.$$

In the case considered at $t_h = 45°\mathrm{C}$

$$a_h = 1.55 \times 10^{-7}\ m^2/\mathrm{s};\quad \lambda_h = 0.641\ \mathrm{W/m \cdot °C};$$

at $t_f = 30°\mathrm{C}$ $\rho_f = 990\ \mathrm{kg/m^3}$;
at $t_w = 60°\mathrm{C}$ $\mu_w = 470 \times 10^{-6}\ \mathrm{Pa \cdot s}$.

$$w = \frac{4G}{\rho_f \pi d^2} = \frac{4 \times 7 \times 10^{-3}}{990 \times 3.14\,(1 \times 10^{-2})^2} = 0.09\ \mathrm{m/s};$$

$$\mathrm{Pe}_h \frac{d}{l} = \frac{wd}{a_h}\frac{d}{l} = \frac{0.09 \times 1 \times 10^{-2}}{1.55 \times 10^{-7}}\,\frac{1 \times 10^{-2}}{1.2} = 48.4;$$

$(\mathrm{GrPr})_h = 1.31 \times 10^6$ and formula (5-4) is applicable.

Substituting the found values in formula (5-4), we find that

$$\mathrm{Nu}_h = 0.8\,(48.4)^{0.4}(1.31 \times 10^6)^{0.1}(470/801)^{-0.14} = 16.6.$$

The coefficient of heat transfer

$$\alpha = \mathrm{Nu}_h \frac{\lambda_h}{d} = 16.6 \frac{0.641}{1 \times 10^{-2}} = 1065\ \mathrm{W/m^2 \cdot °C}.$$

The quantity of heat transferred

$$Q = \alpha\,(t_w - t_f)\,\pi d l = 1065\,(60-30)\,3.14 \times 10^{-2} \times 1.2 = 1200\ \mathrm{W}.$$

5-23. How will the coefficient of heat transfer and the amount of heat transferred under conditions of problem (5-22) change, if the rate of water flow is increased two times, and all other conditions are left unchanged?

Answer

$\alpha = 1850\ \mathrm{W/m^2 \cdot °C}$; $Q = 2090$ W, i.e. α and Q will increase $2^{0.4} \approx 1.32$ times.

5-24. The tubes of a vertical heat exchanger carry water in upward flow. The inner diameter of the tubes $d = 16$ mm and their length $l = 1.2$ m. The rate of water flow through one tube $G = 58$ kg/h. The water temperature at the heat-exchanger inlet $t_{f1} = 30°\mathrm{C}$.

Determine the amount of heat transferred from the wall of one tube to the flowing water and the outlet water temperature, t_{f2}, if the temperature of the tube walls is maintained equal to 80°C.

Answer

$$Q = 1450 \text{ W}; \quad t_{f2} \approx 52°\text{C}.$$

Solution

The rate of water flow

$$G = \frac{58}{3600} = 1.61 \times 10^{-2} \text{ kg/s}.$$

At $t_{f1} = 30°\text{C}$ $\mu_{f1} = 801 \times 10^{-6}$ Pa·s

and

$$\text{Re}_{f1} = \frac{4G}{\pi d \mu_{f1}} = \frac{4 \times 1.61 \times 10^{-2}}{3.14 \times 16 \times 10^{-3} \times 801 \times 10^{-6}} = 1600 < 2300.$$

The pattern of water flow is laminar.

Further calculate the product $(\text{GrPr})_h$. Since the water outlet temperature, t_{f2}, is unknown and, consequently, the mean temperature of water, t_f, cannot be determined, the problem must be solved by the method of successive approximations.

We assume a water outlet temperature $t_{f2} = 50°\text{C}$. Then $t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(30 + 50) = 40°\text{C}$ and the reference temperature $t_h = 0.5\,(t_w + t_f) = 0.5\,(80 + 40) = 60°\text{C}$.

At this reference temperature

$$\beta_h = 5.11 \times 10^{-4} \text{ K}^{-1}; \quad \nu_h = 0.478 \times 10^{-6} \text{ m}^2\text{/s};$$

The Prandtl number $\text{Pr}_h = 2.98$;

$$(\text{GrPr})_h = g\beta_h \frac{(t_w - t_f)\, d^3}{\nu_h^2} \text{Pr}_h =$$

$$= 9.81 \times 5.11 \times 10^{-4} \frac{(80 - 40)\,(16 \times 10^{-3})^3}{(0.478 \times 10^{-6})^2}\, 2.98 =$$

$$= 1.07 \times 10^7 > 8 \times 10^5.$$

The pattern of flow is of the viscous-gravitational kind.

With fluid in viscous-gravitational flow in vertical tubes and coincident direction of forced and free convection at the wall (cooling of the fluid and downward flow or heating of the fluid and upward flow), to calculate the mean rate of heat transfer, the following formula [15] can be used:

$$\mathrm{Nu}_w = 0.35\left(\mathrm{Pe}_h \frac{d}{l}\right)^{0.3}\left[(\mathrm{GrPr})_h \frac{d}{l}\right]^{0.18}, \qquad (5\text{-}5)$$

where the local coefficient of heat transfer is related to the initial temperature difference $t_w - t_f$:

$$\alpha = \frac{q}{t_w - t_{f1}}; \quad \mathrm{Gr}_h = g\beta_h \frac{(t_w - t_f)\, d^3}{\nu_h^2}$$

$$\mathrm{Pe}_h = \frac{wd}{a_h}; \quad t_f = 0.5\,(t_{f1} + t_{f2}).$$

The subscripts "w" and "h" indicate that corresponding physical properties are selected by the wall temperature t_w and the reference temperature $t_h = 0.5\,(t_w + t_f)$.

Formula (5-5) is valid at $\mathrm{Re}_f < 2300$;

$$\left(\mathrm{Pe}_h \frac{d}{l}\right)_{as} \leqslant \mathrm{Pe}_h \frac{d}{l} \leqslant 110; \quad 20 \leqslant \frac{l}{d} \leqslant 130;$$

$$8 \times 10^5 \leqslant (\mathrm{GrPr})_h \leqslant 4 \times 10^6.$$

Here the asymptotic value of the Peclet number:

$$\left(\mathrm{Pe}_h \frac{d}{l}\right)_{as} \approx 1.5\left(\mathrm{GrPr}\frac{d}{l}\right)_h^{0.25}.$$

In the case considered:

at $t_h = 60$°C $a_h = 1.6 \times 10^{-7}$ m²/s; at $t_w = 80$°C $\lambda_w = 0.635$ W/m·°C; at $t_f = 40$°C $\rho_f = 922$ kg/m³.

The mean velocity of water flow

$$w = \frac{4G}{\pi d^2 \rho_f} = \frac{4 \times 1.61 \times 10^{-3}}{3.14\,(16 \times 10^{-3})^2\, 992} = 0.081 \text{ m/s};$$

$$\mathrm{Pe}_h \frac{d}{l} = \frac{wd}{a_h}\frac{d}{l} = \frac{0.081 \times 16 \times 10^{-3}}{1.6 \times 10^{-7}}\,\frac{16 \times 10^{-3}}{1.2} = 108;$$

$$(\mathrm{GrPr})_h \frac{d}{l} = 1.07 \times 10^7\,\frac{16 \times 10^{-3}}{1.2} = 1.43 \times 10^5;$$

$$\left(\mathrm{Pe}_h \frac{d}{l}\right)_{as} \approx 1.5\,(1.43 \times 10^5)^{0.25} = 29.$$

Since all dimensionless terms, or criteria, fall within the limits indicated above, formula (5-5) is applicable.

The Nusselt number Nu_w

$$\mathrm{Nu}_w = 0.35\,(108)^{0.3}(1.43 \times 10^5)^{0.18} = 12.2.$$

The heat-transfer coefficient

$$\alpha = \mathrm{Nu}_w \frac{\lambda_w}{d} = 12.2 \frac{0.635}{16 \times 10^{-3}} = 482 \ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The amount of heat transferred

$$Q = \alpha (t_w - t_f)\pi d l = 482\,(80-30)\,\pi 16 \times 10^{-3} \times$$
$$\times 1.2 = 1450 \ \mathrm{W}.$$

Checking the assumed value of the water outlet temperature

$$t_{f2} = t_{f1} + \frac{Q}{Gc_{pf}} = 30 + \frac{1450}{1.61 \times 10^{-2} \times 4174} =$$
$$= 30 + 21.5 = 51.5\ {}^\circ\mathrm{C},$$

where the heat capacity of water is selected at $t_f = 40^\circ$C;

$$c_{pf} = 4174 \ \mathrm{J/kg \cdot {}^\circ C}.$$

Thus, the first approximation results in $t_{f2} = 51.5^\circ$C. Assuming for the second approximation the water outlet temperature $t_{f2} = 52\ ^\circ$C, we obtain $t_f = 0.5\,(t_{f1} + t_{f2}) = 41^\circ$C and a reference temperature $t_h = 0.5\,(t_w + t_f) = 60.5^\circ$C. The coincidence is sufficiently good, and no more recalculations are required.

5-25. Under conditions of problem 5-24 how will change the amount of heat transferred and the temperature of water leaving the heat exchanger, if water moves downwards, instead of upwards, all other conditions of the problem remaining unchanged?

Answer

$Q = 1860$ W, $t_{f2} = 57.6^\circ$C, i.e. Q will increase by 28 per cent approximately.

Hint. Just as in problem 5-24 the pattern of flow is of the viscous-gravitational kind.

With fluid in viscous-gravitational flow in vertical tubes and oppositely directed forced and free convection at the wall (cooling of the fluid and upward flow or heating of the fluid and downward flow) the mean rate of heat transfer can be calculated with the aid of the following formula [15]:

$$\mathrm{Nu}_f = 0.037\mathrm{Re}_f^{0.37}\mathrm{Pr}_f^{0.4}\left(\frac{\mu_w}{\mu_f}\right)^n, \qquad (5\text{-}6)$$

where in the event of heating $n = -0.11$; with cooling $n = -0.25$.

$$\alpha = \frac{q}{t_w - t_f}; \quad t_f = 0.5\,(t_{f1} + t_{f2}).$$

Formula (5-6) is valid at

$$250 \leqslant \mathrm{Re}_f \leqslant 2 \times 10^4; \quad 1.5 \times 10^6 \leqslant (\mathrm{GrPr})_h \leqslant 12 \times 10^6.$$

Inasmuch as in the problem considered the arithmetic mean temperature of water, t_f, is not known, we may assume the water outlet temperature t_{f2} and solve the problem by the method of successive appoximations.

5-26. How will change the amount of heat transferred and the temperature of water at the outlet, t_{f2}, under conditions of problem 5-24, if the tubes of the heat exchanger are horizontally arranged, with all other conditions remaining unchanged? Compare the answers to problem 5-24, 5-25 and 5-26.

Answer

$$Q = 2300 \text{ W}; \quad t_{f2} = 64.2°\text{C}.$$

Calculation results for the problems 5-24 through 5-26 are tabulated below:

Problem No.	Mutual direction of free and forced convection	Q, W	t_{f2}, °C
5-24	Directions coincide	1450	52
5-25	Oppositely directed	1860	57.6
5-26	Flow in a horizontal tube	2300	64.2

5-27. A vertical water heater handles water having an inlet temperature $t_{f1} = 10°\text{C}$ and flowing upward in tubes of diameter $d = 24$ mm. The temperature of the tube walls is maintained equal to $t_w = 140°\text{C}$. Of what length should the tubes of the heater be to ensure a water outlet temperature $t_{f2} = 70°\text{C}$, at a rate of water flow through a single tube $G = 90$ kg/h?

Answer

$$l = 0.75 \text{ m}.$$

Solution

First find the Reynolds number.

At $t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(10 + 70) = 40°\mathrm{C}$ $\mu_f = 653 \times 10^{-6}$ Pa·s; the rate of water flow $G = 90/3600 = 0.025$ kg/s. Then, the Reynolds number

$$\mathrm{Re}_f = \frac{4G}{\pi d \mu_f} = \frac{4 \times 0.025}{3.14 \times 24 \times 10^{-3} \times 653 \times 10^{-6}} = 2030 < 2300.$$

The fluid is in laminar flow.

At $t_h = 0.5\ (t_w + t_f) = 0.5\,(140 + 40) = 90°\mathrm{C}$, $\beta_h = 6.95 \times 10^{-4}\ \mathrm{K}^{-1}$, $\nu_h = 0.326 \times 10^{-6}\ \mathrm{m^2/s}$, $a_h = 1.68 \times 10^{-7}\ \mathrm{m^2/s}$, $\mathrm{Pr}_h = 1.95$.

$$(\mathrm{GrPr})_h = g\beta_h \frac{(t_w - t_f)\,d^3}{\nu_f^2}\,\mathrm{Pr}_h =$$

$$= 9.81 \times 6.95 \times 10^{-4} \frac{100\,(24 \times 10^{-3})^3}{(0.326 \times 10^{-6})^2}\,1.95 =$$

$$= 1.74 \times 10^6 > 8 \times 10^5.$$

The liquid is in viscous-gravitational pattern of flow and for the case when the directions of forced and free convection coincide heat transfer can be calculated by formula (5-5):

$$\mathrm{Nu}_w = 0.35\left(\mathrm{Pe}_h \frac{d}{l}\right)^{0.3}\cdot\left[(\mathrm{GrPr})_h \frac{d}{l}\right]^{0.18},$$

in which there are two unknown quantities: α and l.

As the second equation we make use of the heat balance equation

$$Q = Gc_{pf}\,(t_{f2} - t_{f1}) = \alpha\,(t_w - t_{f1})\,\pi d l.$$

Substituting the known quantities into the heat-transfer equation, we obtain:

at $t_f = 40°\mathrm{C}$ $\rho_f = 992\ \mathrm{kg/m^3}$, $c_{pf} = 4174$ J/kg·°C; at $t_w = 140°\mathrm{C}$ $\lambda_w = 0.685$ W/m·°C,

$$\mathrm{Pe}_h = \frac{wd}{a_h} = \frac{4G}{\pi d \rho_f a_h} =$$

$$= \frac{4.0 \times 0.025}{3.14 \times 24 \times 10^{-3} \times 992 \times 1.68 \times 10^{-7}} = 7950;$$

$$\alpha = \lambda_w 0.35\,(\mathrm{Pe}_h)_h^{0.3}(\mathrm{GrPr})_h^{0.18} d^{-0.52} l^{-0.48} =$$

$$= 0.685 \times 0.35\,(7950)^{0.3}(1.74 \times 10^6)^{0.18}(24 \times 10^{-3})^{-0.52} l^{-0.48}$$

$$= 745\ l^{-0.48}. \qquad \text{(a)}$$

Substituting the known quantities into the heat-balance equation, we obtain:

$$\alpha = \frac{Gc_{pf}(t_{f2}-t_{f1})}{\pi d(t_w - t_{f1})\, l} = \frac{0.025 \times 4174\,(70-10)}{3.14 \times 24 \times 10^{-3}\,(140-10)\, l} = \frac{638}{l}. \qquad \text{(b)}$$

Solving together equations (a) and (b), we find:

$$l^{0.52} = \frac{638}{745} = 0.857,$$

whence

$$l = 0.75 \text{ m},$$

Now we check whether formula (5-5) is applicable:

$$\frac{l}{d} = \frac{0.75}{24 \times 10^{-3}} = 31.3; \quad \mathrm{Pe}_h \frac{d}{l} = 7950 \frac{1}{31.3} = 253;$$

$$\left(\mathrm{Pe}_h \frac{d}{l}\right)_{as} \approx 1.5 \left[(\mathrm{GrPr})_h \frac{d}{l}\right]^{0.25} =$$

$$= 1.5 \left(1.74 \times 10^{8} \frac{1}{31.3}\right)^{0.25} = 73.$$

Since

$$20 \leqslant \frac{l}{d} \leqslant 130 \quad \text{and} \quad \left(\mathrm{Pe}_h \frac{d}{l}\right)_{as} < \mathrm{Pe}_h \frac{d}{l} < 1100,$$

the used calculation formula is applicable.

5-28. What should the length of tubes of a horizontal heat-exchanger in which water must be heated from a temperature $t_{f1} = 5$°C to a temperature $t_{f2} = 55$°C be, if the diameter of the water-carrying tubes $d = 18$ mm; the temperature of tube walls $t_w = 70$°C, and the rate of water flow through each tube $G = 72$ kg/h?

Answer

$$l = 2 \text{ m}.$$

5-29. Determine the coefficient of heat transfer from the wall of a tube of a steam-turbine condenser to the cooling water, if the lengthwise mean wall temperature of the tube $t_w = 28$°C; the tube inside diameter $d = 16$ mm; the water inlet temperature $t_{f1} = 10$°C, and the water outlet temperature $t_{f2} = 18$°C. The mean velocity of water flow $w = 2$ m/s.

Also determine the amount of heat transferred and the length of the tube.

Answer

$$\alpha = 7340 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad Q = 13.5 \text{ kW}; \quad l = 2.7 \text{ m}.$$

Solution

Since the water inlet and outlet temperatures, t_{f1} and t_{f2}, are given, the Reynolds number can be determined based on the arithmetic mean temperature of the cooling water

$$t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(10 + 18) = 14°\text{C}.$$

At this temperature the kinematic viscosity of water $\nu_f = 1.18 \times 10^{-6}$ m²/s and the Reynolds number

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{2\times 16\times 10^{-3}}{1.18\times 10^{-6}} = 2.71 \times 10^{-4} > 1 \times 10^4.$$

The cooling water is in turbulent flow.

With an incompressible liquid in turbulent flow in tubes and conduits (channels), with Prandtl numbers $\text{Pr} > 0.7$, heat transfer can be calculated by the following formula [13]:

$$\text{Nu}_f = 0.021\text{Re}_f^{0.8}\,\text{Pr}_f^{0.43}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25}\varepsilon_l\,{}^*, \qquad (5\text{-}7)$$

where the subscripts "f" and "w" indicate that the physical properties of the liquid involved are selected based on the arithmetic mean temperature t_f and the wall temperature t_w;

$$\alpha = \frac{q}{\Delta t_{\log}}\,; \quad \Delta t_{\log} = \frac{t_{f2} - t_{f1}}{\ln \dfrac{t_w - t_{f1}}{t_w - t_{f2}}},$$

where ε_l is the correction for the initial section. At $l/d > 50$ $\varepsilon_l = 1$. The values of the correction ε_l are given in the following table:

Re_f	l/d								
	1	2	5	10	15	20	30	40	50
1×10^4	1.65	1.50	1.34	1.23	1.17	1.13	1.07	1.03	1.0
2×10^4	1.51	1.40	1.27	1.18	1.13	1.10	1.05	1.02	1.0
5×10^4	1.34	1.27	1.18	1.13	1.10	1.08	1.04	1.02	1.0
1×10^5	1.28	1.22	1.15	1.10	1.08	1.06	1.03	1.02	1.0

---

* The corrections, accounting for the variation in the physical properties over the cross section of flow in formulae (5-7) and (5-8), $(\text{Pr}_f/\text{Pr}_w)^{0.25}$ and $(\mu_f/\mu_w)^n$, are not valid for gases. See formula (5-13).

With $\mathrm{Re}_f > 1 \times 10^5$ or $\mathrm{Pr}_f > 5$ more accurate results are obtained with the aid of formula [18]:

$$\mathrm{Nu}_f = \frac{\frac{\xi}{8}\mathrm{Re}_f\mathrm{Pr}_f}{12.7\sqrt{\frac{\xi}{8}}\,(\mathrm{Pr}_f^{2/3}-1)+1.07}\left(\frac{\mu_f}{\mu_w}\right)^n, \qquad (5\text{-}8)$$

where $\xi = (1.82 \log \mathrm{Re}_f - 1.64)^{-2}$ is the friction factor for cases of isothermal turbulent flow of fluid in plane tubes. When liquid is heated $n = 0.11$ and when liquid is cooled, $n = 0.25$.

This problem is solved with the aid of formula (5-7).

At $t_f = 14$°C for water $\mathrm{Pr}_f = 8.5$, $\lambda_f = 0.584$ W/m·°C,

$$\rho_f = 999 \text{ kg/m}^3, \quad c_{pf} = 4.187 \text{ kJ/kg·°C}.$$

At $t_w = 28$°C $\mathrm{Pr}_w = 5.7$.

Since the length of the tube is not known, as the first approximation, we assume a correction for the initial section $\varepsilon_l = 1$.

Substituting the known quantities into formula (5-7), we find:

$$\mathrm{Nu}_f = 0.021\,(2.71 \times 10^4)^{0.8}\,(8.5)^{0.43}\left(\frac{8.5}{5.7}\right)^{0.25} = 201.$$

The local heat-transfer coefficient

$$\alpha = \mathrm{Nu}_f\,\frac{\lambda_f}{d} = 201\,\frac{0.584}{16 \times 10^{-3}} = 7320 \text{ W/m}^2\text{·°C.}$$

The rate of water flow through the tube

$$G = w\rho_f\,\frac{\pi d^2}{4} = 2 \times 999\,\frac{3.14\,(16 \times 10^{-3})^2}{4} = 0.403 \text{ kg/s.}$$

The amount of heat transferred to the cooling water

$$Q = Gc_{pf}\,(t_{f2} - t_{f1}) = 0.403 \times 4.187\,(18-10) - 13.5 \text{ kW.}$$

The length of the condenser tube

$$l = \frac{Q}{\alpha \Delta t_{\log} \pi d},$$

where the logarithmic mean temperature difference

$$\Delta t_{\log} = \frac{t_{f2}-t_{f1}}{\ln \frac{t_w - t_{f1}}{t_w - t_{f2}}} = \frac{18-10}{2.3 \log \frac{28-10}{28-18}} = 13.7 \text{ °C}$$

and

$$l = \frac{13\,500}{7320 \times 13.7 \times 3.14 \times 16 \times 10^{-3}} = 2.68 \text{ m}.$$

The correction ε_l was assumed to be unity. The calculated length of the tube $l \approx 2.7$ m, consequently,

$$\frac{l}{d} = \frac{2.7}{16 \times 10^{-3}} = 167.$$

Since $l/d > 50$, then $\varepsilon_l = 1$ and no further calculations are needed.

5-30. With liquid in turbulent flow in a tube how will change the heat-transfer coefficient, if the velocity of the liquid increases 2 and 4 times, while the diameter of the tube and the mean temperatures of the liquid and wall remain unchanged?

Answer

The heat-transfer coefficient will increase respectively 1.74 and 3.04 times.

5-31. With liquid in turbulent flow in a tube how will change the heat-transfer coefficient, if, with the mean temperatures of the liquid and tube wall remaining unchanged, the diameter of the tube is increased 2 and 4 times: (a) maintaining the velocity of flow constant; (b) maintaining the rate of liquid flow constant?

Answer

(a) The coefficient of heat transfer will decrease respectively 1.15 and 1.32 times.

(b) the coefficient of heat transfer will decrease respectively 3.5 and 12 times.

5-32. Determine the ratio of the coefficients of heat transfer from the wall of a tube to water, α_1, and to gas, α_2, with the fluids in turbulent flow in tubes of equal diameters, at equal Reynolds numbers and about equal Prandtl numbers. What shall be this ratio for water and air, if water temperature $t_{f1} = 250°\text{C}$ and air temperature $t_{f2} = 20°\text{C}$?

Answer

$$\frac{\alpha_1}{\alpha_2} \approx \frac{\lambda_1}{\lambda_2}; \quad \frac{\alpha_1}{\alpha_2} \approx 24.$$

5-33. Water flows into a boiler economizer at a temperature $t_{f1} = 165°C$ and leaves the economizer at a temperature $t_{f2} = 215°C$. Calculate the coefficient of heat transfer α from the wall of an economizer tube to the flow of water, if the inner diameter of the water-carrying tubes $d = 36$ mm; the water flow velocity $w = 0.6$ m/s, and the relative length of the tubes $l/d > 50$.

Note. Since the coefficient of heat transfer from the wall of an economizer tube is considerably greater than the coefficient of heat transfer from the flue gas to the wall, the temperature of the inner surface of the tube will be near to the mean temperature of water and the ratio $\mathrm{Pr}_f/\mathrm{Pr}_w \approx 1$. Therefore, under the conditions of the problem considered, we may assume that $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25} = 1$ in formula (5-7).

Answer

$$\alpha = 4750\ \mathrm{W/m^2 \cdot °C}.$$

5-34. A tube having an inside diameter $d = 8$ mm and length $l > 50\,d$ carries water, flowing with a velocity $w = 1.2$ m/s. The external surface of the tube is heated so that the temperature of the internal surface of the tube $t_w = 90°C$. The water is heated from an inlet temperature $t_{f1} = 15°C$ to an outlet temperature $t_{f2} = 45°C$.

Determine the coefficient of heat transfer from the wall of the tube to the water and the mean rate of heat transfer along the tube.

Answer

$$\alpha = 7950\ \mathrm{W/m^2 \cdot °C}; \quad q = 477\ \mathrm{kW/m^2}.$$

5-35. Under conditions of problem 5-34 determine the friction factor. Compare the result obtained with the value of the friction factor ξ_i for a fluid in isothermal flow.

Answer

$$\xi = 0.0263; \quad \xi/\xi_i = 0.875.$$

Solution

With an incompressible liquid in turbulent non-isothermal flow in plane tubes the friction factor can be calculated by the following formula [20]:

$$\xi = \xi_i \left(\frac{\mu_w}{\mu_f}\right)^n \qquad (5\text{-}9)$$

where $\xi_t = (1.82 \log \mathrm{Re}_f - 1.64)^{-2}$ is the friction factor for isothermal flow; μ_w/μ_f is the ratio of the dynamic viscosities taken respectively at the temperature of the wall and the mean temperature of the liquid involved; $n = 0.14$ in the event of heating, and $n=0.28\ \mathrm{Pr}_f^{-0.25}$ when cooling of liquid is involved. The formula is valid at

$$3.3 \times 10^3 \leqslant \mathrm{Re}_f \leqslant 2.5 \times 10^5,$$

$$0.3 \leqslant \frac{\mu_w}{\mu_f} \leqslant 38 \text{ and } 1.3 \leqslant \mathrm{Pr}_f \leqslant 178.$$

From the conditions given in problem 5-34 we have:

$$d = 8 \text{ mm}; \quad w = 1.2 \text{ m/s}; \quad t_f = 30°\mathrm{C}; \quad t_w = 90°\mathrm{C}.$$

For water at $t_f = 30°\mathrm{C}$ $\nu_f = 0.805 \times 10^{-6}$ m²/s; $\mu_f = 802 \times 10^{-6}$ Pa·s and at $t_w = 90°\mathrm{C}$ $\mu_w = 315 \times 10^{-6}$ Pa·s;

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{1.2 \times 8 \times 10^{-3}}{0.805 \times 10^{-6}} = 1.19 \times 10^4.$$

The friction factor for isothermal fluid flow

$$\xi_t = (1.82 \log \mathrm{Re}_f - 1.64)^{-2} = (1.82 \log 1.19 \times 10^4 - 1.64)^{-2} = 0.030.$$

For non-isothermal fluid flow with heating the friction factor

$$\xi = \xi_t \left(\frac{\mu_w}{\mu_f}\right)^{0.14} = 0.03 \left(\frac{315}{802}\right)^{0.14} = 0.0263.$$

Thus

$$\xi/\xi_t = 0.875.$$

5-36. Under conditions of problem 5-34 how will change the local heat-transfer coefficient, if the water-carrying tube is made in the form of a coil having a diameter $D = 2R = 200$ mm (Fig. 5-6)?

Answer

$\alpha_{bent} = 9100$ W/m²·°C, i.e. the heat-transfer coefficient will increase by 14.5 per cent.

Solution

With fluid flowing in bent tubes at $\mathrm{Re}'_{cr} < \mathrm{Re}_f < \mathrm{Re}''_{cr}$ the local heat-transfer coefficient can be calculated with the aid of formula (5-7) in which the dimensionless terms Re'_{cr} and Re''_{cr} are equal to

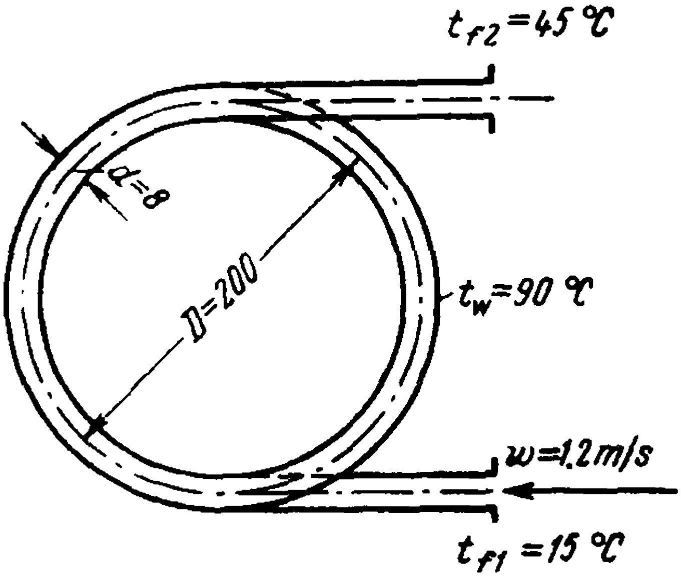

Fig. 5-6. Solution to problem 5-36

$$\mathrm{Re}'_{cr} = \frac{16.4}{\sqrt{dR}}; \quad \mathrm{Re}''_{cr} = 18\,500\left(\frac{d}{2R}\right)^{0.28}, \qquad (5\text{-}10)$$

where d = tube inside diameter;
R = tube bending radius.

The relationships (5-10) are true for $d/R \geqslant 8 \times 10^{-4}$ [4].

If $\mathrm{Re}_f > \mathrm{Re}''_{cr}$, the heat-transfer coefficient can be calculated by the same formula (5-7), but the obtained value of the coefficient should be multiplied by ε_R, which for coiled tubes is determined by the following formula

$$\varepsilon_R = 1 + 1.8\ d/R. \qquad (5\text{-}11)$$

In the case considered (see problem 5-34), for water at $t_f = 30$°C $\nu_f = 0.805 \times 10^{-6}$ m²/s and

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{1.2 \times 8 \times 10^{-3}}{0.805 \times 10^{-6}} = 1.19 \times 10^4;$$

$$R''_{cr} = 18\,500\left(\frac{d}{2R}\right)^{0.28} = 18\,500\left(\frac{8}{2 \times 100}\right)^{0.28} = 7500.$$

Since $\mathrm{Re}_f \gg \mathrm{Re}_{cr}$, the coefficient is calculated with the aid of formula (5-7), introducing the correction (5-11). From the solution of problem 5-34, we know that for a straight tube the heat-transfer coefficient $\alpha = 7950$ W/m²·°C.

$$\varepsilon_R = 1 + 1.8\,\frac{8}{100} = 1.145 \text{ and}$$

$$\alpha_{bent} = \alpha\varepsilon_R = 7950 \times 1.145 = 9100 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

5-37. Determine the amount of heat removed from water flowing through a coil having a radius $R = 160$ mm and bent from a tube with a diameter $d = 18$ mm. The rate of water flow through the coil $G = 0.24$ kg/s; the lengthwise mean water temperature $t_f = 120$°C; the lengthwise constant temperature of the inner surface of the tube $t_w = 110$°C. The length of the tube from which the coil is bent $l = 3$ m.

Answer

$$Q = 14 \text{ kW}.$$

5-38. It is planned to construct a heat-exchange apparatus from straight round tubes having a diameter $d = 30$ mm, which should carry a coolant. The inner surface temperature of tube walls is given, $t_w = 120$°C.

The coolant has a mean temperature $t_f = 70$°C and it must remove heat in the amount $Q = 300$ kW.

Determine the cooling surface, if used, as a coolant are: (a) water; (b) transformer oil; (c) air at atmospheric pressure.

The mean velocity of water and oil flow is assumed equal to $w = 2$ m/s, and the velocity of air flow $w = 10$ m/s.

In calculating assume for the three cases that $l > 50\,d$ and a logarithmic mean temperature difference $\Delta t_{\log} \approx \approx t_w - t_f$.

Answer

	α, W/m²·°C	F, m²
For water	10 700	0.56
For oil	1140	5.25
For air	38.4	156

5-39. How will change the heat-transfer coefficients and the heating surfaces for water, oil and air, obtained in problem 5-38, if at the same mean temperature of the coolant ($t_f = 70$°C), the wall temperature will be not 120°C but 20°C, i.e. if cooling of the heat-carrying fluid will take place at the same temperature difference as in problem 5-38?

Answer

(a) The local coefficient of heat transfer from water to the wall will decrease by 32.4 per cent and $F = 0.83$ m^2.

(b) The local coefficient of heat transfer from transformer oil to the wall will decrease by 41.6 per cent and $F = 9.0$ m^2.

(c) At relatively small temperature differences the local coefficient of heat transfer from air to the wall is practically independent of the temperature difference and the surface will be the same as in problem 5-38: $F = 156$ m^2.

5-40. A straight tube having a diameter $d = 30$ mm and length $l = 2.5$ m carries a heat-carrying agent moving with a velocity $w = 4$ m/s, and its mean temperature $t_f = 50$°C.

Calculate the pressure drop across the tube, if used as a heat-carrying agent are: (a) water; (b) transformer oil. Calculate for the case of cooling of the heat-carrying agent at a tube wall temperature t_w 20°C and for the case of heating of the heat-carrying agent at a wall temperature $t_w = 80$°C.

Answer

If water is cooled, the pressure drop $\Delta p = 11.5$ kPa; if transformer oil is cooled, the pressure drop $\Delta p = 17.6$ kPa; when water is heated $\Delta p = 9.57$ kPa and when transformer oil is heated $\Delta p = 14.3$ kPa.

5-41. A tube having a diameter $d = 38$ mm carries water flowing with a velocity $w = 9$ m/s. The temperature of the tube inner surface $t_w = 50$°C and the flowing water is heated from an inlet temperature $t_{f1} = 16$°C to an outlet temperature $t_{f2} = 24$°C.

Determine the local coefficient of heat transfer from the wall to water and the length of the tube.

Answer

$$\alpha = 28\,400 \text{ W/m}^2\cdot\text{°C}; \quad l = 3.35 \text{ m}.$$

Solution

Determine the pattern of water flow:

$$t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(16 + 24) = 20\text{°C};$$

at $t_f = 20°\mathrm{C}$ the kinematic viscosity of water $\nu_f = 1 \times \times 10^{-6}$ m^2/s and

$$\mathrm{Re}_f = \frac{9 \times 38 \times 10^{-3}}{1 \times 10^{-6}} \times 3.42 \times 10^5 > 10^5.$$

Water is in turbulent flow; the heat-transfer coefficient can be calculated with the aid of formula (5-8). At $t_f = = 20°\mathrm{C}$ $\mathrm{Pr}_f = 7.02$;

$$\mu_f = 1004 \times 10^{-6}\ \mathrm{Pa \cdot s}; \quad \lambda_f = 0.599\ \mathrm{W/m \cdot °C};$$

$$\rho_f = 998\ \mathrm{kg/m^3}; \quad c_{pf} \approx 4187\ \mathrm{J/kg \cdot °C}.$$

At $t_w = 50°\mathrm{C}$ $\mu_w = 549 \times 10^{-6}$ Pa·s.

$$\xi = (1.82 \log \mathrm{Re}_f - 1.64)^{-2} = [1.82 \log (3.42 \times 10^5 - -1.64]^{-2} = 0.0141;$$

$$\mathrm{Nu}_f = \frac{\frac{\xi}{8} \mathrm{Re}_f \mathrm{Pr}_f}{12.7 \sqrt{\frac{\xi}{8}} (\mathrm{Pr}_f^{2/3} - 1) + 1.07} \left(\frac{\mu_f}{\mu_w}\right)^n,$$

where for heating $n = 0.11$; consequently

$$\mathrm{Nu}_f = \frac{\frac{0.0141}{8} \times 3.42 \times 10^5 \times 7.02}{12.7 \left(\frac{0.0141}{8}\right)^{0.5} (7.02^{2/3} - 1) + 1.07} \times \left(\frac{1004}{549}\right)^{0.11} = 1800$$

and the heat-transfer coefficient

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 1800 \frac{0.599}{38 \times 10^{-3}} = 28\,400\ \mathrm{W/m^2 \cdot °C}.$$

The length of the tube is determined from the heat balance equation

$$Q = \alpha (t_w - t_f) \pi d l = G c_{pf} (t_{f2} - t_{f1}).$$

The rate of water flow and the amount of heat transferred to water

$$G = \rho_f w \frac{\pi d^2}{4} = 998 \times 9 \frac{3.14 (38 \times 10^{-3})^2}{4} = 10.15\ \mathrm{kg/s};$$

$$Q = G c_{pf} (t_{f2} - t_{f1}) = 10.15 \times 4187 \times 8 = 241\ \mathrm{kW};$$

then

$$l = \frac{Q}{\alpha (t_w - t_f) \pi d} = \frac{3.41 \times 10^5}{2.84 \times 10^4 (50 - 20)\, 3.14 \times 38 \times 10^{-3}} = 3.35\ \mathrm{m}.$$

5-42. In a heat-exchange apparatus water must add heat in the amount $Q = 460$ kW, moving along a straight tube of an inside diameter $d = 50$ mm. The inner surface temperature of the tube is maintained constant and equal to 20 °C. The rate of water flow $G = 11$ kg/s, and the water inlet temperature $t_{f1} = 75$ °C.

Determine the length of the tube required.

Answer

$$l = 3.1 \text{ m}.$$

5-43. Determine the local heat-transfer coefficient and the rate of heat flow on the inner surface of a tube having a diameter $d = 12$ mm and carrying water with velocity $w = 6.5$ m/s, if the mean water temperature $t_f = 160$°C and the temperature at the inner surface of the tube t_w is maintained equal to 185 °C.

Answer

$$\alpha = 4 \times 10^4 \text{ W/m}^2\cdot\text{°C}; \quad q = 1 \times 10^6 \text{ W/m}^2.$$

5-44. Determine the velocity with which water having the arithmetic mean temperature $t_f = 150$ °C must be pumped through a tube with a diameter $d = 20$ mm and length $l = 2.3$ m, to ensure the removal of heat in the amount of 9 kW, provided water is in turbulent flow and the temperature of the inner surface of the tube $t_w = 170$ °C.

Also determine the water inlet and outlet temperatures.

Note. Solving the problem, it should be borne in mind that in formula (5-7) the heat-transfer coefficient is related to the logarithmic mean difference between the temperatures of the wall and liquid.

Answer

$$w = 0.36 \text{ m/s}; \quad t_{f1} = 140\text{ °C}; \quad t_{f2} = 160\text{ °C}.$$

Solution

Let us find the value of the heat-transfer coefficient required, assuming as the first approximation that $\Delta t_{\log} = (t_w - t_f)$:

$$\alpha = \frac{Q}{\pi d l (t_w - t_f)} = \frac{9000}{3.14\times2\times10^{-2}\times2.3\,(170-150)} = 3100 \text{ W/m}^2\cdot\text{°C}.$$

At $t_f = 150$ °C $\nu_f = 0.202 \times 10^{-6}$ m²/s, $\rho_f = 917$ kg/m³, $\lambda_f = 0.685$ W/m·°C, $c_{pf} = 4313$ J/kg·°C, $\mathrm{Pr}_f = 1.17$.

At $t_w = 170\,°C$ $Pr_w = 1.05$.

Determine the Nusselt number Nu_f and the required Reynolds number, Re_f, by formula (5-7):

$$Nu_f = \frac{\alpha d}{\lambda_f} = \frac{3100 \times 2 \times 10^{-2}}{0.685} = 90.5;$$

$$Re_f^{0.8} = \frac{Nu_f}{0.021 Pr_f^{0.43} \left(\frac{Pr_f}{Pr_w} \right)^{0.25}} =$$

$$= \frac{90.5}{0.021\,(1.17)^{0.43} \left(\frac{1.17}{1.05} \right)^{0.25}} = 0.3930,$$

whence

$$Re_f = 3.11 \times 10^4.$$

As the first approximation determine the velocity of water required:

$$w = Re_f \frac{\nu_f}{d} = 3.11 \times 10^4 \frac{0.202 \times 10^{-6}}{2 \times 10^{-2}} = 0.314 \text{ m/s}.$$

The rate of water flow

$$G = \rho_f w \frac{\pi d^2}{4} = 917 \times 0.314 \frac{3.14\,(2 \times 10^{-2})^2}{4} = 0.0903 \text{ kg/s}$$

and the temperature drop along the tube

$$\delta t = \frac{Q}{G c_{pf}} = \frac{9000}{0.0903 \times 4313} = 23\,°C.$$

Consequently, the initial and final water temperatures

$$t_{f1} = t_f - 0.5\delta t = 150 - 11.5 = 138.5\,°C;$$

$$t_{f2} = t_f + 0.5\delta t = 150 + 11.5 = 161.5\,°C$$

The logarithmic mean temperature difference

$$\Delta t_{\log} = \frac{t_{f2} - t_{f1}}{2.3 \log \frac{t_w - t_{f1}}{t_w - t_{f2}}} = \frac{23}{2.3 \log \frac{31.5}{8.5}} = 17.6\,°C.$$

As the second approximation we assume $\Delta t_{\log} = 17.6\,°C$;

$$\alpha = \frac{9000}{3.14 \times 2 \times 10^{-2} \times 2.3 \times 17.6} = 3530 \text{ W/m}^2 \cdot °C.$$

$$Re_f^{0.8} = 3930 \frac{3530}{3100} = 4470;$$

$$Re_f = 3.66 \times 10^4; \quad w = 0.37 \text{ m/s}.$$

$G = 0.106$ kg/s, $\delta t = 19.7$ °C, then $t_{f1} = 140.2$, $t_{f2} = 159.8$ °C, and the logarithmic mean temperature difference $\Delta t_{\log} = 18.2$ °C.

As the third approximation assume $\Delta t_{\log} = 18.2$°C; $\alpha = 3420$ W/m$^2\cdot$°C; $\mathrm{Re}_f = 3.55 \times 10^4$, and $w = 0.36$ m/s. Then $\delta t = 20$°C, $t_{f1} = 140$°C, $t_{f2} = 160$°C, and $\Delta t_{\log} = 18.2$°C which coincides with the previously assumed value.

5-45. Water at a temperature $t_{f1} = 30$°C flows into a tube having a diameter $d = 12$ mm and length $l = 2.2$ m.

Determine the temperature of water at the tube outlet, t_{f2}, if it is known that the rate of water flow $G = 0.083$ kg/s and the temperature of the tube inner surface $t_w = 60$°C.

Answer

$$t_{f2} = 50\text{°C}.$$

Solution

In order to calculate the rate of heat transfer, we must know the mean temperature of water along the tube. Since the water outlet temperature is not known, the problem must be solved by the method of successive approximations.

Assume a water outlet temperature $t_{f2} = 40$°C. Then, the mean temperature $t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(30 + 40) = 35$°C. At that temperature the kinetic viscosity of water $\mu_f = 7.28 \times 10^{-4}$ Pa·s;

$$\mathrm{Re}_f = \frac{4G}{\pi d \mu_f} = \frac{4 \times 8.3 \times 10^{-2}}{3.14 \times 1.2 \times 10^{-2} \times 7.28 \times 10^{-4}} = 12\,100 > 10^4.$$

Water is in turbulent flow in the tube.

At $t_f = 35$°C $\lambda_f = 0.626$ W/m$^2\cdot$°C, $\mathrm{Pr}_f = 4.85$; at $t_w = 60$°C $\mathrm{Pr}_w = 3.00$.

Substituting the found values into formula (5-7), we find the Nusselt number Nu_f and the heat-transfer coefficient:

$$\mathrm{Nu}_f = 0.021\,\mathrm{Re}_f^{0.8}\,\mathrm{Pr}_f^{0.43}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25} =$$

$$= 0.21\,(1.21 \times 10^4)^{0.8}\,(4.85)^{0.43}\left(\frac{4.85}{3.00}\right)^{0.25} = 86;$$

$$\alpha = \mathrm{Nu}_f\,\frac{\lambda_f}{d} = 86\,\frac{0.626}{1.2 \times 10^{-2}} = 4490 \text{ W/m}^2\cdot\text{°C}.$$

The temperature of water at the tube outlet, t_{f2}, we find from the heat-balance equation

$$\alpha \Delta t_{\log} \pi d l = G c_{pf} (t_{f2} - t_{f1}).$$

Allowing for

$$\Delta t_{\log} = \frac{t_{f2} - t_{f1}}{2.3 \log \frac{t_w - t_{f1}}{t_w - t_{f2}}},$$

we obtain

$$\log (t_w - t_{f2}) = \log (t_w - t_{f1}) - \frac{\alpha \pi d l}{2.3 G c_{pf}};$$

$$\log (60 - t_{f2}) = \log (60 - 30) - \frac{4490 \times 3.14 \times 1.2 \times 10^{-2} \times 2.2}{2.3 \times 0.083 \times 4187},$$

whence t_{f2} = 49.7°C.

As the second approximation we assume t_{f2} = 50°C. Then, t_f = 40°C; $\mu_f = 6.54 \times 10^{-4}$ Pa·s; λ_f = 0.634 W/m·°C; Pr_f = 4.30; Re_f = 13 500; Nu_f = 87 and α = 4600 W/m^2·°C.

The water outlet temperature (second approximation)

$$\log (60 - t_{f2}) = \log (60 - 30) - \frac{4600 \times 3.14 \times 1.2 \times 10^{-2} \times 2.2}{2.3 \times 0.083 \times 4187};$$

$$t_{2f} = 50\ ^\circ\mathrm{C}.$$

5-46. A heat exchanger is built-up of parallel straight tubes of diameter d = 18 mm and length l = 2.2 m. The tubes carry heating water (Fig. 5-7). The number of tubes n = 30. The total rate of water flow through the heat exchanger $G = 2.4 \times 10^4$ kg/h. The water temperature at the heat-exchanger inlet t_{f1} = 90°C.

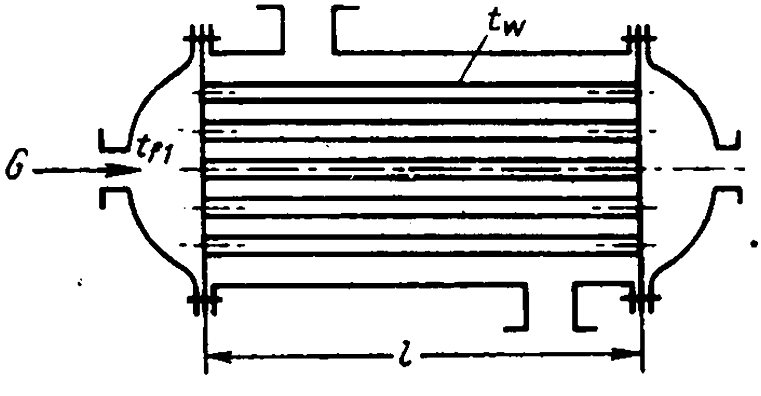

Fig. 5-7. Solution to problem 5-46

Determine the amount of heat rejected from the water, if the temperature of the tube inner surface $t_w = 50°C$.

Answer

$$Q = 558 \text{ kW}.$$

5-47. Water flows through the channel of the fuel element of a nuclear reactor at pressure $p = 8$ MPa. The diameter of the channel $d = 8$ mm and its length $l = 2.5$ m. The rate of water flow $G = 0.12$ kg/s. Water inlet temperature $t_{f1} = 190°C$.

Determine the temperature of water and the surface temperature of the channel on the outlet side, t_{f2} and t_{w2}, if the rate of heat flow on the wall is assumed approximately constant along the length of the channel and equal to $q_w = 620$ kW/m^2.

Note. With the temperatures and pressure considered, account should be taken of the dependence of the heat capacity of water on temperature and pressure. Water outlet temperature should, therefore, be determined based on the change in enthalpy along the channel:

$$i_2 = i_1 + \frac{q_w \pi \, dl}{G}, \text{ kJ/kg}.$$

Answer

$$t_{f2} = 260°C; \quad t_{w2} = 285°C.$$

5-48. The temperature and pressure of water at the inlet of the cooling channel of a nuclear reactor $t_{f1} = 180°C$, $p = 10$ MPa. The channel has a circular cross section with an inner diameter $d = 6$ mm and length $l = 3$ m.

What rate of water flow through the channel must be ensured so that the temperature of the wall at the channel outlet would be 20°C below the water saturation temperature at the given pressure, if the rate of heat flow on the inner surface of the channel is assumed approximately constant along the length of the channel and equal to $q_w = 740$ kW/m^2?

Note. To determine the rate of water flow, it is necessary to calculate the local coefficient of heat transfer from the wall of the channel to the cooling water and the temperature of water at the channel outlet, α_2 and t_{f2}, which in turn depend on the rate of water flow. The problem can be solved, therefore, by the method of successive approximations, assuming a velocity of water flow in the channel ranging from $w = 3$ to $w = 6$ m/s.

Answer

$$G = 0.1 \text{ kg/s}.$$

5-49. In an experimental outfit, used to investigate heat transfer for fluids in turbulent flow, water flows in a stainless tube having a diameter $d = 5$ mm and wall thickness $\delta = 0.5$ mm. The tube is heated with electric current and all the heat liberated in the wall due to current flow is transferred to the water through the inner surface of the tube.

Determine the local coefficient of heat transfer from the inner surface of the wall to water, α_{ex}, W/m²·°C, at a distance $l = 600$ mm from the inlet, if the test results are as follows: the current intensity $I = 400$ A; the rate of water flow $G = 0.1$ kg/s; the water delivery pressure $p = 16$ mPa; water temperature at the tube inlet $t_{f1} = 300$°C; the temperature of the outer surface of the tube at a distance $l = 600$ mm from the inlet $t_{w.o} = 350$°C.

Assume the electric resistance and thermal conductivity to be constant and equal respectively to $\rho = 0.85$ Ohm-mm²/m. and $\lambda = 19.8$ W/m·°C.

Answer

$$\alpha_{ex} = 48\,500 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

5-50. Determine the error admitted in calculating the heat-transfer coefficient based on test results, under conditions assumed in problem 5-49, if: (a) the temperature drop across the wall of the test tube is not accounted for; and (b) the thermal conductivity of the material of the tube is overestimated by 10 per cent, i.e. it is assumed that $\lambda = 21.8$ W/m·°C.

Compare the value of α_{ex}, based on test results, with the results obtained calculating by formula (5-7), i.e. with α_{cal}.

Answer

(a) $\alpha = 30\,000$ W/m²·°C; the error is equal to 38 per cent.
(b) $\alpha = 45\,600$ W/m²·°C; the error is equal to 5 per cent.
$\alpha_{cal} = 50\,300$ W/m²·°C; divergence by 3 per cent.

5-51. A square-section channel with a side $a = 10$ mm and length $l = 1600$ mm carries water flowing with a velocity $w = 4$ m/s. Calculate the coefficient of heat transfer from the wall of the channel to water, if the lengthwise mean water temperature $t_f = 40°\text{C}$ and the temperature of the inner surface of the channel $t_w = 90°\text{C}$.

Answer

$$\alpha = 20\,300 \ \text{W/m}^2\cdot°\text{C}.$$

Solution

At the mean temperature $t_f = 40°\text{C}$ the physical properties of water are as follows:

$\nu_f = 0.659 \times 10^{-6}\ \text{m}^2/\text{s}$; $\lambda_f = 0.634\ \text{W/m}\cdot°\text{C}$; $\text{Pr}_f = 4.3$.

At the wall temperature $t_w = 90°\text{C}$ $\text{Pr}_w = 1.95$.

The equivalent diameter of the channel

$$d_{eq} = 4\,\frac{f}{u} = \frac{4a^2}{4a} = a = 0.01\ \text{m},$$

where f = cross-sectional area of the channel, m^2;
u = perimeter of the channel, m.

The Reynolds number

$$\text{Re}_f = \frac{wd_{eq}}{\nu_f} = \frac{4 \times 0.01}{0.659 \times 10^{-6}} = 6.07 \times 10^4 > 10^4.$$

The water is in turbulent flow.

For fluids (liquids) with Prandtl numbers $\text{Pr} \geqslant 0.7$ in turbulent flow in channels with a non-circular cross section heat transfer can be approximately calculated by formula (5-7), introducing the equivalent diameter as the reference dimension. Consequently,

$$\text{Nu}_f = 0.021\,\text{Re}_f^{0.8}\,\text{Pr}_f^{0.43} = \left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25} =$$

$$= 0.021\,(6.07 \times 10^4)^{0.8}\,(4.3)^{0.43}\left(\frac{4.3}{1.95}\right)^{0.25} = 320$$

and the heat-transfer coefficient

$$\alpha = \text{Nu}_f\,\frac{\lambda_f}{d_{eq}} = 320\,\frac{0.634}{0.01} = 20\,300\ \text{W/m}^2\cdot°\text{C}.$$

5-52. How will change the heat-transfer coefficient and the amount of heat through length of 1 m of the channel under conditions of problem 5-51, if the square-section channel is replaced with: (a) a plate-type channel with sides in a 1 : 25 proportion; (b) a channel with an equilateral-triangular section, with the cross-sectional area of the channel and water flow velocity remaining unchanged?

Answer

(a) the heat-transfer coefficient will increase by about 20 per cent. The amount of heat transferred through length of 1 m of the channel will increase 3.16 times.

(b) the local heat-transfer coefficient will increase by 2.5 per cent, and the amount of heat transferred, by 17 per cent.

5-53. The external annular channel of a tube-in-tube heat exchanger (Fig. 5-8) carries water flowing with a velocity $w = 3$ m/s. The mean temperature of the water along the channel $t_f = 40$°C.

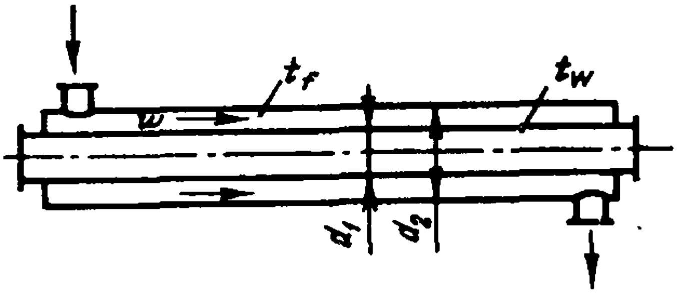

Fig. 5-8. Solution to problem 5-53

Determine the lengthwise-mean heat-transfer coefficient and the total amount of heat transferred in the heat exchanger, if the temperature of the outer surface of the inside tube $t_w = 70$°C. The outer and inner diameters of the annular channel are respectively $d_2 = 26$ mm and $d_1 = 20$ mm; the length of the channel $l = 1.4$ m.

Answer

$$\alpha = 7600\ \text{W/m}^2\cdot{}^\circ\text{C};\quad Q = 20\ \text{kW}.$$

Solution

At a mean water temperature $t_f = 40$°C $\nu_f = 0.659 \times 10^{-6}\ \text{m}^2/\text{s}$. The equivalent diameter of the annular channel (annulus)

$$d_{eq} = \frac{4f}{u} = d_2 - d_1 = 26 - 20 = 6\ \text{mm}$$

and

$$\text{Re}_f = \frac{wd_{eq}}{\nu_f} = \frac{3 \times 6 \times 10^{-3}}{0.659 \times 10^{-6}} = 27\,300 > 10^4.$$

The mean heat-transfer coefficient on the inner surface of the wall, in the event of turbulent fluid flow (of liquids and gases) in annular channels, can be calculated by the following formula [4]:

$$\mathrm{Nu}_f = 0.017\,\mathrm{Re}_f^{0.8}\,\mathrm{Pr}_f^{0.4}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25}\left(\frac{d_2}{d_1}\right)^{0.18}, \qquad (5\text{-}12)$$

where the subscripts "f" and "w" indicate that the physical properties of the fluid (liquid or gas) are, respectively, selected by the mean arithmetic temperature t_f and the wall temperature t_w; taken as the reference dimension is the equivalent diameter $d_{eq} = d_2 - d_1$ (d_1 and d_2 are the inner and outer diameters of the annular channel, respectively).

Formula (5-12) is valid at d_2/d_1 ranging from 1.2 to 1.4; l/d_{eq} ranging from 50 to 460 and Pr_f falling within 0.7 to 100.

In the case considered:

$$\frac{d_2}{d_1} = \frac{26}{20} = 1.3; \quad \frac{l}{d_{eq}} = \frac{2}{6 \times 10^{-3}} = 333$$

so that calculations are done with the aid of formula (5-12).

At $t_f = 40°\mathrm{C}$ $\mathrm{Pr}_f = 4.31$; $\lambda_f = 0.635$ W/m·°C.

At $t_w = 70°\mathrm{C}$ $\mathrm{Pr}_w = 2.55$.

$$\mathrm{Nu}_f = 0.017\,(2.73 \times 10^4)^{0.8}\,(4.31)^{0.4}\left(\frac{4.31}{2.55}\right)^{0.25}\left(\frac{26}{20}\right)^{0.18} = 72;$$

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d_{eq}} = 72\,\frac{0.635}{6 \times 10^{-3}} = 7600\ \mathrm{W/m^2 \cdot °C}.$$

$$Q = \alpha\,(t_w - t_f)\,\pi d l = 7600\,(70-40)\,3.14 \times 2 \times 10^{-2} \times \times\, 1.4 = 20\ \mathrm{kW}.$$

5-54. How will change the local heat-transfer coefficient and the amount of heat transferred in the heat exchanger, under the conditions of problem 5-53, if the outer diameter of the annulus $d_2 = 32$ mm, i.e. if the width of the annulus is doubled, on condition that (a) water flow velocity and all other conditions remain unchanged; (b) the rate of water flow and all other conditions remain unchanged?

Answer

(a) $\alpha_a = 6900$ W/m²·°C; $\alpha_a/\alpha = 0.91$; $Q = 18.2$ kW.

(b) $\alpha_b = 3580$ W/m²·°C; $\alpha_b/\alpha = 0.47$; $Q = 9.4$ kW.

5-55. A boiler bank of tubes is exposed to a longitudinal flow of flue gas. The tubes of the bank have an outside diameter $d = 80$ mm and length $l = 3$ m. The tubes are in-line arranged with pitches $s_1 = 200$ mm and $s_2 = 200$ mm

(Fig. 5-9). The mean temperature of the flue gas $t_f = 750°C$; the mean temperature of the outside surface of the tubes $t_w = 250°C$, and the mean velocity of gas flow $w = 6$ m/s. The volumetric composition of the flue gas (relative partial pressures) $\bar{P}_{CO_2} = 13$, $\bar{P}_{H_2O} = 11$, $\bar{P}_{N_2} = 76$.

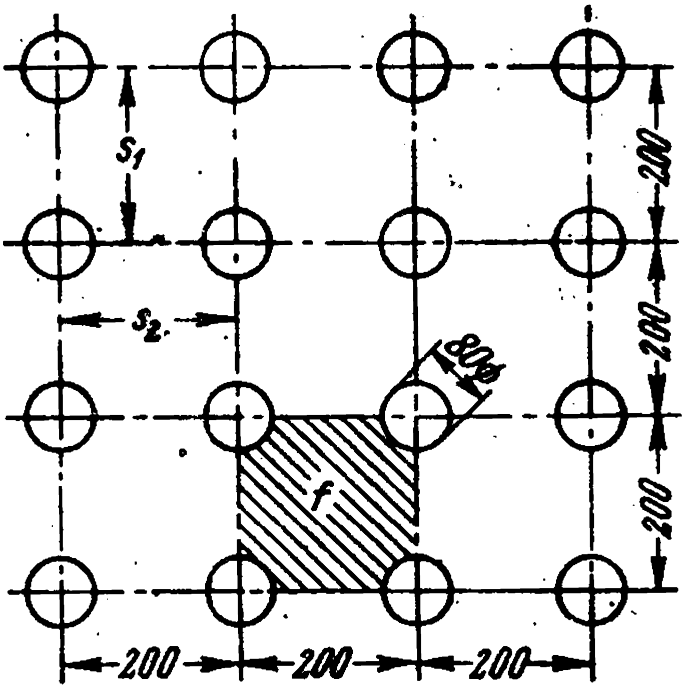

Fig. 5-9. Solution to problem 5-55

Determine the local coefficient of heat transfer by convection from the flue gas to the heating surface of the tube bank.

Answer

$$\alpha = 13 \text{ W/m}^2\cdot°\text{C}.$$

Solution

The equivalent diameter of the bank of tubes

$$d_{eq} = \frac{4f}{u} = \frac{4\left(s_1 s_2 - \frac{\pi d^2}{4}\right)}{\pi d} = 4\frac{s_1 s_2}{\pi d} - d.$$

Hence,

$$d_{eq} = 4\frac{0.2 \times 0.2}{3.14 \times 0.08} - 0.08 = 0.557 \text{ m}.$$

At the mean temperature $t_f = 750°\mathrm{C}$ the physical properties of the flue gas of the given composition are respectively:

$$\nu_f = 122 \times 10^{-6}\ \mathrm{m^2/s};\quad \lambda_f = 8.71 \times 10^{-2}\ \mathrm{W/m\cdot °C};$$

$$\mathrm{Pr}_f = 0.6$$

and the Reynolds number

$$\mathrm{Re}_f = \frac{wd_{eq}}{\nu_f} = \frac{6 \times 0.557}{122 \times 10^{-6}} = 2.74 \times 10^4 > 10^4.$$

With gases in turbulent flow in tubes and conduits and in the case of longitudinal flow past a bank of tubes heat transfer can be calculated with the aid of formula (5-7), but, then, the correction for the variation of physical properties with temperature, $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}$, is not valid.

For gases the effect of the variation of physical properties on the intensity of heat transfer can be accounted for by introducing the so-called temperature factor [8]:

$$\Theta = \frac{T_w}{T_f} = \frac{t_w + 273}{t_f + 273};$$

$$\left.\begin{array}{l}\text{with cooling and } 0.5 \leqslant \Theta \leqslant 1 \\ \qquad \mathrm{Nu} = \mathrm{Nu}_f(1.27 - 0.27\Theta); \\ \text{with heating and } 1 \leqslant \Theta \leqslant 3.5 \\ \qquad \mathrm{Nu} = \mathrm{Nu}_f \Theta^{-0.55},\end{array}\right\} \qquad (5.13)$$

where Nu_f is the Nusselt number at constant physical properties:

$$\mathrm{Nu}_f = 0.021 \mathrm{Re}_f^{0.8} \mathrm{Pr}_f^{0.43} \varepsilon_l.$$

In the case considered

$$\frac{l}{d_{eq}} = \frac{3}{0.557} = 5.4$$

and from the table for formula (5-7) we find $\varepsilon_l = 1.24$. Hence,

$$\mathrm{Nu}_f = 0.021\ (2.74 \times 10^4)^{0.8} \times 0.6^{0.43} \times 1.24 = 73.8;$$

$$\Theta = \frac{250 + 273}{750 + 273} = 0.51;$$

$$\mathrm{Nu} = \mathrm{Nu}_f\,(1.27 - 0.27\,\theta) = 73.3\,(1.27-0.27 \times 0.51)=83,$$

and the heat-transfer coefficient

$$\alpha = \mathrm{Nu}\frac{\lambda_f}{d_{eq}} = 83\,\frac{0.0871}{0.557} = 13\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

5-56. Under conditions of problem 5-55 how will change the coefficient of heat transfer by convection, if the pitch s_1 is increased two and four times. The pitch s_2 and all other conditions of the problem remain unchanged.

Answer

$$\alpha_{2s1} = 11.3\ \mathrm{W/m^2 \cdot {}^\circ C};\quad \alpha_{4s1} = 9.9\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

5-57. Under conditions of problem 5-55 how will change the local coefficient of heat transfer by convection, if the diameter of the tubes is diminished two times. All other conditions of the problem remain unchanged.

Answer

$$\alpha' = 11.3\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

5-58. A tube having an inside diameter $d = 46$ mm carries air flowing at a high velocity. The rate of air flow $G = 0.2$ kg/s.

The thermodynamic temperature of the air at the tube inlet $t_1 = 1200$°C. The tube wall temperature $t_w = 350$°C. The inlet air pressure $p_1 = 750$ mm Hg and the outlet air pressure $p_2 = 510$ mm Hg.

How long should the tube be, to ensure a thermodynamic outlet temperature $t_2 = 750$°C? Also determine the Mach numbers at the inlet and outlet of the tube.

Answer

$$l = 2.5\ \mathrm{m};\quad M_1 = 0.66;\quad M_2 = 0.81.$$

Solution

The inlet and outlet air pressure

$$p_1 = 750 \times 13.6 \times 9.81 = 1 \times 10^5\ \mathrm{Pa};\quad p_2 = 510 \times 13.6 \times 9.81 = 6.8 \times 10^4\ \mathrm{Pa}.$$

The density of air at the tube inlet

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1 \times 10^5}{287.4\,(1200+273)} = 0.236\ \mathrm{kg/m^3},$$

and at the tube outlet

$$\rho_2 = \frac{p_2}{RT_2} = \frac{6.8\times 10^4}{287.4\,(750+273)} = 0.231 \text{ kg/m}^3,$$

where for air the gas constant R = 287.4 J/kg·°C.

The air velocity at the inlet

$$w_1 = \frac{4G}{\rho_1 \pi d^2} = \frac{4\times 0.2}{0.236\times 3.14\,(4.6\times 10^2)^2} = 520 \text{ m/s},$$

at the outlet

$$w_2 = w_1 \frac{\rho_1}{\rho_2} = 510\ \frac{0.236}{0.231} = 510 \text{ m/s}.$$

The velocity of sound and the Mach numbers:

$$a_1 = \sqrt{kRT_1} \approx 20.1\sqrt{T_1} = 20.1\sqrt{1473} = 770 \text{ m/s};$$

$$M_1 = \frac{w_1}{a_1} = \frac{510}{770} \approx 0.66;$$

$$a_2 = 20.1\sqrt{T_2} = 20.1\sqrt{1023} = 642 \text{ m/s};$$

$$M_2 = \frac{w_2}{a_2} = \frac{520}{642} \approx 0.81.$$

Stagnation temperature

$$\nu_1 = T_1 + \frac{w_1^2}{2c_{p1}} = 1473 + \frac{510^2}{2\times 1210} = 1473 + 107 = 1580 \text{ K},$$

where at t_1 = 1200°C c_{p1} = 1.21 kJ/kg·°C

$$\nu_2 = T_2 + \frac{w_2^2}{2c_{p2}} = 1023 + \frac{520^2}{2\times 1145} = 1023 + 118 = 1141 \text{ K},$$

where at t_2 = 750°C c_{p2} = 1.145 kJ/kg·°C.

The logarithmic mean difference between stagnation temperatures

$$\Delta\nu_{\log} = \frac{\nu_1 - \nu_2}{2.3\log\dfrac{\nu_1 - T_w}{\nu_2 - T_w}} = \frac{1580 - 1141}{2.3\log\dfrac{1580-623}{1141-623}} = 717\ ^\circ\text{C}.$$

The mean arithmetic air temperature

$$\bar{T} - 0.5\,(T_1 + T_2) = 0.5\,(1473 + 1023) = 1250 \text{ K}$$

or

$$\bar{t} = 1250 - 273 = 977^\circ\text{C}.$$

Under the given conditions the heat-transfer coefficient can be calculated approximately by formula (5-13), where taken as the reference temperature is $\bar{t}$, and the heat-transfer coefficient is related to the temperature difference $\Delta v_{\log}$. At $\bar{t} = 977°\mathrm{C}$ $\lambda_f = 7.97 \times 10^{-2}$ W/m·°C, $\mu_f = 48.5 \times 10^{-6}$ Pa·s, $c_{pf} = 1.182$ kJ/kg·°C, and the Prandtl number $\mathrm{Pr}_f = 0.719$.

$$\mathrm{Re}_f = \frac{4G}{\pi d \mu_f} = \frac{4\times 0.2}{3.14\times 4.6\times 10^{-2}\times 48.5\times 10^{-6}} = 1.14\times 10^5.$$

Assuming that the ratio $l/d > 50$, we find:

$$\mathrm{Nu}_f = 0.021\mathrm{Re}_f^{0.8}\mathrm{Pr}_f^{0.43} = 0.021(1.14\times 10^5)^{0.8}(0.719)^{0.43} = 204.$$

The correction for the temperature factor by formula (5-14): with cooling of the gas

$$\mathrm{Nu} = \mathrm{Nu}_f\,(1.27-0.27\Theta),$$

where

$$\Theta = \frac{T_w}{\bar{T}} = \frac{623}{1250} \approx 0.5.$$

Thus,

$$\mathrm{Nu} = 204\,(1.27-0.27\times 0.5) = 204\times 1.13 = 231;$$

$$\alpha = \mathrm{Nu}\frac{\lambda_f}{d} = 231\,\frac{7.97\times 10^{-2}}{4.6\times 10^{-2}} = 400 \text{ W/m}^2\cdot°\mathrm{C}.$$

The rate of heat flow

$$q = \alpha\Delta v_{\log} = 400\times 717 = 2.87\times 10^5 \text{ W/m}^2;$$

The amount of heat transferred

$$Q = Gc_{pf}\,(v_1 - v_2) = 0.2\times 1185\,(1580 - 1141) = 1.04\times 10_5 \text{ W}.$$

The heat-transfer surface

$$F = \frac{Q}{q} = \frac{1.04\times 10^5}{2.87\times 10^5} = 0.362 \text{ m}^2.$$

The unknown length of the tube

$$l = \frac{F}{\pi d} = \frac{0.362}{3.14\times 4.6\times 10^{-2}} = 2.5 \text{ m}.$$

5-59. A tube having a diameter $d = 14$ mm and length $l = 900$ mm carries mercury flowing with velocity $w = 2.5$ m/s. The mean temperature of mercury $t_f = 250°\mathrm{C}$.

Determine the coefficient of heat transfer from the mercury to the wall of the tube, the rate of heat flow and the quantity of heat transferred per unit time, on condition that the mean temperature of the wall $t_w = 220°C$.

Answer

$\alpha = 24\,000$ W/m$^2\cdot$°C; $q = 7.2 \times 10^5$ W/m^2; $Q = 28.4$ kW.

Solution

At the mean mercury temperature $t_f = 250°C$ its physical properties are:

$\nu_f = 7.55 \times 10^{-8}$ m^2/s; $\lambda_f = 11$ W/m$\cdot$°C; $\mathrm{Pr}_f = 1.24 \times 10^{-2}$.

The Reynolds number

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{2.5 \times 14 \times 10^{-3}}{7.55 \times 10^{-8}} = 4.63 \times 10^5 > 10^4.$$

The mercury is in turbulent flow.

With pure liquid metals in turbulent flow in tubes the heat-transfer coefficient can be calculated by the following formula [16]:*

$$\mathrm{Nu}_f = 5 + 0.025\mathrm{Pe}_f^{0.8}. \qquad (5.14)$$

In the case considered

$$\mathrm{Pe}_f = \mathrm{Re}_f\mathrm{Pr}_f = 4.63 \times 10^5 \times 1.24 \times 10^{-2} = 5750;$$

$$\mathrm{Nu}_f = 5 + 0.025\,(5750)^{0.8} = 30.5;$$

$$\alpha = \mathrm{Nu}_f\frac{\lambda_f}{d} = 30.5\frac{11}{14 \times 10^{-3}} = 24\,000 \text{ W/m}^2\cdot\text{°C}.$$

The rate of heat flow

$$q = \alpha\,(t_f - t_w) = 2.4 \times 10^4\,(250 - 220) = 7.2 \times 10^5 \text{ W/m}^2.$$

The amount of heat transferred

$$Q = q\pi dl = 7.2 \times 10^5 \times 3.14 \times 14 \times 10^{-3} \times 0.9 = 28.4 \text{ kW}.$$

5-60. The heating surface of the heat-exchange apparatus of an atomic power plant is built-up of tubes having the inside diameter $d = 12$ mm and length $l = 2400$ mm. The tubes carry liquid sodium at the mean temperature $t_f = 400°C$, flowing with the mean velocity $w = 2.5$ m/s.

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* In the case of contaminated liquid metals and in the presence of an additional contact resistance the heat-transfer coefficient diminishes, and it can be calculated by formula $\mathrm{Nu}_f = 4.5 + 0.014\mathrm{Pe}_f^{0.8}$.

Determine the coefficient of heat transfer from the sodium coolant to the wall of a tube.

Answer

$$\alpha = 46\,000 \text{ W/m}^2\cdot°\text{C}.$$

5-61. Under conditions of problem 5-60 determine the heat-transfer coefficient if, instead of sodium, lithium or an alloy is used, of a composition 25% Na + 75% K.

Answer

$$\alpha_{Li} = 47\,400 \text{ W/m}^2\cdot°\text{C}; \quad \alpha_{Na+K} = 23\,600 \text{ W/m}^2\cdot°\text{C}.$$

5-62. In an experimental plant, used to determine heat transfer in liquid metals, a tube having a diameter $d = 12$ mm and length $l = 1$ m carries liquid bismuth. The tube is heated with the aid of an electric heater; the rate of heat flow on the wall is constant along the tube and amounts to $q_w = 6 \times 10^5$ W/m².

Determine the temperature of the wall at the tube outlet, if bismuth inlet temperature $t_{f1} = 300°C$ and the rate of bismuth flow $G = 2.2$ kg/s.

Answer

$$t_{w2} = 396°\text{C}.$$

Solution

At a constant rate of heat transfer through the wall of the tube the bismuth outlet temperature is determined from equation

$$t_{f2} = t_{f1} + \frac{q_w \pi dl}{G c_{pf}}.$$

The heat capacity of bismuth depends but slightly on temperature; at $t_{f1} = 300°C$ $c_{pf1} = 151$ kJ/kg·°C.

Substituting the known values, we find:

$$t_{f2} = 300 + \frac{6 \times 10^5 \times 3.14 \times 1.2 \times 10^{-2} \times 1.0}{2.2 \times 151} = 300 + 68 = 368\ °\text{C}.$$

At the outlet temperature $t_{f2} = 368°C$ the physical properties of bismuth are as follows: $\mu_{f2} = 15 \times 10^{-4}$ Pa·s, $\lambda_{f2} = 14$ W/m·°C, $c_{pf2} = 151$ J/kg·°C, $\text{Pr}_{f2} = 1.62 \times 10^{-2}$.

The Reynolds number at the tube outlet

$$\mathrm{Re}_{f2}=\frac{4G}{\pi d\mu_{f2}}=\frac{4\times 2.2}{3.14\times 1.2\times 10^{-2}\times 15\times 10^{-4}}=$$
$$=1.56\times 10^{5}>10^{4};$$

the pattern of flow is turbulent, and the coefficient of heat transfer at the tube outlet is calculated with the aid of formula (5-14):

$$\mathrm{Pe}_{f2}=\mathrm{Re}_{f2}\mathrm{Pr}_{f2}=1.56\times 10^{5}\times 1.62\times 10^{-2}=2530;$$
$$\mathrm{Nu}_{f2}=5\times 0.025\,(2.53\times 10^{3})^{0.8}=18.2;$$
$$\alpha_2=\mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d}=18.2\,\frac{14}{12\times 10^{-3}}=21\,200\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The temperature of the wall at the tube outlet

$$t_{w2}=t_{f2}+\frac{q_w}{\alpha_2}=368+\frac{6\times 10^{5}}{2.12\times 10^{4}}=368+28.3=396.3\,^\circ\mathrm{C}.$$

5-63. A tube having an inside diameter $d = 10$ mm and length $l = 1$ m carries sodium. The tube is heated with an electric heater; the rate of heat flow is constant along the tube and amounts to $q_w = 1 \times 10^6$ W/m². The temperature of sodium at the tube inlet $t_{f1} = 300$°C.

What should the rate of liquid sodium flow be, to ensure a wall temperature t_w, at the end of the tube not exceeding 400°C?

Answer

$$G \geqslant 0.3\ \mathrm{kg/s}.$$

5-64. Compare the Nusselt numbers and the heat-transfer coefficients calculated for water, air and liquid sodium in turbulent flow in a round tube at Reynolds numbers ranging from 10^4 to 10^6.

Carry out the comparison considering flow in a tube having a diameter $d = 20$ mm; water flows with a mean temperature along the tube $t_f = 20$°C; air flows at atmospheric pressure and a mean temperature $t_f = 100$°C; and the mean temperature of sodium $t_f = 350$°C.

Answer

Calculation results are tabulated below and presented graphically in Fig. 5-10.

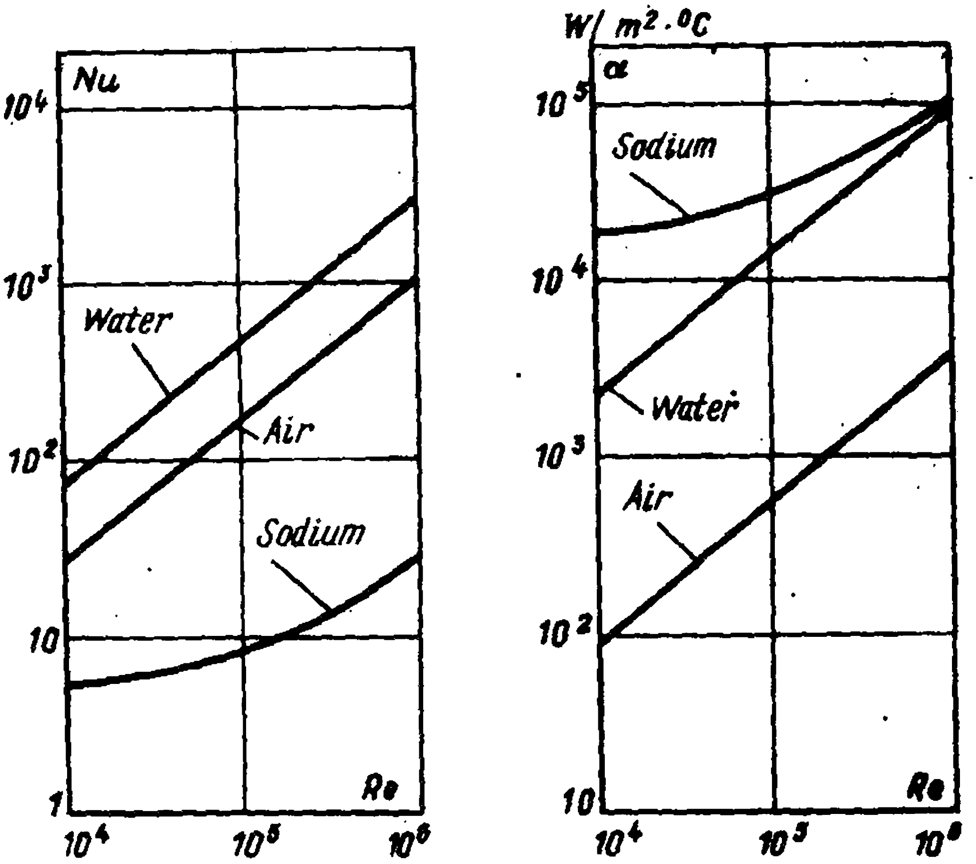

Fig. 5-10. Solution to problem 5-64

	Nu			α, W/m²·°C		
Re	10^4	10^5	10^6	10^4	10^5	10^6
Water	77.3	487	3070	2310	14 600	91 800
Air	28.4	179	1130	91.3	575	3 630
Sodium	5.66	9.16	31.3	19 000	30 800	105 000

5-65. A horizontal steel pipeline having a diameter $d_1/d_2 = 50/57$ mm carries water flowing with a velocity $w = 0.15$ m/s. The mean water temperature $t_{f1} = 100$°C. The pipeline is lagged (insulated) with asbestos, the outside diameter of the lagging $d_3 = 89$ mm (Fig. 5-11).

Determine the loss of heat from length of 1 m of the pipeline q_l, W/m, if the temperature of the still air surrounding

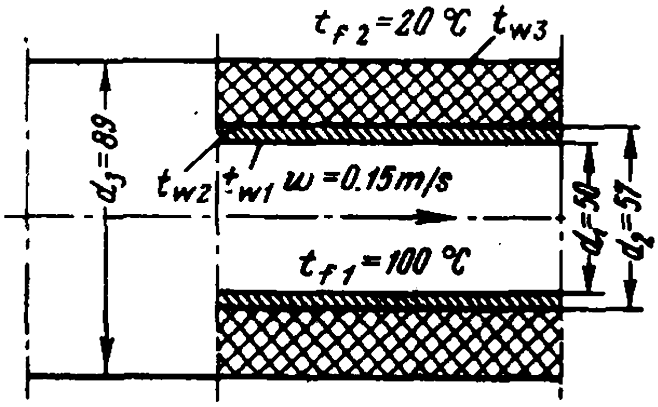

Fig. 5-11. Solution to problem 5-65

the pipeline $t_{f2} = 20°C$. Also determine the surface temperatures of the pipeline and asbestos lagging t_{w1}, t_{w2} and t_{w3}.

Answer

$$q_l = 66 \text{ W/m}; \quad t_{w1} \approx t_{w2} \approx 99.7°C; \quad t_{w3} = 59.3°C.$$

Solution

The coefficient of heat transfer from water to the inner surface of the pipeline:

at $t_{f1} = 100°C$ $\nu_1 = 0.295 \times 10^{-6}$ m²/s, $\lambda_{f1} = 0.683$ W/m·°C, $Pr_{f1} = 1.75$;

$$Re_{f1} = \frac{wd_1}{\nu_{f1}} = \frac{0.15 \times 5 \times 10^{-2}}{0.295 \times 10^{-6}} = 2.54 \times 10^4.$$

Water is in turbulent flow and calculation formula (5-7) is used.

Taking into account that the coefficient of heat transfer from water to the inside surface of the tube will exceed considerably the local coefficient of heat transfer from the outer surface of the insulating lagging to the surrounding air, and allowing for the presence of the asbestos insulation, we can assume that $t_{w1} \approx t_{f1}$ and, consequently, $Pr_{w1} \approx Pr_{f1}$, then

$$Nu_{f1} = 0.021\, Re_f^{0.8} Pr_f^{0.43} = 0.021(2.54 \times 10^4)^{0.8}\, 1.75^{0.43} = 90$$

and

$$\alpha_1 = \mathrm{Nu}_{f1}\frac{\lambda_{f1}}{d_1} = 90\,\frac{0.683}{5\times 10^{-2}} = 1230\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Heat is transferred from the outside surface of the insulation to the surrounding air by free convection. Since the heat-transfer coefficient depends then on the temperature difference $\Delta t = t_{w3} - t_{f2}$, and the temperature t_{w3} is unknown, the calculations are carried out by the method of successive approximations.

Allowing for $\alpha_1 \gg \alpha_2$, we assume as the first approximation $t_{w3} = 65^\circ\mathrm{C}$.

The heat-transfer coefficient is determined by formula (7-1) derived for a horizontal tube:

$$\mathrm{Nu}_f = 0.5\,(\mathrm{GrPr})_f^{0.25}.$$

At. $t_{f2} = 20^\circ\mathrm{C}$

$$\beta_{f2} = \frac{1}{293 + t_{f2}} = \frac{1}{293}\,K^{-1};$$

$$\nu_{f2} = 15.06\times 10^{-6}\ \mathrm{m^2/s};\quad \lambda_{f2} = 2.59\times 10^{-2}\ \mathrm{W/m\cdot{}^\circ C};$$

$$\mathrm{Pr}_{f2} = 0.703;$$

$$(\mathrm{GrPr})_{f2} = g\beta_{f2}\frac{(t_{w3} - t_{f2})}{\nu_{f2}^2}\mathrm{Pr}_{f2} =$$

$$= \frac{9.81(65-20)\,(8.9\times 10^{-2})^3}{293\,(15.06\times 10^{-6})^2}\,0.703 = 3.3\times 10^6;$$

$$\mathrm{Nu}_{f2} = 0.5\,(3.3\times 10^6)^{0.25} = 21.2;$$

$$\alpha_2 = \mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d_3} = 21.2\,\frac{2.59\times 10^{-2}}{8.9\times 10^{-2}} = 6.17\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The overall heat-transfer coefficient (first approximation

$$k_1 = \frac{1}{\dfrac{1}{\alpha_1 d_1} + \dfrac{1}{2\lambda_1}\dfrac{\ln d_2}{d_1} + \dfrac{1}{2\lambda_2}\dfrac{\ln d_3}{d_2} + \dfrac{1}{\alpha_2 d_3}} =$$

$$= \frac{1}{\dfrac{1}{1230\times 5\times 10^{-2}} + {}^{2}\dfrac{2.3}{\times 46}\log\dfrac{57}{50} + \dfrac{2.3}{2\times 0.116}\log\dfrac{89}{57} + \dfrac{1}{6.17\times 8.9\times 10^{-2}}} =$$

$$= 0.266\ \mathrm{W/m\cdot{}^\circ C},$$

where for steel $\lambda_1 = 46\ \mathrm{W/m\cdot{}^\circ C}$; and for asbestos $\lambda_2 = 0.116\ \mathrm{W/m\cdot{}^\circ C}$.

With $k_l = 0.266$ W/m·°C the rate of heat flow from length of 1 m of the tube will amount to:

$$q_l = k_l \pi (t_{f1} - t_{f2}) = 0.266 \times 3.14 (100-20) = 67 \text{ W/m},$$

where as a first approximation, the temperature t_{w3} will be:

$$t_{w3} = t_{f2} + \frac{q_l}{\pi} \frac{1}{\alpha_2 d_3} =$$

$$= 20 + \frac{67}{3.14 \times 6.17 \times 8.9 \times 10^{-2}} = 58.9 \text{ °C}.$$

As the second approximation we assume $t_{w3} = 59$°C, then $(\mathrm{GrPr})_{f2} = 2.86 \times 10^6$; $\mathrm{Nu}_{f2} = 20.6$; $\alpha_2 = 6.0$ W/m²·°C.

Substituting into the formula for k_l this value of α_2, we get:

$$k_l = 0.263.$$

Since, as the second approximation, the value of the overall heat-transfer coefficient coincides practically with the previously obtained value, no further calculations are necessary.

Now we find the rate of heat flow and surface temperatures:

$$q_l = 0.263 \times 3.14 \times 80 = 66 \text{ W/m};$$

$$t_{w3} = t_{f2} + \frac{q_l}{\pi} \frac{1}{\alpha_2 d_3} =$$

$$= 20 + \frac{66}{3.14 \times 6 \times 8.9 \times 10^{-2}} = 59.3 \text{ °C};$$

$$t_{w2} = t_{w3} + \frac{q_l}{\pi} \frac{1}{2\lambda_2} \ln \frac{d_3}{d_2} =$$

$$= 59.3 + \frac{66}{3.14 \times 2 \times 0.116} 2.3 \log \frac{89}{57} \approx 99.7 \text{ °C};$$

$$t_{w1} = t_{w2} + \frac{q_l}{\pi} \cdot \frac{1}{2\lambda_1} \ln \frac{d_2}{d_1} =$$

$$= 99.7 + \frac{66}{3.14 \times 2 \times 46} 2.3 \log \frac{57}{50} \approx 99.73 \text{ °C}.$$

5-66. How will change the loss of heat q_l, W/m, and the temperature of the outside surface of the lagging t_{w3}, under conditions of problem 5-65, if the thickness of the asbestos lagging is doubled, and all other conditions remain unchanged?

Answer

$q_l = 51.5$ W/m. The loss of heat from the surface of the asbestos lagging will diminish by about 22 per cent; t_{w3} will be 46.8°C, i.e. the temperature of the outside surface of the insulation will decrease by about 12.5°C.

5-67. A pipe having a diameter $d_i = 12$ mm carries water at a pressure $p = 24$ MPa. The rate of water flow $G = 0.15$ kg/s. The bulk temperature of water in the section x at a distance $x > 15\, d$ from the inlet into the heated section of the tube $t_{fx} = 380$°C.

Determine the local heat-transfer coefficient α_x and the local value of the rate of heat flow on the wall, q_{wx}, in the section of the tube considered, if it is known that the local temperature of the wall in this section $t_{wx} = 390$°C.

It may be assumed that heat transfer takes place under regular-regime conditions, i.e. that $q/\rho w < (q/\rho w)_{cr}$ and, consequently, there is no local deterioration of heat transfer (see problem 5-70) and free convection is immaterial for the process of heat transfer.

Compare the value of α_x with the heat-transfer coefficient α_0, calculated at constant physical properties by the formula which is valid for the region at a distance from the critical $(p \ll p_{cr})$.

Answer

$\alpha_x = 3.3 \times 10^4$ W/m²·°C; $q_{wx} = 3.3 \times 10^5$ W/m²;

$\alpha_x/\alpha_0 \approx 0.7$.

Solution

With water in turbulent flow in tubes at supercritical pressure under heating conditions heat transfer can be calculated with the aid of the following formula [5]:

$$\mathrm{Nu}_f = \mathrm{Nu}_0 \left(\frac{\bar{c}_p}{c_{pf}}\right)^{0.4} \left(\frac{\rho_w}{\rho_f}\right)^m, \tag{5-15}$$

where Nu_0 is the Nusselt number at constant physical properties of the fluid considered, calculated by formula (5-8),

$$\mathrm{Nu}_0 = \frac{\frac{\xi}{8}\mathrm{Re}_f\mathrm{Pr}_f}{12.7\sqrt{\frac{\xi}{8}}(\mathrm{Pr}_f^{2/3} - 1) + 1.07};$$

where $\xi = (1.82 \log \mathrm{Re} - 1.64)^{-2}$ is the friction factor;

$$\bar{c}_p = \frac{i_w - i_f}{t_w - t_f}$$

is the integral mean heat capacity in the temperature interval from t_f to t_w; i_w, i_f are the enthalpies of the fluid respectively at temperatures t_w and t_f;

$$m = 0.35 - 0.05 \frac{p}{p_{cr}},$$

where $p_{cr} = 22.12$ MPa is the critical pressure of water.

Formula (5-15) is valid for water at $x/d \geqslant 15$, $1.03 \leqslant p/p_{cr} \leqslant 1.4$, $0.6 \leqslant T_f/T_m \leqslant 1.2$, $0.6 \leqslant T_w/T_m \leqslant 1.2$, when free convection does no affect materially the process of heat transfer, which agrees with the condition $\mathrm{Gr}/\mathrm{Re}^2 < 0.6$ for the so-called regular regimes of heat transfer as well.

T_f and T_w are the temperatures of the fluid (liquid or gas) and wall, measured in degrees K; T_m is the pseudocritical temperature, K, i.e. the temperature at which heat capacity is maximum at a given pressure.

When $T_f < T_m < T_w$ formula (5-15) is valid at $q_w/\rho_w < (q_w/\rho_w)_{cr}$, where $(q_w/\rho_w)_{cr}$ is the critical ratio of the rate of heat flow to mass velocity, and when the ratio q_w/ρ_w exceeds the critical, the local deterioration of heat transfer may take place.

Under the conditions assumed, at $p = 24$ MPa, the pseudocritical temperature $t_m = 380.7$°C; at $t_f = 380$°C the physical properties of water are as follows:

$v_f = 2.59 \times 10^{-3}$ m^3/kg; $i_f = 2028$ kJ/kg; $c_{pf} = 6838$ kJ/kg·°C;.

$\lambda_f = 0.269$ W/m·°C; $\mu_f = 4.68 \times 10^{-5}$ Pa·s; $\mathrm{Pr}_f = 11.9$.

At $t_{wx} = 390$°C $v_w = 5.619 \times 10^{-3}$ m^3/kg, $i_w = 2505$ kJ/kg.

The local Reynolds number

$$\mathrm{Re}_f = \frac{4G}{\pi d \mu_f} = \frac{4 \times 0.15}{\pi \times 12 \times 10^{-3} \times 4.68 \times 10^{-5}} = 3.4 \times 10^5;$$

the water is in turbulent flow.

The relative values of the wall and fluid temperatures are

$$\frac{T_f}{T_m}=\frac{380+273}{380.7+273}=\frac{653}{653.7}\approx 0.999;$$

$$\frac{T_w}{T_m}=\frac{663}{653.7}=1.03;$$

Thus, $T_f < T_m < T_w$. Taking into account the fact that the conditions of the problem prescribe $q_w/\rho_w < (q_w/\rho_w)_{cr}$ and $\mathrm{Gr}/\mathrm{Re}^2 < 0.6$, calculation formula (5-15) is used.

First we determine Nu_{0x} and α_{0x} by formula (5-18):

$$\xi=[1.82\log(3.4\times 10^5)-1.64]^{-2}=1.41\times 10^{-2};$$

$$\frac{\xi}{8}=1.76\times 10^{-3};\quad \sqrt{\frac{\xi}{8}}=4.2\times 10^{-2};$$

$$\mathrm{Nu}_{0x}=\frac{1.76\times 10^{-3}\times 3.4\times 10^5\times 11.9}{12.7\times 4.2\times 10^{-2}(11.9^{2/3}-1)+1.07}=2155;$$

$$\alpha_{0x}=\mathrm{Nu}_{0x}\frac{\lambda_f}{d}=2155\,\frac{0.269}{12\times 10^{-3}}=4.83\times 10^4\ \mathrm{W/m^2\cdot {}^\circ C}.$$

We determine the multiplier in formula (5-15), allowing for the effect of variation of physical properties of water over the cross section of the flow:

$$\varphi=\left(\frac{\bar{c}_p}{c_{pf}}\right)^{0.4}\left(\frac{\rho_c}{\rho_f}\right)^m;$$

$$\bar{c}_p=\frac{i_w-i_f}{t_w-t_f}=\frac{2505-2028}{390-380}=47.7\ \mathrm{kJ/kg\cdot {}^\circ C}.$$

$$\frac{\bar{c}_p}{c_{pf}}=\frac{47.7}{68.4}=0.70;$$

$$\frac{\rho_w}{\rho_f}=\frac{v_f}{v_w}=\frac{2.596\times 10^{-3}}{5.619\times 10^{-3}}=0.461;$$

$$m=0.35-\frac{0.05p}{p_{cr}}=0.35-0.05\,\frac{24}{22.12}=0.296;$$

$$\varphi=(0.7)^{0.4}(0.461)^{0.296}=0.69.$$

The local Nusselt number and the heat-transfer coefficient

$$\mathrm{Nu}_{fx}=\mathrm{Nu}_{0x}\varphi=2155\times 0.69=1485;$$

$$\alpha_x=\mathrm{Nu}_{fx}\frac{\lambda_f}{d}=1485\,\frac{0.269}{12\times 10^{-3}}=3.33\times 10^4\ \mathrm{W/m^2\cdot {}^\circ C}.$$

The local rate of heat flow

$$q_{wx} = \alpha_x(t_{wx} - t_{fx}) = 3.33 \times 10^4\ (390-380) = 3.33 \times 10^5\ \text{W/m}^2.$$

The ratio of the heat-transfer coefficients

$$\frac{\alpha_x}{\alpha_0} = 0.69.$$

5-68. Determine the local heat-transfer coefficient and the local rate of heat flow for water flowing in a tube at supercritical pressure, as treated in problem 5-67, if the local wall temperature in the section considered, $t_{wx} = 420°\text{C}$, and all other conditions of the problem remain unchanged as in problem 5-67. Compare calculation results with the answer to problem 5-67.

Answer

$$\alpha_x = 2.05 \times 10^4\ \text{W/m}^2\cdot°\text{C};\quad q_{wx} = 8.15 \times 10^5\ \text{W/m}^2;\quad \alpha_x/\alpha_0 = 0.43.$$

An increase in the temperature difference results in a lower heat-transfer coefficient, but the rate of heat flow on the wall rises.

5-69. Determine the local heat-transfer coefficient and the local rate of heat flow for water flowing in a tube at subcritical pressure under conditions of problem 5-67, but if the water is at a pressure $p = 30$ MPa.

Calculate for the same rate of water flow $G = 0.15$ kg/s, assuming a water temperature $t_f = 400°\text{C}$ and a wall temperature $t_w = 440°\text{C}$, i.e. at about the same ratios T_f/T_m and T_w/T_m as in problem 5-68. Compare calculation results with the answer to problem 5-68.

Answer

$$\alpha_x = 1.9 \times 10^4\ \text{W/m}^2\cdot°\text{C};\quad q_{wx} = 7.6 \times 10^5\ \text{W/m}^2;\quad \alpha_0 = 3 \times 10^4\ \text{W/m}^2\cdot°\text{C}.$$

Thus, at about the same Reynolds numbers Re_f and ratios T_f/T_m and T_w/T_m at a pressure $p = 24$ MPa $(p/p_{cr} \approx 1.08)$ $\alpha_x/\alpha_0 \approx 0.43$, and at a pressure $p = 30$ MPa $(p/p_{cr} \approx 1.36)$ $\alpha_x/\alpha_0 \approx 0.64$.

5-70. Water, flowing through a heat exchanger at a rate $G = 1.0$ kg/s and a pressure $p = 24$ MPa, must be heated from a temperature $t_{f1} = 375°C$ to a temperature $t_{f2} = 390°C$. The water is in upward flow through $n = 20$ vertical tubes having a diameter $d = 10$ mm and length $l = 2.5$ m. Water is to be heated under the conditions of a constant rate of heat flow on the wall, $q_w =$ const.

Determine the magnitude of q_w, W/m², required, and check whether at this value of q_w the local deterioration of heat transfer may take place in any section along the tubes.

The effect of natural convection on heat transfer is to be ignored.

Answer

$q_w = 4 \times 10^5$ W/m². Heat transfer will not be impaired, since $q_w < q_{cr} \approx 5.7 \times 10^5$ W/m².

Solution

At $p = 24$ MPa and inlet and outlet temperatures $t_{f1} = 375°C$ and $t_{f2} = 390°C$ the inlet and outlet enthalpies of water are respectively:

$$i_{f1} = 1875 \text{ kJ/kg}, \quad i_{f2} = 2505 \text{ kJ/kg};$$

$$\Delta i = i_{f2} - i_{f1} = (2505-1875) \times 10^3 = 630 \times 10^3 \text{ J/kg}.$$

The required amount of heat transferred in one tube

$$Q = \frac{G}{n}\,\Delta i = \frac{1.0}{20} \times 630 \times 10^3 = 3.15 \times 10^4 \text{ W}.$$

The required rate of heat flow

$$q_w = \frac{Q}{\pi d l} = \frac{3.15 \times 10^4}{\pi \times 1 \times 10^{-2} \times 2.5} = 4 \times 10^5 \text{ W/m}^2.$$

Heat transfer may be deteriorated at temperatures $t_f < t_m < t_w$ and if the ratio q_w/ρ_w exceeds some critical value $(q_w/\rho_w)_{cr}$. The value of the ratio $(q_w/\rho_w)_{cr}$ can be approximately evaluated by the following formula [21]:

$$\left(\frac{g_w}{\rho_w}\right)_{cr} \approx 0.034 \left(\frac{c_p}{\beta_p}\right)_{t_m} \sqrt{\frac{\xi}{8}}, \tag{5-16}$$

where $(c_p/\beta_p)t_m$ is determined at the temperature t_m.

Formula (5-16) is true for the case of upward fluid flow in vertical tubes*.

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* More detailed data on the application limits for formula (5-16) can be found in [21].

At $p = 24$ MPa $t_m = 380.7°C$ and, since according to the conditions of the problem, $t_{f1} < t_m$, and $t_{f2} > t_m$, on some section along the tubes $t_f < t_m < t_w$. It is, therefore, necessary to check with the aid of formula (5-16) whether q_w exceeds the critical value.

At the given pressure the maximum heat capacity $(c_p)_m$ and the quantity $(\beta_\rho)_m$ cannot be determined with sufficient accuracy from tables (see the Appendix). Therefore, allowing that

$$\beta_\rho = -\frac{1}{\rho}\frac{\partial\rho}{\partial t},$$

it is convenient to present the ratio c_p/β_ρ in the following form

$$\frac{c_p}{\beta_\rho} = -\rho\frac{c_p\,\partial t}{\partial\rho} = -\rho\frac{\partial_i}{\partial\rho} \approx -\rho\frac{\Delta i}{\Delta\rho}:$$

Taking into account that $\rho = \frac{1}{v}$, we find from tables that $t_m = 380.7°C$, $\rho_m = 344$ kg/m³;
at $t = 380.7 - 1 = 379.7°C$ $\rho = 395.6$ kg/m³, $i = 2012.4$ kJ/kg;
at $t = 380.7 + 1 = 381.7°C$ $\rho = 286.8$ kg/m³, $i = 2201.3$ kJ/kg.

$$\Delta i = (2201.3 - 2012.4) \approx 189 \text{ kJ/kg};$$

$$\Delta\rho = 286.8 - 395.6 = -108.8 \text{ kg/m}^3.$$

$$\left(\frac{c_p}{\beta_\rho}\right)_{t_m} \approx -\left(\rho\frac{\Delta i}{\Delta\rho}\right)_{t_m} = 344\,\frac{189}{108.8} = 598 \text{ J/kg}.$$

To calculate the friction factor ξ, we determine with some margin the greatest value of the Reynolds number, i.e. we determine Re_f at the outlet from the tubes. At $t_{f2} = 390°C$ $\mu_f = 3.24 \times 10^{-5}$ Pa·s and

$$\mathrm{Re}_f = \frac{4G_1}{\pi d\mu_f} = \frac{4 \times 0.05}{\pi \times 1 \times 10^{-2} \times 3.24 \times 10^{-5}} = 1.97 \times 10^5,$$

where the rate of water flow through one tube

$$G_1 = \frac{G}{n} = \frac{1.0}{20} = 0.05 \text{ kg/s}.$$

$$\xi = (1.82\log \mathrm{Re} - 1.64)^{-2} = [1.82\log(1.97 \times 10^5) - 1.64]^{-2} = 1.57 \times 10^{-2};$$

$$\sqrt{\frac{\xi}{8}} = \sqrt{\frac{1.57 \times 10^{-2}}{8}} = 4.43 \times 10^{-2}.$$

Substituting the determined values into Eq. (5-16), we find:

$$\left(\frac{q_w}{\rho w}\right)_{cr} \approx 0.034 \times 5.98 \times 10^5 \times 4.43 \times 10^{-2} = 900 \text{ J/kg}.$$

The mass velocity

$$\rho w = \frac{4G_1}{\pi d^2} = \frac{4 \times 0.5}{\pi \times 1 \times 10^{-4}} = 637 \text{ kg/m}^2\cdot\text{s}.$$

The critical heat flux

$$q_{wcr} \approx 900 \times 637 \approx 5.7 \times 10^5 \text{ W/m}^2.$$

Since $q_w < q_{wcr}$, no local deterioration of heat transfer shall take place.

5-71. Determine the rate of heat flow required, q_w, W/m², and check whether at that rate of heat flow there may occur a local deterioration of heat transfer in the heat exchanger considered in problem 5-70, if the rate of water flow and the diameter of tubes are increased respectively to $G = 2$ kg/s and $d = 16$ mm. The length of tubes and the water inlet and outlet temperatures are not changed.

Answer

$$q_w = 5 \times 10^5 \text{ W/m}^2; \quad q_{cr} \approx 4.4 \times 10^5 \text{ W/m}^2.$$

Since $q_w > q_{cr}$ heat transfer may deteriorate and a dangerous local increase in the wall temperature may take place.

5-72. Determine (approximately) the magnitude of the ratio $(q_w/\rho w)$ below which there will be no local deterioration of heat transfer on sections of a heated tube, carrying water in upward flow at pressures $p = 24$ MPa and $p = 30$ MPa with the Reynolds number ranging from 3×10^4 to 3×10^5.

Answer

At $\mathrm{Re} = 3 \times 10^4$ and 3×10^5 and $p = 24$ MPa $(q_w/\rho w)_{cr} \approx 1100$ and 865 J/kg; at $p = 30$ MPa $(q_w/\rho w)_{cr} \approx 1300$ and 1000 J/kg respectively.

5-73. A vertical tube having a diameter $d = 14$ mm carries water in upward flow at a pressure $p = 24$ MPa. The rate of water flow $G = 0.2$ kg/s. Water is heated, maintaining a constant rate of heat flow through the wall $q_w = 7 \times 10^5$ W/m^2.

Determine the tube wall temperature t_{wx} at a distance $x > 15\,d$ from the inlet into the heated section, if it is known that the mean bulk temperature of water in the section of interest $t_{fx} = 370°C$.

Answer

$$t_{wx} = 408°C.$$

Solution

At a pressure $p = 24$ MPa and $t_{fx} = 370°C$

$$\mu_{fx} = 6.34 \times 10^{-5}\ \text{Pa}\cdot\text{s}$$

and the Reynolds number

$$\mathrm{Re}_{fx} = \frac{4G}{\pi d \mu_{fx}} = \frac{4 \times 0.2}{\pi \times 14 \times 10^{-3} \times 6.34 \times 10^{-5}} = 2.87 \times 10^5.$$

The water is in turbulent flow and, since $p > p_{cr}$, the rate of heat transfer can be calculated with the aid of formula (5-15):

$$\mathrm{Nu}_{fx} = \mathrm{Nu}_0 \left(\frac{\bar{c}_p}{c_{pf}}\right)^{0.4} \left(\frac{\rho_w}{\rho_f}\right)^m.$$

However, since to determine the Nusselt number from formula (5-15), the wall temperature must be known, the above equation must be solved together with the following expression:

$$\alpha_x = \frac{q_{wx}}{t_{wx} - t_{fx}}. \tag{a}$$

Calculations can be carried out by the method of successive appoximations or graphically. Let us use the graphical method.

Let us assume a number of values $t_{wx} = 390$, 400, 410 and 420°C. Then, we determine the rate of heat flow for each of the assumed values of t_{wx} by formula (5-15) and by formula (a).

Let $t_{wx} = 390°C$. At $p = 24$ MPa and $t_{wx} = 390°C$ $v_w = 5.619 \times 10^{-3}$ m^3/kg, $i_w = 2505$ kJ/kg. At $t_{fx} = 370°C$ $v_f = 1.89 \times 10^{-3}$ m^3/kg, $i_f = 1805$ kJ/kg, $\lambda_f = 0.396$ W/m·°C, $c_{pf} = 11.8$ kJ/kg, $\mathrm{Pr}_f = 1.89$.

The resistance factor.

$$\xi = (1.82\ \log\ \mathrm{Re} - 1.64)^{-2} = [1.82\ \log (2.87 \times 10^5) - 1.64]^{-2} = 1.46 \times 10^{-2}.$$

According to formula (5-8), the Nusselt number

$$\mathrm{Nu}_0 = \frac{\frac{\xi}{8}\mathrm{Re}_f \mathrm{Pr}_f}{12.7\sqrt{\frac{\xi}{8}}(\mathrm{Pr}_f^{2/3}-1)+1.07} =$$

$$= \frac{\frac{1.46 \times 10^{-2}}{8} 2.87 \times 10^5 \times 1.89}{12.7\sqrt{\frac{1.46 \times 10^{-2}}{8}}(1.89^{2/3}-1)+1.07} = 733.$$

The integral mean capacity

$$\bar{c}_p = \frac{i_w - i_f}{t_w - t_f} = \frac{2505-1805}{390-370} = 35\ \mathrm{kJ/kg \cdot {}^\circ C},$$

$$\frac{\bar{c}_p}{c_{pf}} = \frac{35}{11.8} = 2.97.$$

Relative temperatures of the fluid and wall. At $p = 24$ MPa and $t_m = 380.7°C$ and

$$\frac{T_f}{T_m} = \frac{370+273}{380.7 \times 173} = \frac{643}{653.7} = 0.984; \quad \frac{T_w}{T_m} = \frac{663}{653.7} \approx 1.013.$$

The ratio of densities

$$\frac{\rho_w}{\rho_f} = \frac{v_f}{v_w} = \frac{1.89}{5.62} = 0.336;$$

$$m = 0.35 - 0.05\frac{p}{p_{cr}} = 0.35 - 0.05\frac{24}{22.12} = 0.296.$$

According to formula (5-15), the Nusselt number

$$\mathrm{Nu}_x = 733\,(2.97)^{0.4}\,(0.336)^{0.296} = 733 \times 1.12 = 820.$$

At $t_w = 390°C$ the heat-transfer coefficient is found from formula (5-15) to be equal to

$$\alpha_x = \mathrm{Nu}_x\frac{\lambda_f}{d} = 820\frac{0.369}{14 \times 10^{-3}} = 2.32 \times 10^4\ \mathrm{W/m^2 \cdot {}^\circ C},$$

and from formula (a)

$$\alpha'_x = \frac{q_{wx}}{t_{wx} - t_{fx}} = \frac{7 \times 10^5}{390 - 370} = 3.5 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Similar calculation results for $t_{wx} = 400$, 410 and 420°C are presented in the following table:

t_w, °C	i_w, kJ/kg	v_w, m³/kg	$\dfrac{\bar{c}_p}{c_{pf}}$	$\dfrac{\rho_w}{\rho_f}$	Nu_x	$\alpha_x \times 10^{-4}$, W/m²·°C	$\alpha'_x \times 10^{-4}$, W/m²·°C
390	2505	5.619	2.97	0.336	820	2.32	3.50
400	2642	6.738	2.37	0.280	714	2.02	2.33
410	2735	7.544	1.98	0.250	641	1.81	1.75
420	2808	8.205	1.70	0.230	587	1.66	1.40

The calculation results are used to plot α_x and α'_x as a function of the temperature t_{wx} (Fig. 5-12). The unknown value of $t_{wx} = 408.5$°C is represented by the point at which the two curves intersect.

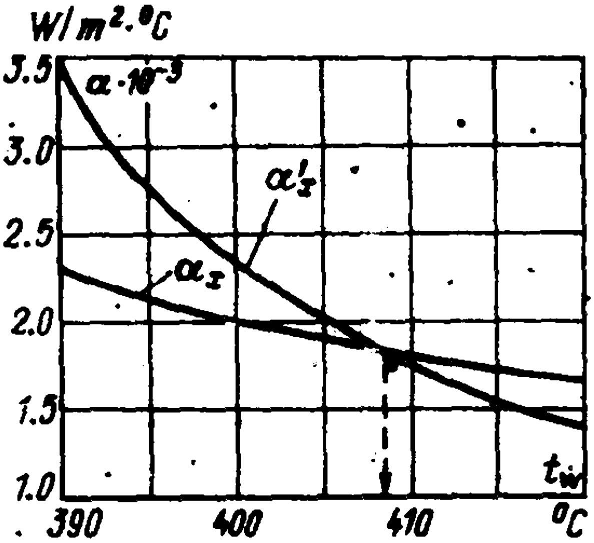

Fig. 5-12. Solution to problem 5-73

At $p = 24$ MPa the pseudocritical temperature $t_m = 380.7$°C. Since, according to calculation results, $t_f < t_m < t_w$, it is necessary to check whether the given rate of heat flow does not exceed the critical rate of flow.

Using the solution for problem 5-70, we find that at $p = 24$ MPa, formula (5-16) yields for water

$$\left(\frac{c_p}{\beta_p}\right)_{tm} = 5.98 \times 10^5 \text{ J/kg},$$

whence it follows that

$$\left(\frac{q}{\rho w}\right)_{cr} \approx 0.034 \left(\frac{c_p}{\beta_p}\right)_{t_m} \sqrt{\frac{\xi}{8}} =$$

$$= 0.034 \times 5.98 \times 10^5 \sqrt{\frac{\xi}{8}} \approx 2 \times 10^4 \sqrt{\frac{\xi}{8}},$$

In the problem considered $\mathrm{Re}_f = 2.87 \times 10^5$;

$$\sqrt{\frac{\xi}{8}} = 4.27 \times 10^{-2}, \quad \rho w = \frac{4G}{\pi d^2} = \frac{4 \times 0.2}{\pi \times 1.4^2 \times 10^{-4}} = 1300$$

and $q_{cr} \approx 2.10^4 \times 4.27 \times 10^{-2} \times 1.3 \times 10^3 \approx 1.1 \times \times 10^6$ W/m².

It follows that $q_w = 7 \times 10^5 < q_{cr}$, no local deterioration of heat transfer shall take place and the use of formula (5-15) is justified.

5-74. A tube having a diameter $d = 4$ mm carries carbon dioxide at a pressure $p = 10$ MPa. The tube is being heated at an approximately constant rate of heat flow on the wall. In sections x at a distance of $x > 20d$ from the inlet into the heated section of the tube the local Reynolds numbers, mean bulk fluid temperature and wall temperature are respectively $\mathrm{Re}_f = 2 \times 10^5$, $t_{fx} = 22$°C, $t_{wx} = 227$°C.

Determine the ratio of the local Nusselt number to the Nusselt number at constant physical properties of the fluid involved, $\mathrm{Nu}_f/\mathrm{Nu}_0$, and the magnitude of the local heat transfer coefficient in the cross section considered α_x, W/m²·°C. It is to be assumed that natural convection does not affect materially heat transfer.

Answer

$$\mathrm{Nu}_f/\mathrm{Nu}_0 \approx 0.49; \quad \alpha_x = 7100 \ \mathrm{W/m^2 \cdot {}^\circ C}.$$

Solution

The critical pressure of carbon dioxide $p_{cr} = 7.39$ MPa, consequently, heat transfer proceeds at $p > p_{cr}$.

With carbon dioxide in turbulent flow in tubes under supercritical pressure, for the case of heating, heat transfer can be calculated by the following formulae [5, 6]:

$$\mathrm{Nu}_f = \mathrm{Nu}_0 \left(\frac{\bar{c}_p}{c_{pf}}\right)^n \left(\frac{\rho_w}{\rho_f}\right) \varepsilon_x, \qquad (5\text{-}17)$$

where Nu_0, the Nusselt number at constant physical properties, the ratio of heat capacities $(\bar{c}_p/c_{pf})$, the ratio of densities ρ_w/ρ_f and the exponent m are determined the same as in formula (5-15);

n is the exponent depending on the relative wall and fluid temperatures, T_w/T_m and T_f/T_m, where T_m is the pseudocritical temperature, K.

At $T_w/T_m \leqslant 1$ or $T_f/T_m \geqslant 1.2$ $n = 0.4$;

at $1 \leqslant T_w/T_m \leqslant 2.6$ and $T_f/T_m \leqslant 1$ $n = n_1 = 0.22 + 0.18\ T_w/T_m$;

at $1 \leqslant T_w/T_m \leqslant 2.6$ and $1 \leqslant T_f/T_m \leqslant 1.2$ $n = n_1 + (5n_1 - 2)(1 - T_f/T_m)$.

The values of n, as a function of the ratio of absolute temperatures, are given in the following table:

T_f/T_m	T_w/T_m								
	1	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
1	0.4	0.436	0.472	0.508	0.544	0.580	0.616	0.652	0.688
1.04	0.4	0.429	0.458	0.486	0.515	0.544	0.573	0.602	0.630
1.08	0.4	0.423	0.443	0.465	0.486	0.508	0.530	0.551	0.573
1.12	0.4	0.414	0.423	0.443	0.458	0.472	0.486	0.501	0.515
1.16	0.4	0.407	0.414	0.422	0.429	0.436	0.443	0.450	0.458
1.20	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

ε_x is the correction for the initial section. If the heated section is preceded by a section of hydrodynamic stabilization, then [6]

$$\varepsilon_x = 0.86 + 0.54\left(\frac{d}{x}\right) \cdot 0.4.$$

The above formula is valid at $2 \leqslant x/d \leqslant 20$. At $x/d \geqslant 20$ $\varepsilon_x = 1$.

Formula (5-17) is valid for carbon dioxide at $1.06 \leqslant p/p_{cr} \leqslant 5.25$, $0.9 \leqslant T_f/T_m \leqslant 1.2$, and $0.9 \leqslant T_w/T_m \leqslant 2.6$, when natural convection does not affect materially heat transfer which corresponds to the condition $\mathrm{Gr}/\mathrm{Re}^2 < 0.6$*.

Under the conditions considered, at $p = 10$ MPa, the pseudocritical temperature $t_m = 45°\mathrm{C}$; at $t_{fx} = 22°\mathrm{C}$ ($T_{fx} =$

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* More detailed data on the application limits for formula (5-17) can be found in [5].

$= 295$ K) the physical properties of carbon dioxide are:

$$\rho_f = 842.9\ \text{kg/m}^3;\quad c_{pf} = 2.7\ \text{kJ/kg}\cdot{}^\circ\text{C};$$

$$i_f = 550\ \text{kJ/kg};\quad \lambda_f = 0.0977\ \text{W/m}\cdot{}^\circ\text{C};\quad \text{Pr}_f = 2.21.$$

At $t_{wx} = 227^\circ$C ($T_{wx} = 500$ K) $i_w = 965.8$ kJ/kg; $\rho_w = 113.1$ kg/m^3.

Calculations are carried out with the aid of formula (5-17), first determining Nu_0 by formula (5-8).

$$\xi = (1.82 \log \text{Re} - 1.64)^{-2} =$$

$$= [1.82 \log (2 \times 10^5) - 1.64]^{-2} = 1.56 \times 10^{-2};$$

$$\xi/8 = 1.95 \times 10^{-3};\quad \sqrt{\xi/8} = 4.41 \times 10^{-2}.$$

$$\text{Nu}_0 = \frac{\frac{\xi}{8}\text{Re}_f\text{Pr}_f}{12.7\sqrt{\frac{\xi}{8}}(\text{Pr}_f^{2/3} - 1) + 1.07} =$$

$$= \frac{1.95 \times 10^{-3} \times 2 \times 10^5 \times 2.21}{12.7 \times 4.41 \times 10^{-2}(2.21^{2/3} - 1) + 1.07} = 590.$$

Let us determine in formula (5-17) the multiplier, accounting for the variation of the physical properties of carbon dioxide over the cross section of flow:

$$\varphi = \left(\frac{\bar{c}_p}{c_{pf}}\right)^n \left(\frac{\rho_w}{\rho_f}\right)^m;$$

$$\bar{c}_p = \frac{i_w - i_f}{t_w - t_f} = \frac{965.8 - 550}{227 - 22} = 2.03\ \text{kJ/kg}\cdot{}^\circ\text{C};$$

$$\frac{\bar{c}_p}{c_{pf}} = \frac{2.03}{2.7} = 0.753;$$

$$T_f/T_m = \frac{22 + 273}{45 + 273} = \frac{295}{318} = 0.928;\quad T_w/T_m = 500/318 = 1.57.$$

Since $T_f/T_m < 1$ and $1 < T_w/T_m < 2.6$ then

$$n = 0.22 + 0.18 T_w/T_m = 0.22 + 0.18 \times 1.57 \approx 0.5;$$

$$\frac{\rho_w}{\rho_f} = \frac{113.1}{842.9} = 0.134;$$

$$m = 0.35 - 0.05\frac{p}{p_{cr}} = 0.35 - 0.05 \times \frac{10}{7.39} = 0.282;$$

$$\varphi = (0.735)^{0.5}(0.134)^{0.282} = 0.494.$$

Under the conditions of the problem $x > 20\ d$ and, consequently, $\varepsilon_x = 1$. The local Nusselt number and the heat-transfer coefficient

$$\mathrm{Nu}_{fx} = \mathrm{Nu}_0\varphi = 590 \times 0.494 = 291;$$

$$\alpha_x = \mathrm{Nu}_{fx}\frac{\lambda_f}{d} = 291\,\frac{9.77\times 10^{-3}}{4.10^{-3}} = 7100\ \mathrm{W/m^2{\cdot}{}^\circ C}.$$

5-75. Determine the ratio of the local Nusselt number to the Nusselt number at constant physical properties, $\mathrm{Nu}_{fx}/\mathrm{Nu}_0$, and the magnitude of the local heat-transfer coefficient α_x under the conditions of problem 5-74, but assuming the mean bulk temperature of carbon dioxide to be $t_{fx} = 43^\circ\mathrm{C}$ and $t_{fx} = 67^\circ\mathrm{C}$.

Answer

At $t_{fx} = 43^\circ\mathrm{C}$ $\alpha_x = 4610\ \mathrm{W/m^2{\cdot}{}^\circ C}$, $\mathrm{Nu}_f/\mathrm{Nu}_0 = 0.319$, at $t_{fx} = 67^\circ\mathrm{C}$, $\alpha_x = 3570\ \mathrm{W/m^2{\cdot}{}^\circ C}$, $\mathrm{Nu}_f/\mathrm{Nu}_0 = 0.61$. Based on the solutions of problems 5-74 and 5-75, the variation of $\mathrm{Nu}_f/\mathrm{Nu}_0$ and α_x, depending on T_f/T_m, is illustrated graphically in Fig. 5-13.

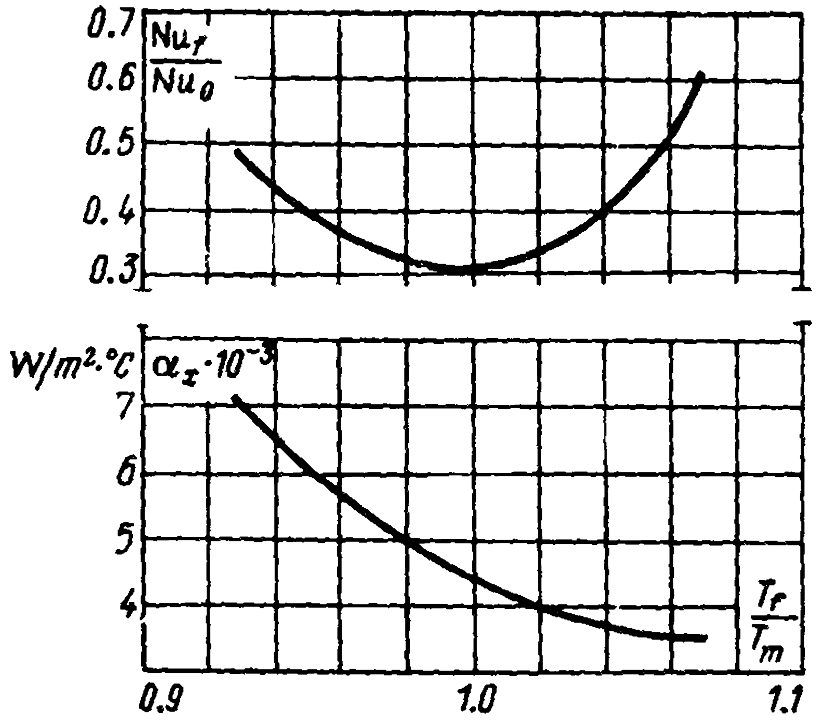

Fig. 5-13. Solution to problem 5-75

5-76. Find the dependence of the ratio of the local Nusselt number to the Nusselt number at constant physical properties, Nu_f/Nu_0, on the relative wall tempertatre T_w/T_m for carbon dioxide in turbulent flow in a round tube.

The calculations should be carried out assuming a pressure $p = 10$ MPa, for a cross section at a distance from the inlet, the mass temperature of the carbon dioxide $t_f = 17°C$ and the following wall temperatures: $t_w = 27$, 44, 67, 127 and 227°C.

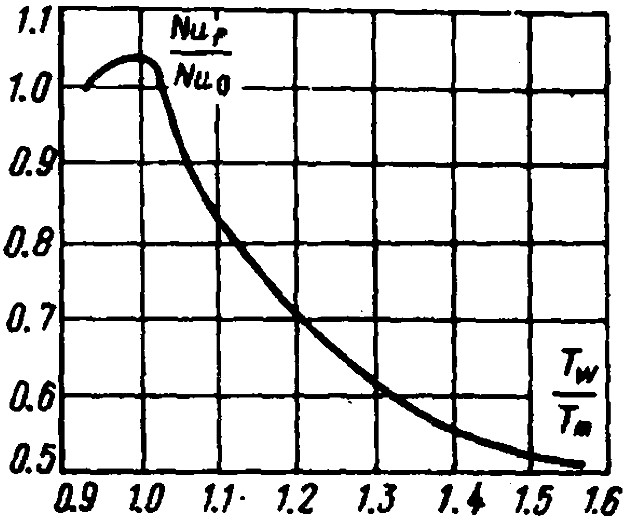

Fig. 5-14. Solution to problem 5-76

Answer

Calculation results are tabulated below and illustrated in Fig. 5-14:

t_w, °C	27	44	67	127	227
T_w/T_m	0.944	0.997	1.07	1.26	1.57
Nu_f/Nu_0	1.005	1.035	0.874	0.646	0.511

5-77. Determine the heat-transfer coefficient and wall temperature with air flowing along a one-side heated annular channel. The outside and inside diameters of the channel are respectively $d_2 = 40$ mm and $d_1 = 8$ mm. In the section considered, arranged downstream of the section of thermal stabilization ($x > l_{in.t}$), the mean mass temperature $t_f = 100°C$ and air velocity $w = 55$ m/s.

Solve the problem for two cases: (a) with heat being transferred to the air only through the inside wall of the channel ($q_{w2} = 0$) and (b) with heat being transferred to the air only through the outside wall of the channel ($q_{w1} = 0$).

In the two cases the rate of heat flow on a corresponding wall is to be assumed equal to $q_w = 1.5 \times 10^4$ W/m^2.

Answer

(a) $\alpha_{1.1} = 141$ W/m$^2\cdot$°C $t_{w1} = 206$°C;
(b) $\alpha_{2.1} = 121$ W/m$^2\cdot$°C $t_{w2} = 224$°C.

Solution

Physical properties of air at $t_f = 100$°C:

$\nu_f = 23.13 \times 10^{-6}$ m^2/s; $\lambda_f = 3.21 \times 10^{-2}$ W/m$\cdot$C; $\mathrm{Pr}_f = 0.688$.

The equivalent diameter of the annular channel

$$d_{eq} = d_2 - d_1 = 40 - 8 = 32 \text{ mm}.$$

The Reynolds number

$$\mathrm{Re}_f = \frac{wd_{eq}}{\nu_f} = \frac{55 \times 32 \times 10^{-3}}{23.13 \times 10^{-6}} = 7.6 \times 10^4;$$

the pattern of air flow in the channel is turbulent.

With air in turbulent flow in one-side heated annular channels [22] under conditions of thermal stabilization ($x > l_{to.t}$), constant rate of heat flow on the wall ($q_w = \text{const}$) and heating only of the inside surface of the annulus, $q_{w2} = 0$,

$$\mathrm{Nu}_{1,1} = \mathrm{Nu}_t\, 0.86 \left(\frac{d_1}{d_2}\right)^{-0.16} \xi, \qquad (15\text{-}18)^*$$

where $\mathrm{Nu}_{1.1} \doteq d_{1.1} d_{eq}/\lambda_f$ is the Nusselt number for the inside surface in the event of one-side heating;

$$\alpha_{1,1} = \frac{q_{w1}}{t_{w1} - t_f},$$

Nu_t is the Nusselt number for air flowing inside a round tube, calculated by the equivalent diameter d_{eq}.

With $d_1/d_2 < 0.2$,

$$\xi = 1 + 7.5 \left(\frac{d_2/d_1 - 5}{\mathrm{Re}_f}\right)^{0.6};$$

with $d_1/d_2 \geqslant 0.2$ $\xi = 1$.

Formula (5-18) is valid at $0.07 \leqslant d_1/d_2 \leqslant 1$.

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[*] For air formula (5-18) gives almost the same results as formula (5-12).

When only the outside surface is heated, $q_{w1} = 0$:

$$\mathrm{Nu}_{2,1} = \mathrm{Nu}_t\left[1 - 0.14\left(\frac{d_1}{d_2}\right)^{0.6}\right], \tag{5-19}$$

where $\mathrm{Nu}_{2,1} = \frac{\alpha_{,21} d_{eq}}{\lambda_f}$ is the Nusselt number for the outside surface in the event of one-side heating;

$$\alpha_{2,1} = \frac{q_{w2}}{t_{w2} - t_f}.$$

Forumla (5-19) is valid at $0 \leqslant d_1/d_2 \leqslant 1$. Formulae (5-18) and (5-19) are valid at $10^4 \leqslant \mathrm{Re}_f \leqslant 3 \times 10^5$ and $\mathrm{Pr}_f \approx 0.7$.

With $d_1/d_2 = 1$ these formulae give the value of the Nusselt number for the case of one-side heating in a plate-type fuel element:

$$\mathrm{Nu}_{1,1} = \mathrm{Nu}_{2,1} = 0.86\mathrm{Nu}_t.$$

In the case considered the Nusselt number Nu_t is calculated by formula (5-7) and a correction is introduced by formula (5-13):

$$\mathrm{Nu}_t = 0.021\,\mathrm{Re}_f^{0.8}\,\mathrm{Pr}_f^{0.43}\Theta_w^{-0.55}.$$

Assuming, as the first approximation, that the correction for the temperature factor $\Theta_w^{-0.55} = (T_w/T_f)^{-0.55} = 1$, we have:

$$\mathrm{Nu}_t = 0.021 \times (7.6 \times 10^4)^{0.8}(0.688)^{0.43} = 145.$$

(a) $q_{w1} = 1.5 \times 10^4$ W/m², $q_{w2} = 0$,

$$\mathrm{Nu}_{1,1} = \mathrm{Nu}_t\,0.86\left(\frac{d_1}{d_2}\right)^{-0.16}\xi =$$

$$= 145 \times 0.86\left(\frac{8}{40}\right)^{-0.16} \times 1 = 162,$$

where $\frac{d_1}{d_2} = \frac{8}{40} = 0.2$ and $\xi = 1$.

$$\alpha_{1,1} = \mathrm{Nu}_{1,1}\frac{\lambda_f}{d_{eq}} = 162\,\frac{3.21 \times 10^{-2}}{32 \times 10^{-3}} = 163\ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$t_{w1} = t_f + \frac{q_{w1}}{\alpha_{1,1}} = 100 + \frac{1.5 \times 10^4}{163} = 192\,^\circ\mathrm{C}.$$

Having determined t_{w1} as the first appoximation, let us introduce a correction for the temperature factor:

$$\Theta_{w1}=\frac{T_{w1}}{T_f}=\frac{192+273}{100+273}=1.25;$$

$$\Theta_{w1}^{-0.55}=(1.25)^{-0.55}=0.885;$$

$\mathrm{Nu}_f = 0.885 \times 145 = 128$ and, correspondingly,

$$\alpha_{1,1} = 0.885 \times 163 = 144\ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$t_{w1}=100+\frac{1.5\times 10^4}{144}=204\ {}^\circ\mathrm{C}.$$

The final, more precise, results, obtained with the aid of the method of successive approximations, are $\alpha_{1,1} = 141\ \mathrm{W/m^2 \cdot {}^\circ C}$ and $t_{w1} = 206\,{}^\circ\mathrm{C}$.

(b) $q_{w2} = 1.5 \times 10^4\ \mathrm{W/m^2 \cdot {}^\circ C}$, $q_{w1} = 0$.

The Nusselt number Nu_f remains equal to 145 as above and

$$\mathrm{Nu}_{2,1}=\mathrm{Nu}_f\left[1-0.14\left(\frac{d_1}{d_2}\right)^{0.6}\right]=$$

$$=145\,[1-0.14\,(0.2)^{0.6}]=140.$$

As the first approximation the heat-transfer coefficient $\alpha_{2,1}$ and wall temperature t_{w2} are equal to

$$\alpha_{2,1}=\mathrm{Nu}_{2,1}\,\frac{\lambda_f}{d_{eq}}=140\,\frac{3.21\times 10^{-2}}{32\times 10^{-3}}=141\ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$t_{w2}=t_f+\frac{q_{w2}}{\alpha_{2,1}}=100+\frac{1.5\times 10^4}{141}\approx 206\ {}^\circ\mathrm{C}$$

and the correction for the temperature factor

$$\Theta_{w2}=\frac{T_{w2}}{T_f}=\frac{206+273}{373}=1.28;$$

$$\Theta_{w2}^{-0.55}=(1.28)^{-0.55}=0.875;$$

$$\alpha_{2,1}=0.875\times 141=123\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

$$t_{w2}=100+\frac{1.5\times 10^4}{123}=222\ {}^\circ\mathrm{C}$$

and further calculations give the following final results:

$$\alpha_{2,1} = 121\ \mathrm{W/m^2 \cdot {}^\circ C} \text{ and } t_{w2} = 224\,{}^\circ\mathrm{C}.$$

5-78. Air flows through the annulus of an inside diameter $d_1 = 5.4$ mm and an outside diameter $d_2 = 60$ mm. The rate of air flow $G = 0.12$ kg/s and its mean mass temperature in the section considered $t_f = 80°C$.

Determine the inner surface temperature of the annulus, t_{w1}, if heat is added only through this surface and the rate of heat flow $q_{w1} = 2 \times 10^4$ W/m^2.

Answer

$$t_{w1} = 262°C.$$

5-79. An annulus having an inside diameter $d_1 = 12$ mm and an outside diameter $d_2 = 30$ mm carries air at a rate of $G = 5 \times 10^{-3}$ kg/s. Air is heated with heat added only through the inside surface of the annulus and the lengthwise constant rate of heat flow through the wall of the annulus $q_{w1} = 2 \times 10^4$ W/m^2. At the inlet into the heated section the air temperature $t_f = 20°C$.

Determine the heat-transfer coefficient and the temperature of the inside wall of the annulus, α_x and t_{wx}, at distances $x_1=90$ mm and $x_2=720$ mm from the inlet into the heated section. The effect of the temperature factor on the rate of heat transfer can be ignored.

Answer

$$x_1 = 90 \text{ mm}, \quad \alpha_{x1} = 275 \text{ W/m}^2\cdot°\text{C}, \quad t_{wx1} = 94°\text{C};$$

$$x_2 = 720 \text{ mm}, \quad \alpha_{x2} = 223 \text{ W/m}^2\cdot°\text{C}, \quad t_{wx2} = 120°\text{C}.$$

Solution

The variation of air bulk temperature along the annulus is determined from the heat balance equation

$$t_{fx} = t_{f1} + \frac{q_{w1}\pi d_1}{Gc_{pf}}\, x = 20 + \frac{2 \times 10^4 \pi \times 12 \times 10^{-3}}{5 \times 10^{-3} \times 1.01 \times 10^3}\, x = 20 + 14.9x,$$

where, in the temperature interval

$$t_f = 20 \text{ to } 140°\text{C}, \quad c_{pf} \approx 1.01 \times 10^3 \text{ J/kg}\cdot°\text{C}.$$

At $x_1 = 90$ mm $t_{fx1} = 20 + 14.9 \times 90 \times 10^{-3} \approx 21.3°C$ and at that temperature the physical properties of air:

$$\mu_{x1} = 18.2 \times 10^{-6} \text{ Pa}\cdot\text{s}; \quad \lambda_{x1} = 2.60 \times 10^{-2} \text{ W/m}\cdot°\text{C};$$

$$\text{Pr}_{x1} = 0.703.$$

The Reynolds number

$$\mathrm{Re}_x = \frac{w_x d_{eq}}{\nu_x} = \frac{G d_{eq}}{f \mu_x},$$

where $d_{eq} = d_2 - d_1 = 30 - 12 = 18$ mm, and the ratio of the equivalent diameter to the cross-sectional area of the annulus

$$\frac{d_{eq}}{f} = \frac{4(d_2 - d_1)}{\pi (d_2^2 - d_1^2)} = \frac{4}{\pi (d_2 + d_1)} = \frac{4}{\pi (30 + 12) \times 10^{-3}} = 30.4\ 1/\mathrm{m},$$

consequently,

$$\mathrm{Re}_x = \frac{30.4 \times 5 \times 10^{-2}}{\mu_x} = \frac{1.52}{\mu_x};$$

at $x_1 = 90$ mm

$$\mathrm{Re}_{x1} = \frac{1.52}{18.2 \times 10^{-6}} = 8.35 \times 10^4.$$

With air in turbulent flow along a one-side heated annular channel the length of the section of thermal stabilization can be calculated from the following relationship [22]:

$$\frac{l_{in.t}}{d_{eq}} = 15\left(1 + 1.2\frac{d_1}{d_2}\right), \tag{5-20}$$

and the Nusselt number at $1 \leqslant x/d_{eq} \leqslant l_{in.t}/d_{eq}$ is found by formula

$$\frac{\mathrm{Nu}_{1,l}}{\mathrm{Nu}_{1,l\infty}} = \frac{\mathrm{Nu}_{2,l}}{\mathrm{Nu}_{2,l\infty}} = 0.86 + 0.54\left[\left(1 + 1.2\frac{d_1}{d_2}\right) \times \right.$$
$$\left. \times \left(\frac{x}{d_{eq}}\right)^{-0.4} - 0.188\frac{d_1}{d_2}\right]. \tag{5-21}$$

Formula (5-21) is valid for $\mathrm{Nu}_{1,l}$ at $0.14 \leqslant d_1/d_2 \leqslant 1$ and for $\mathrm{Nu}_{2,l}$ at $0 \leqslant d_1/d_2 \leqslant 1$.

In the formula, $\mathrm{Nu}_{1,l\infty}$ and $\mathrm{Nu}_{2,l\infty}$ represent the limiting Nusselt numbers at $x > l_{in.t}$, determined by formulae (5-18) and (5-19).

In the case considered $d_1/d_2 = 12/30 = 0.4$,

$$\frac{l_{in.t}}{d_{eq}} = 15(1 + 1.2 \times 0.4) = 22.2$$

and

$$\frac{x_1}{d_{eq}} = \frac{90}{18} = 5 < \frac{l_{in.t}}{d_{eq}};$$

$$\frac{x_2}{d_{eq}} = \frac{720}{18} = 40 > \frac{l_{in.t}}{d_{eq}}.$$

That is why, having calculated $Nu_{1,1\infty}$ by formula (5-18), it is necessary to calculate $Nu_{1,1}$ for the cross section x_1 by formula (5-21). The cross section x_2 requires no correction for the initial section.

With air flowing in a round tube the Nusselt number is determined by formula (5-7) without a correction for the temperature factor

$$Nu_{t1} = 0.021 Re_{x1}^{0.8} Pr_{x1}^{0.43} = 0.021 (8.35 \times 10^4)^{0.8} \times \times (0.703)^{0.43} = 153.$$

With air flowing in a one-side heated annular channel at $x > l_{in.t}$ the Nusselt number is calculated by means of formula (5-18)

$$Nu_{1,1\infty} = Nu_t 0.86 \left(\frac{d_1}{d_2}\right)^{-0.16} \xi,$$

since $d_1/d_2 = 0.4 > 0.2$, then $\xi = 1$ and the correction factor $0.86 (d_1/d_2)^{-0.16} = 0.86 (0.4)^{-0.16} \approx 1$, i.e. in the given case

$$Nu_{1,1\infty} = Nu_t = 153.$$

Since $x_1 < l_{in.t}$, then by formula (5-21)

$$\frac{Nu_{x1}}{Nu_{1,1\infty}} = 0.86 + 0.54 \left[(1 + 1.2 \times 0.4) \left(\frac{x}{d_{eq}}\right)^{-0.4} - 0.188 \times \times 0.4\right] = 0.819 + 798 (x/d_{eq})^{-0.4} \text{ and at } x/d_{eq} = 5$$

$$\frac{Nu_{x1}}{Nu_{1,1\infty}} = 0.819 + 0.798 \times 5^{-0.4} = 1.24;$$

$$Nu_{x1} = 1.24 \times 153 = 190.$$

The heat-transfer coefficient

$$\alpha_{x1} = Nu_{x1} \frac{\lambda_{x1}}{d_{eq}} = 190 \frac{2.6 \times 10^{-2}}{18 \times 10^{-3}} = 275 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

At a distance $x_1 = 90$ mm from the inlet the wall temperature

$$t_{wx1} = t_{fx1} + \frac{q_{wi}}{\alpha_{x1}} = 21.3 + \frac{2\times 10^4}{275} = 94.1\ ^\circ\mathrm{C}.$$

At a distance $x_2 = 720$ mm the wall temperature is determined in a similar way, but, since $x_2/d_{eq} = 40 > l_{in.t}/d_{eq}$, the correction calculated by formula (5-21) is not introduced. Then

$$t_{fx2} = 20 + 14.9x_2 = 20 + 14.9 \times 720 \times 10^{-3} = 30.7^\circ\mathrm{C}.$$

At $t_{fx2} = 30.7^\circ$C $\mu_{x2} = 18.7 \times 10^{-6}$ Pa·s, $\lambda_{x2} = 2.68 \times$ $\times\ 10^{-2}$ W/m·°C, $\mathrm{Pr}_{x2} = 0.701$,

$$\mathrm{Re}_{x2} = \frac{1.52}{\mu_{x2}} = 8.12 \times 10^4;$$

$\mathrm{Nu}_t = 0.021\ (8.12 \times 10^4)^{0.8}\ (0.701)^{0.43} = 150$ and, since at $d_1/d_2 = 0.4$ $\mathrm{Nu}_{1,1} \approx \mathrm{Nu}_t$, then $\mathrm{Nu}_{x2} = \mathrm{Nu}_{1,1} = 150$;

$$\alpha_{x2} = \mathrm{Nu}_{x2}\frac{\lambda_{x2}}{d_{eq}} = 150\,\frac{2.68\times 10^{-2}}{18\times 10^{-3}} = 223\ \mathrm{W/m^2\cdot{}^\circ C};$$

$$t_{wx2} = t_{fx2} + \frac{q_{wi}}{\alpha_{x2}} = 30.7 + \frac{2\times 10^4}{223} = 120.4\ ^\circ\mathrm{C}.$$

5-80. Determine the heat-transfer coefficient and the temperature of the inside wall of an annular channel, α_x and t_{wx}, under the conditions of problem 5-79, at distances $x = 45$, 180, 360 and 1500 mm from the inlet into the heated section. Plot the variation graph of α_x, t_{wx} and air temperature t_{fx} along the annular channel, making use of the quantities obtained in solving problem 5-79.

Answer

With account taken of the data obtained in solving problem 5-79, calculation results are tabulated below and illustrated in Fig. 5-15 graphically.

x, mm	45	90	180	360	720	1500
x/d_{eq}	2.5	5	10	20	40	83.3
α_x, W/m²·°C	310	275	252	235	223	222
t_{wx}, °C	85	94	102	111	120	132
t_{fx}, °C	20.7	21.3	22.7	25.4	30.7	42.4

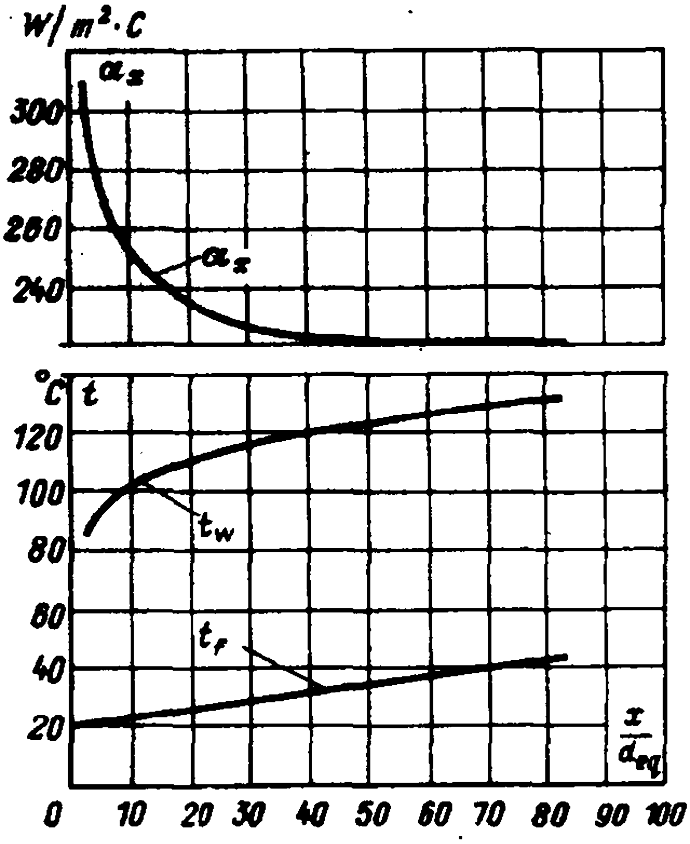

Fig. 5-15. Solution to problem 5-80

5-81. Determine the temperatures of the inside and outside wall surfaces, t_{w1} and t_{w2}, at a distance $x = 1400$ mm from the inlet into a non-symmetrically heated annular channel, the inside and outside diameters of which are $d_1 = 6$ mm and $d_2 = 20$ mm. The annulus carries water flowing at a rate $G = 0.3$ kg/s. The water inlet temperature $t_{f1} = 125$°C. The lengthwise-constant rates of heat flow on the inside and outside walls of the concentric annular channel are respectively equal to $q_{w1} = 1 \times 10^5$ W/m² and $q_{w2} = 2 \times 10^5$ W/m².

Answer

$$t_{w1} = 148°\text{C}; \quad t_{w2} = 165°\text{C}.$$

Solution

The heat balance equation is used to find the mean bulk temperature of water in a particular cross section

$$Gc_{pf}(t_f - t_{f1}) = q_{w1}\pi d_1 x + q_{w2}\pi d_2 x,$$

whence

$$t_f = t_{f1} + \frac{(q_{w1}d_1 + q_{w2}d_2)\pi x}{Gc_{pf}} =$$

$$= 125 + \frac{(1\times10^5\times6\times10^{-3}+2\times10^5\times20\times10^{-3})\pi\times1.4}{0.3\times4.27\times10^3} \approx 141\ ^\circ\mathrm{C},$$

where, in the 125 to 141°C temperature interval, the heat capacity of water $c_{pf} = 4.27\times10^3$ J/kg·°C.

At $t_f = 141$°C the physical properties of water are as follows:

$$\rho_f = 925\ \mathrm{kg/m^3};\quad \lambda_f = 0.685\ \mathrm{W/m\cdot{}^\circ C};$$
$$\nu_f = 0.216\times10^{-6}\ \mathrm{m^2/s};\quad \mathrm{Pr}_f = 1.25.$$

The clear passage and the equivalent diameter of the annulus

$$f = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(20^2 - 6^2)\times10^{-6} = 2.86\times10^{-4}\ \mathrm{m^2};$$

$$d_{eq} = d_2 - d_1 = 20 - 6 = 14\ \mathrm{mm}.$$

Water velocity and the Reynolds number

$$w = \frac{G}{\rho_f f} = \frac{0.3}{925\times2.86\times10^{-4}} = 1.13\ \mathrm{m/s};$$

$$\mathrm{Re}_f = \frac{wd_{eq}}{\nu_f} = \frac{1.13\times14\times10^{-3}}{0.216\times10^{-6}} = 7.33\times10^4.$$

With fluid flowing in non-symmetrically heated annular coolant channels under conditions of thermal stabilization and constant rates of heat flow the Nusselt numbers on corresponding surfaces can be determined by means of the following formulae [22]:

$$\mathrm{Nu}_1 = \frac{\mathrm{Nu}_{1,1}}{1+\dfrac{q_{w2}}{q_{w1}}\theta_{aw1}\mathrm{Nu}_{1,1}}; \tag{5-22}$$

$$\mathrm{Nu}_2 = \frac{\mathrm{Nu}_{2,1}}{1+\dfrac{q_{w1}}{q_{w2}}\theta_{aw2}\mathrm{Nu}_{2,1}}, \tag{5-23}$$

where $\mathrm{Nu}_{1,1}$ and $\mathrm{Nu}_{2,1}$ = Nusselt numbers for the one-side heated inside and outside surfaces, respectively;

θ_{aw1} and θ_{aw2} = dimensionless adiabatic temperatures (at $q_{w1} = 0$ and $q_{w2} = 0$).

For liquids and gases at $0.5 \leqslant \mathrm{Pr} \leqslant 100$

$$\frac{\mathrm{Nu}_{1,1}}{\mathrm{Nu}_t} = (1-\varphi)\left(\frac{d_1}{d_2}\right)^n \xi; \tag{5-24}$$

$$\frac{\mathrm{Nu}_{2,1}}{\mathrm{Nu}_t} = (1-0.14)\left(\frac{d_1}{d_2}\right)^{0.6}, \tag{5-25}$$

where

$$\varphi = \frac{0.255}{1+1.1\,\mathrm{Pr}^{0.9}}; \quad n = -0.16\,\mathrm{Pr}^{-0.15};$$

$$\Theta_{aw1} = 22\left[0.27\left(\frac{d_1}{d_2}\right)^2 - 1\right]\mathrm{Re}^{-0.87}\,\mathrm{Pr}^{-1.05}; \tag{5-26}$$

$$\Theta_{aw2} = \Theta_{aw1}\frac{d_1}{d_2}; \tag{5-27}$$

Nu_t is the Nusselt number for water flowing in a round tube and calculated by the equivalent diameter d_{eq}.

Formulae (5-24) and (5-25) are valid at

$$10^4 \leqslant \mathrm{Re} \leqslant 10^6 \text{ and } 0.1 \leqslant d_1/d_2 \leqslant 1.$$

In the case considered the Nusselt number Nu_t is determined with the aid of formula (5-7):

$$\mathrm{Nu}_t = 0.021\,\mathrm{Re}_f^{0.8}\,\mathrm{Pr}_f^{0.43}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25} =$$
$$= 0.021\,(7.33\times 10^4)^{0.8}(1.25)^{0.43} = 179,$$

where $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}$ is assumed equal to unity as the first approximation.

When only the inside surface is heated the Nusselt number is calculated by formula (5-24):

$$\varphi = \frac{0.255}{1+1.1\,(1.25)^{0.9}} = 0.109;$$

$$n = -0.16\,(1.25)^{-0.15} = -0.154;$$

$$\frac{d_1}{d_2} = \frac{6}{20} = 0.3;$$

$$\frac{\mathrm{Nu}_{1,1}}{\mathrm{Nu}_t} = (1-0.109)\,(0.3)^{-0.154} = 1.07;$$

$$\mathrm{Nu}_{1,1} = 179 \times 1.07 = 192.$$

With $q_{w2}/q_{w1} = 2$ the Nusselt number is determined by formula (5-22) for which purpose Θ_{aw1} is first calculated by formula (5-26):

$$\Theta_{aw1} = 22\,[0.27\,(0.3)^2 - 1]\,(7.33 \times 10^4)^{-0.87}(1.25)^{-1.06} =$$
$$= -1 \times 10^{-3};$$

$$\mathrm{Nu}_1 = \frac{192}{1 - 2 \times 1 \times 10^{-3} \times 192} = 312.$$

The heat-transfer coefficient and the temperature of the inside surface of the annular coolant channel

$$\alpha_1 = \mathrm{Nu}_1 \frac{\lambda_f}{d_{eq}} = 312\,\frac{0.685}{14 \times 10^{-3}} = 1.53 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$t_{w1} = t_f + \frac{q_{w1}}{\alpha_1} = 141 + \frac{1 \times 10^5}{1.53 \times 10^4} = 147.5\ {}^\circ\text{C}.$$

Since the difference $t_f - t_w$ is small, the ratio $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}$ is assumed approximately equal to unity, just as in calculating Nu_f.

In the event of one-side heating the Nusselt number is calculated with the aid of formula (5-25)

$$\frac{\mathrm{Nu}_{2,1}}{\mathrm{Nu}_f} = 1 - 0.14\,(0.3)^{0.6} = 0.932;$$

$$\mathrm{Nu}_{2,1} = 0.932 \times 179 = 167.$$

The dimensionless adiabatic temperature is found by formula (5-27) to be

$$\Theta_{aw2} = \Theta_{aw1}\frac{d_1}{d_2} = -1 \times 10^{-3} \times 0.3 = -0.3 \times 10^{-3}.$$

The Nusselt number by formula (5-23)

$$\mathrm{Nu}_2 = \frac{167}{1 - 0.5 \times 0.3 \times 10^{-3} \times 167} = 171.$$

The heat-transfer coefficient and the temperature of the outside surface of the annular channel are

$$\alpha_2 = \mathrm{Nu}_2 \frac{\lambda_f}{d_{eq}} = 171\,\frac{0.685}{14 \times 10^{-3}} = 8.35 \times 10^3 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$t_{w2} = t_f + \frac{q_{w2}}{\alpha_2} = 141 + \frac{2 \times 10^5}{8.35 \times 10^3} = 165\ {}^\circ\text{C}.$$

5-82. Determine the temperatures of the inside and outside walls of an annular channel, t_{w1} and t_{w2}, under the conditions of problem 5-81, but with $t_{f1} = 120°C$, and $q_{w1} = 2 \times 10^5$, $q_{w2} = 1 \times 10^5$ W/m^2.

Answer

$$t_{w1} = 160°C; \quad t_{w2} = 152°C.$$

5-83. A plate-type fuel element of the core of a nuclear reactor has sodium as a coolant. The width of the coolant channel $b = 3$ mm. The velocity of the sodium coolant $w = 3$ m/s. The mean bulk temperature of sodium in the cross section of the channel considered, $t_f = 400°C$.

Determine the temperatures of the inside surfaces of the coolant channel, t_{w1} and t_{w2}, if the rate of heat flow on one of them is $q_{w1} = 7 \times 10^5$ W/m^2, and on the other, $q_{w2} = 0$.

Answer

$$t_{w1} = 412°C; \quad t_{w2} = t_{aw2} = 395°C.$$

Solution

At $t_f = 400°C$ the physical properties of sodium are as follows:

$$\lambda_f = 63.9 \text{ W/m}\cdot°\text{C}; \quad \nu_f = 33 \times 10^{-8} \text{ m}^2/\text{s}; \quad \text{Pr}_f = 5.6 \times 10^{-3}.$$

The equivalent diameter of the channel

$$d_{eq} = 2b = 2 \times 3 = 6 \text{ mm}.$$

The Reynolds number

$$\text{Re}_f = \frac{w d_{eq}}{\nu_f} = \frac{3 \times 6 \times 10^{-3}}{33 \times 10^{-8}} = 5.45 \times 10^4.$$

With liquid-metal coolants in turbulent flow through annular and plate-type channels the Nusselt numbers and the adiabatic temperatures of the walls can be determined approximately, in the event of one-side heating, with the aid of the following formulae:

$$\frac{\text{Nu}_{1,t}}{\text{Nu}_t} = 0.57 \left(\frac{d_1}{d_2}\right)^{-0.4} + \frac{0.18 d_1}{d_2}; \tag{5-28}$$

$$\frac{\text{Nu}_{2,t}}{\text{Nu}_t} = 1 - 0.25 \left(\frac{d_1}{d_2}\right)^{0.3}, \tag{5-29}$$

where, according to formula (5-14), $Nu_t = 5 + 0.025\ Pe^{0.8}$ is the Nusselt number for liquid metals in flow through round tubes, calculated by the equivalent diameter;

$$\Theta_{aw1} Nu_{1,1} = \frac{1}{a + b\,Pe^{0.8}}, \qquad (5\text{-}30)$$

where $a = 2.2\,(d_1/d_2)^{0.5}$ and $b = 4.5 \times 10^{-5}\,(d_1/d_2)^{0.12}$;

$$\Theta_{aw2} = \Theta_{aw1} \frac{d_1}{d_2},$$

where

$$\Theta_{aw1} = \frac{(t_{aw1} - t_f)\,\lambda_f}{q_{w2} d_{eq}} \quad \text{and} \quad \Theta_{aw2} = \frac{(t_{aw2} - t_f)\,\lambda_f}{q_{w1} d_{eq}}.$$

For the case of flow in a flat-type channel ($d_2/d_1 = 1$) formulae (5-28) and (5-29) yield:

$$Nu_{1,1} = Nu_{2,1} = 0.75\ Nu_t.$$

In the case considered

$$Pe_f = Re_f Pr_f = 5.45 \times 10^4 \times 5.6 \times 10^{-3} = 305;$$

$$Nu_t = 5 + 0.025\,(305)^{0.8} = 7.4.$$

With one-side heating of the coolant channel the Nusselt number and the heat-transfer coefficient are:

$$Nu_{1,1} = 0.75 \times 7.4 = 5.55;$$

$$\alpha_{1,1} = Nu_{1,1} \frac{\lambda_f}{d_{eq}} = 5.55 \frac{63.9}{6 \times 10^{-3}} = 5.9 \times 10^4\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

On the heated side of the channel the wall temperature

$$t_{w1} = t_f + \frac{q_{w1}}{\alpha_1} = 400 + \frac{7 \times 10^5}{5.9 \times 10^4} \approx 412\,^\circ\text{C}.$$

Since $q_{w2} = 0$, then $t_{w2} = t_{aw2}$. For the coolant channel of a flat-type fuel element $\Theta_{aw2} = \Theta_{aw1}$, $Nu_{1,2} = Nu_{2,1}$ and by formula (5-30)

$$\Theta_{aw2} Nu_{2,1} = \frac{1}{a + b\,Pe^{0.8}};$$

$a = 2.2$; $b = 4.5 \times 10^{-4}$ and

$$\Theta_{aw2}\,\mathrm{Nu}_{2,1} = \frac{1}{2.2 + 4.5 \times 10^{-4}\,(305)^{0.8}} = -0.445;$$

$$\mathrm{Nu}_{2,1} = \mathrm{Nu}_{1,1} = 5.55;$$

$$\Theta_{aw2} = \frac{0.445}{5.55} = -0.08.$$

The wall temperature on the non-heated side of the coolant channel

$$t_{w2} = t_{aw2} = t_f + \Theta_{aw2}\frac{q_{w1} d_{eq}}{\lambda_f} =$$

$$= 400 - 0.08\,\frac{7 \times 10^5 \times 6 \times 10^{-3}}{63.9} = 400 - 5.25 \approx 395\,^\circ\mathrm{C}.$$

Thus, $t_{w2} < t_f$, although $q_{w2} = 0$. This is traced to the fact that although the temperature of the wall is less than the mean mass temperature of the fluid, the fluid temperature gradient on the wall is equal to zero, as shown in Fig. 5-16c.

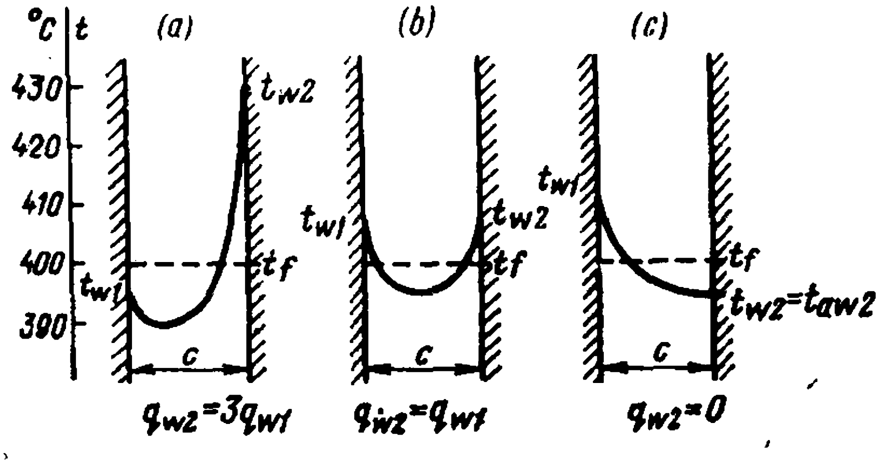

Fig. 5-16. Solution to problem 5-84

5-84. Determine wall temperatures t_{w1} and t_{w2} for the case of liquid sodium flowing in the coolant channel of a flat-type fuel element under conditions of problem 5-83, if (a) the rate of heat flow $q_{w2} = 3q_{w1} = 2.1 \times 10^6$ W/m^2 and (b) $q_{w2} = q_{w1} = 7 \times 10^5$ W/m^2.

Hint. Make use of calculation formulae (5-22) and (5-23).

Answer

(a) $t_{w1} = 396°C$, $t_{w2} = 430°C$.
(b) $t_{w1} = t_{w2} = 407°C$.

Under the conditions assumed in problems 5-83 and 5-84 the variation in liquid sodium temperature over the cross section of the channel is shown graphically in Fig. 5-16.

5-85. Determine the distribution of temperature in water along the length of the external and internal channels of a fuel element with two coolant passes (of the Field's tube type), illustrated in Fig. 5-17. The coolant water flows downward through the external annular channel, turns around, then flows upward through the internal annular channel till it leaves the fuel element.

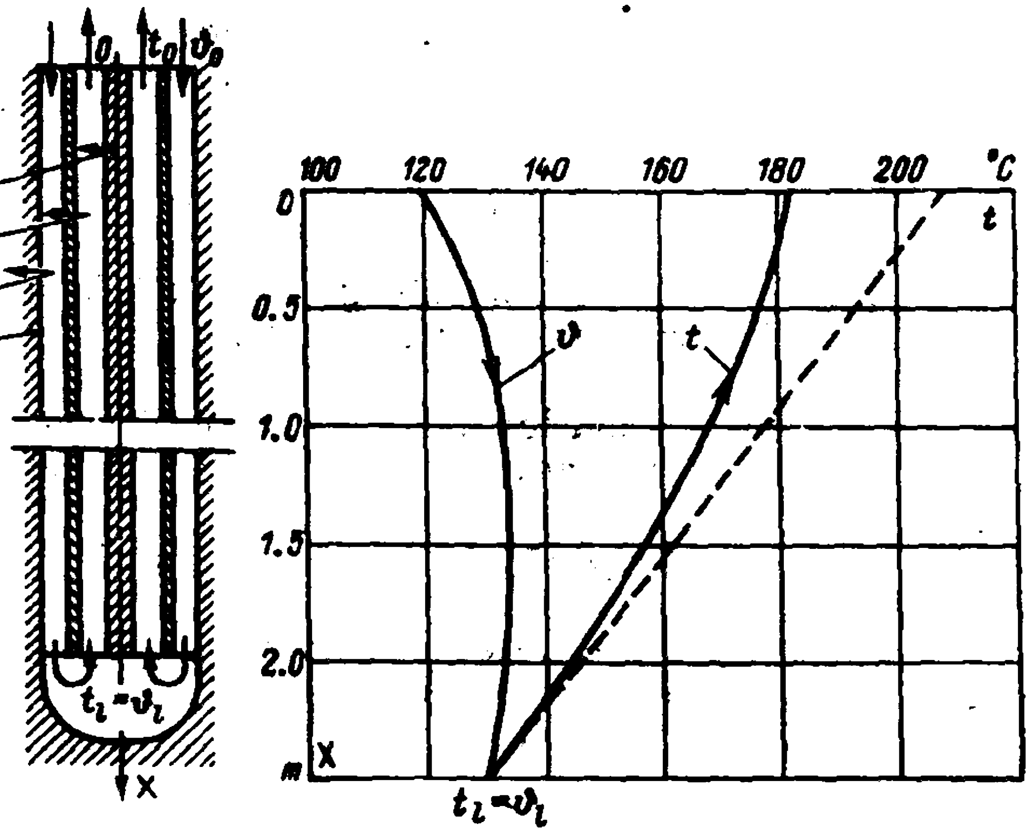

Fig. 5-17. Solution to problem 5-85

Solve the problem for the following conditions: the length of each pass $l = 2.5$ m; the inlet water temperature $\vartheta_0 = 120°C$; the rate of water flow $G = 0.22$ kg/s; the rate of heat transfer from length of 1 m of the central fuel element $q_l = 3.10^4$ W/m; the temperature of the outside surface of the external channel is constant along the length of the channel and equal to $T = 116°C$; the overall coefficient of heat transfer through the wall separating the two coolant channels $k_1 = 350$ W/m·°C; the coefficient of heat transfer to the external wall (or from the external wall) $\alpha_2 = 450$ W/m·°C; k_1 and α_2 are lengthwise constant and their magnitudes are reduced to unit length. The heat capacity of water is assumed constant: $c_p = 4.3 \times 10^3$ J/kg·°C.

Determine the temperature of water at the end of the first pass, ϑ_l, and at the outlet of the second pass, t_0, and also the coordinate x_m and the magnitude of ϑ_m which is the maximum water temperature in the first pass.

Answer

$$\vartheta_l = t_l = 130°C; \quad t_0 = 182°C; \quad x_m = 1.54\ m; \quad \vartheta_m = 134°C.$$

Temperature distribution along the first and second passes is shown graphically in Fig. 5-17.

Solution

Temperature distribution along the external annular coolant channel is determined by the following equation:

$$\vartheta = A_1 e^{\varepsilon_1 x} + A_2 e^{\varepsilon_2 x} + T + \frac{q_l}{k_2} \tag{5-31}$$

where ϑ = temperature of coolant in the external channel;
x = coordinate measured from the inlet into the external channel.

$$A_1 = \frac{C_1 \varepsilon_1 e^{\varepsilon_2 l} - \frac{q_l}{W}}{\varepsilon_1 e^{\varepsilon_2 l} - \varepsilon_2 e^{\varepsilon_1 l}};$$

$$A_2 = -\frac{C_1 \varepsilon_2 e^{\varepsilon_1 l} - \frac{q_l}{W}}{\varepsilon_1 e^{\varepsilon_2 l} - \varepsilon_2 e^{\varepsilon_1 l}};$$

$$C_1 = \vartheta_0 - T - \frac{q_l}{k_2};$$

$$\varepsilon_{1,2} = \frac{1}{2W}\left(-k_2 \pm \sqrt{k_2^2 + 4k_1k_2}\right);$$

where $W = Gc_p$ W/°C; k_1 and k_2 are the lengthwise constant overall heat-transfer coefficients reduced to unit length, W/m·°C. In the private case considered it is assumed that $k_2 = \alpha_2$ and T is the temperature of the heat-transfer surface.

With $x = l$ formula (5-31) gives the temperature of the coolant at the end of the first pass, which, naturally, is equal to that at the beginning of the second pass, $\vartheta_l = t_l$.

Determine the quantities present in Eq. (5-31):

$$W = Gc_p = 0.22 \times 4.3 \times 10^3 = 947 \ \mathrm{W/°C};$$

$$\frac{q_l}{k_2} = \frac{q_l}{\alpha_2} = \frac{3 \times 10^4}{450} = 66.7\ °\mathrm{C};$$

$$\frac{q_l}{W} = \frac{3 \times 10^4}{947} = 31.7\ °\mathrm{C/m};$$

$$C_1 = \vartheta_0 - T - \frac{q_l}{k_2} = 120 - 116 - 66.7 = -62.7\ °\mathrm{C};$$

$$\varepsilon_1 = \frac{1}{2W}\left(-k_2 \pm \sqrt{k_2^2 + 4k_1k_2}\right) =$$

$$= \frac{1}{2 \times 947}\left(-450 + 10^2\sqrt{4.5^2 + 4 \times 3.5 \times 4.5}\right) = 0.244\ 1/\mathrm{m};$$

$$\varepsilon_2 = \frac{1}{2W}\left(-k_2 - \sqrt{k_2^2 + 4k_1k_2}\right) = -0.72\ 1/\mathrm{m};$$

$$e^{\varepsilon_2 l} = e^{-0.72 \times 2.5} = 0.165;$$

$$e^{\varepsilon_1 l} = e^{0.244 \times 2.5} = 1.84;$$

$$A_1 = \frac{-62.7 \times 0.244 \times 0.165 - 31.7}{0.244 \times 0.165 + 0.72 \times 1.84} = -\frac{34.22}{1.365} = -25.1\ °\mathrm{C};$$

$$A_2 = -\frac{62.7 \times 0.72 \times 1.84 - 31.7}{1.365} = -37.6\ °\mathrm{C}.$$

The water temperature at the end of the first pass at $x = l$

$$\vartheta_l = t_l = A_1 e^{\varepsilon_1 l} + A_2 e^{\varepsilon_2 l} + T + \frac{q_l}{k_2} =$$

$$= -25.1 \times 1.84 - 37.6 \times 0.165 + 116 + 66.7 = 130.2\ °\mathrm{C}.$$

Under the given conditions the distribution of coolant temperature along the internal channel is determined by the following equation

$$t = B_1 e^{\varepsilon_1 x} + B_2 e^{\varepsilon_2 x} + T + \frac{q_l}{k_1} + \frac{q_l}{k_2}, \qquad (5\text{-}32)$$

where

$$B_1 = -\frac{C_2\varepsilon_2 + \frac{q_l}{W}}{(\varepsilon_1 - \varepsilon_2)\,e^{\varepsilon_1 l}}; \quad B_2 = \frac{C_2\varepsilon_1 + \frac{q_l}{W}}{(\varepsilon_1 - \varepsilon_2)\,e^{\varepsilon_2 l}};$$

$$C_2 = t_l - T - \frac{q_l}{k_1} - \frac{q_l}{k_2}.$$

Determine the quantities present in formula (5-32):

$$\frac{q_l}{k_1} = \frac{3 \times 10^4}{350} = 85.7\,^\circ\mathrm{C};$$

$$C_2 = 130.2 - 116 - 85.7 - 66.7 = -138.2\,^\circ\mathrm{C};$$

$$B_1 = -\frac{138.2 \times 0.72 + 31.7}{(0.244 + 0.72)\,1.84} = -74\,^\circ\mathrm{C};$$

$$B_2 = \frac{-138.2 \times 0.244 + 31.7}{(0.244 + 0.72)\,0.165} = -12.6\,^\circ\mathrm{C}.$$

According to formula (5-32), at $x = 0$, water outlet temperature

$$t_0 = B_1 + B_2 + T + \frac{q_l}{k_1} + \frac{q_l}{k_2} = -74 - 12.6 + 116 +$$
$$+ 85.7 + 66.7 = 181.8\,^\circ\mathrm{C}.$$

The coordinate of maximum temperature in the first pass is determined with the aid of formula

$$x_m = \frac{1}{\varepsilon_1 - \varepsilon_2} \ln\left(-\frac{A_2\varepsilon_2}{A_1\varepsilon_1}\right). \tag{5-33}$$

Substituting into formula (5-33) the values of the corresponding quantities present in it, we obtain:

$$x_m = \frac{1}{0.244 + 0.72} \ln\left(-\frac{37.6 \times 0.72}{-25.1 \times 0.244}\right) =$$
$$= \frac{1}{0.964} \ln(4.43) = 1.54 \text{ m}.$$

The maximum water temperature in the first pass at $x = x_m$ is determined from formula (5-31):

$$\vartheta_m = A_1 e^{\varepsilon_1 x_m} + A_2 e^{\varepsilon_2 x_m} + T + \frac{q_l}{k_2} = -25.1e^{0.244 \times 1.54} -$$
$$-37.6e^{-0.72 \times 1.54} + 116 + 66.7 =$$
$$= -36.5 - 12.4 + 182.7 = 133.8\,^\circ\mathrm{C}.$$

Temperature distribution in water along the first and second passes is shown in Fig. 5-17. The dotted line represents a linear distribution of temperature in the internal channel in the absence of heat transfer into the external channel: $t' = t_l + q_l x/W$.

5-86. Determine the distribution of water temperature along annular coolant channels of the fuel element with two coolant passes, considered in problem 5-85, if the length of the channels is increased from 2.5 to 3 m and leaving all other conditions unchanged. Compare calculation results with the answer to problem 5-85.

Answer

$\vartheta_l = 133°C$, $t_0 = 190°C$; $x_m = 1.77$ m, $\vartheta_m = 138°C$. An increase of the length and, consequently, of the amount of heat transferred by 20 per cent results in an increase in the water outlet temperature t_0 by 8°C, which corresponds to an increase in the amount of heat added to the water only by 13 per cent. This is explained by a higher rate of heat transfer from the water in the external channel.

5-87. Determine the distribution of water temperature along the coolant channels of the fuel element with two coolant passes, considered in problem 5-85, if the overall heat-transfer coefficient k_1 was increased to 600 W/m·°C by increasing the clear passage of the internal channel, maintaining the same rate of water flow, $G = 0.22$ kg/s, and leaving all other conditions unchanged. Compare calculation results with the answer to problem 5-85.

Answer

$\vartheta_l = 137°C$; $t_0 = 175°C$; $x_m = 1.57$ m; $\vartheta_m = 141.5°C$. An increase of the coefficient of heat transfer through the wall, separating the two channels, results in a more intensive heating of the water in the external channel and, consequently, leads to a greater heat loss to surroundings. That is why the outlet temperature t_0 is lower than under the conditions considered in problem 5-85.

5-88. Determine the distribution of temperature in the coolant and wall along the channel in the core of a nuclear reactor. The cylindrical fuel element has an outside diameter $d = 15$ mm and a length $l = 2.5$ m, and is made from uranium with a thermal conductivity $\lambda = 31$ W/m·°C. The fuel element has a tightly fitting cladding made from stainless steel with a thermal conductivity $\lambda = 21$ W/m·°C. The cladding thickness $\delta = 0.5$ mm.

Assume the volumetric rate of heat flow, q_v, to be constant over the cross section of the fuel element and vary along the element obeying a cosine law (the reactor has no end closures or caps). If the origin of coordinates is placed midlength of the fuel element, then at $x = 0$ $q_{v0} = 2.2 \times 10^8$ W/m³.

The fuel element is cooled with liquid sodium, flowing in the coolant channel of the element at a rate $G = 0.6$ kg/s with an inlet temperature $t_{f1} = 250$°C. The coefficient of heat transfer from the surface of the steel cladding to the liquid sodium $\alpha = 1 \times 10^5$ W/m²·°C.

Carry out calculations and determine: the temperature of sodium at midlength of the channel ($x = 0$) and at the channel outlet ($x = l/2$); the temperatures of the external and internal surfaces of the cladding at $x = 0$ and at the centreline of the fuel element at $x = 0$ ($t_{w,0}$, $t_{w1,0}$, $t_{ax,0}$); the coordinates and magnitudes of maximum temperatures $t_{w\,m}$, $t_{w1,m}$, and $t_{ax,m}$.

Answer

$$t_{f,0} = 290\,^\circ\text{C};\quad t_{f2} = 330\,^\circ\text{C};$$

$$t_{w,0} = 298\,^\circ\text{C};\quad x_{w,m} = 1.1\ \text{m};\quad t_{w,m} = 331\,^\circ\text{C};$$

$$t_{w1,0} = 316\,^\circ\text{C};\quad x_{w1,m} = 0.79\ \text{m};\quad t_{w1,m} = 338\,^\circ\text{C};$$

$$t_{ax,0} = 416\,^\circ\text{C};\quad x_{ax,m} = 0.245\ \text{m};\quad t_{ax,m} = 422\,^\circ\text{C}.$$

Solution

If heat evolves in the reactor following the cosine distribution law, the variation in coolant temperature along the channel is determined with the aid of equation

$$t_f - t_{f1} = \frac{q_0 l}{\pi G c_p}\left(\sin\frac{\pi x}{l} + 1\right), \qquad (5\text{-}34)$$

where x is the coordinate measured from the middle of the channel and q_0 represents the heat-transfer rate from unit length of the channel at $x = 0$, measured in W/m.

The outlet temperature of liquid sodium ($x = l/2$):

$$t_{f2} - t_{f1} = \frac{2}{\pi} \frac{q_0 l}{G c_p},$$

$$q_0 = q_v \frac{\pi d_1^2}{4} = 2.2 \times 10^8 \; \frac{\pi \times 15^2 \times 10^{-6}}{4} = 3.9 \times 10^4 \text{ W/m}.$$

In the temperature interval from $t_{f1} = 250°\text{C}$ to $300°\text{C}$ the heat capacity of liquid sodium $c_p = 1.29 \times 10^3$ J/kg·°C and

$$t_{f2} - t_{f1} = \frac{2}{\pi} \frac{3.9 \times 10^4 \times 2.5}{0.6 \times 1.29 \times 10^3} = 80\,°\text{C},$$

whence

$$t_{f2} = t_{f1} + 80 = 250 + 80 = 330\text{ °C}.$$

The temperature of liquid sodium at midlength of the channel ($x = 0$):

$$t_{f,0} = t_{f1} + \frac{q_0 l}{\pi G c_p} = t_{f1} + \frac{t_{f2} - t_{f1}}{2} = 250 + \frac{80}{2} = 290\,°\text{C}.$$

The difference between the temperature at the axis of the fuel element and sodium temperature at $x = 0$

$$t_{ax,0} - t_{f,0} = \frac{q_0}{\pi} \left(\frac{1}{4\lambda} + \frac{1}{2\lambda_w} \ln \frac{d_1 + 2\delta}{d_1} + \frac{1}{\alpha (d_1 + 2\delta)} \right) =$$

$$= \frac{3.9 \times 10^4}{\pi} \left(\frac{1}{4 \times 31} + \frac{1}{2.21} \ln \frac{15 + 2 \times 0.5}{15} + \right.$$

$$\left. + \frac{1}{1 \times 10^5 (15 + 2 \times 0.5) 10^{-3}} \right) = 100 + 18.4 + 7.8 = 126.2\,°\text{C}.$$

Thus,

$$\Delta t_{w,0} = t_{w,0} - t_{f,0} = 7.8°\text{C};$$

$$\Delta t_{w1,0} = t_{w1,0} - t_{f,0} = 26.2°\text{C};$$

$$\Delta t_{ax,0} = t_{ax,0} - t_{f,0} = 126.2°\text{C}.$$

At $x = 0$ the temperatures of the external and internal surfaces of the steel cladding and at the axis of the fuel element at $x = 0$ are:

$$t_{w,0} = t_{f,0} + \Delta t_{w,0} = 290 + 7.8 \approx 298°\text{C};$$

$$t_{w1,0} = t_{f,0} + \Delta t_{w1,0} = 290 + 26.2 \approx 316°\text{C};$$

$$t_{ax,0} = t_{f,0} + \Delta t_{ax,0} = 290 + 126.2 \approx 416°\text{C}.$$

The maximum temperature of the heat-transfer surface is determined from the following formula

$$\frac{t_{w,m}-t_{f1}}{t_{f2}-t_{f1}}=\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{\Delta t_{w,0}}{t_{f2}-t_{f1}}\right)^2}. \qquad (5\text{-}35)$$

Substituting into formula (5-35) the values of $\Delta t_{w,0}$ and $t_{f2}-t_{f1}$, we obtain:

$$\frac{t_{w,m}-t_{f1}}{t_{f2}-t_{f1}}=\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{7.8}{80}\right)^2}=1.01, \text{ whence}$$

$$t_{w,m}=t_{f1}+1.01\,(t_{f2}-t_{f1})=250+1.01\times 80\approx 331\,^\circ\text{C}.$$

The coordinate at which the temperature of the heat-transfer surface is maximum is determined by formula

$$x_{w,m}=\frac{l}{\pi}\arctan\left(\frac{t_{f2}-t_{f1}}{2\Delta t_{w,0}}\right); \qquad (5\text{-}36)$$

$$x_{w,m}=\frac{2.5}{\pi}\arctan\left(\frac{80}{2\times 7.8}\right)=1.1 \text{ m}.$$

The magnitudes and coordinates of maximum temperatures at the axis of the fuel element and on the internal surface of the steel cladding are determined by formulae (5-35) and (5-36), substituting in them the temperature difference $\Delta t_{w,0}$ respectively with $\Delta t_{ax,0}$ and $\Delta t_{w1,0}$:

$$\frac{t_{ax,m}-t_{f1}}{t_{f2}-t_{f1}}=\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{\Delta t_{ax,0}}{t_{f2}-t_{f1}}\right)^2}=$$

$$=0.5+\sqrt{0.25+\left(\frac{126.2}{80}\right)^2}=2.15;$$

$$t_{ax,m}=250+2.15\times 80=422\,^\circ\text{C}.$$

$$\frac{t_{w1,m}-t_{f1}}{t_{f2}-t_{f1}}=\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{\Delta t_{w1,0}}{t_{f2}-t_{f1}}\right)^2}=$$

$$=0.5+\sqrt{0.25+\left(\frac{26.2}{80}\right)^2}=1.1;$$

$$t_{w1,m}=250+1.1\times 80=338\,^\circ\text{C};$$

$$x_{ax,m}=\frac{l}{\pi}\arctan\left(\frac{t_{f2}-t_{f1}}{2\Delta t_{ax,0}}\right)=$$

$$=\frac{2.5}{\pi}\arctan\left(\frac{80}{2\times 126.2}\right)=0.245 \text{ m};$$

$$x_{w1,m}=\frac{2.5}{\pi}\arctan\left(\frac{80}{2\times 26.2}\right)=0.632 \text{ m}.$$

5-89. Determine the distribution of the coolant temperature and the wall temperature along the fuel element considered in problem 5-88, if water is used as a coolant in the nuclear reactor.

The rate of water flow $G = 0.6$ kg/s, and water inlet temperature $t_{f1} = 180°\mathrm{C}$. The coefficient of heat transfer from the surface of the fuel-element cladding to water $\alpha = 2.5 \times 10^{-4}$ W/m$^2\cdot$°C. All other conditions are as in problem 5-88.

Calculate the magnitudes of quantities as in problem 5-88.

Answer

$$t_{f,0} = 191.6°\mathrm{C}; \quad t_{f2} = 203°\mathrm{C};$$

$$t_{w,0} = 223°\mathrm{C}; \quad x_{w,m} = 0.263\ \mathrm{m}; \quad t_{w,m} = 225°\mathrm{C};$$

$$t_{w1,0} = 241°\mathrm{C}; \quad x_{w1,m} = 0.183\ \mathrm{m}; \quad t_{w1,m} = 243°\mathrm{C};$$

$$t_{ax,0} = 341°\mathrm{C}; \quad x_{ax,m} = 0.0615\ \mathrm{m}; \quad t_{ax,m} = 341°\mathrm{C}.$$

Chapter Six

HEAT TRANSFER WITH TUBES IN FORCED CROSS FLOW

6-1. A copper bus bar of round cross section, $d = 15$ mm, is cooled with a cross flow of dry air (Fig. 6-1). The velocity and temperature of the free-stream air flow are respectively $w = 1$ m/s and $t_f = 20$°C.

Calculate the coefficient of heat transfer from the surface of the copper bus bar to the cooling air and the admissible current intensity for the bus bar on condition that its surface temperature should not exceed $t_w = 80$°C.

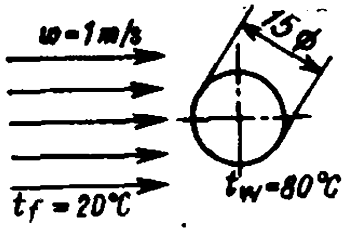

Fig. 6-1. Solution to problem 6-1

The resistivity of copper $\rho = 0.0175$ Ohm·mm²/m.

Answer

$$\alpha = 23.8\ \text{W/m}^2\cdot{}^\circ\text{C};\quad I = 825\ \text{A}.$$

Solution

At a temperature $t_f = 20$°C the physical properties of air are: $\nu_f = 15.06 \times 10^{-6}$ m²/s, $\lambda_f = 2.59 \times 10^{-2}$ W/m·°C.

The Reynolds number

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{1 \times 0.015}{15.06 \times 10^{-6}} = 995.$$

For a single cylinder placed in cross air flow heat transfer can be calculated with the aid of the following formulae [3]:

$$\left.\begin{aligned}&\text{at } 10 \leqslant \mathrm{Re}_f \leqslant 1\times10^3 \qquad \mathrm{Nu}_f = 0.44\mathrm{Re}_f^{0.5};\\ &\text{at } 1\times10^3 \leqslant \mathrm{Re}_f \leqslant 2\times10^5 \qquad \mathrm{Nu}_f = 0.22\mathrm{Re}_f^{0.6},\end{aligned}\right\} \quad (6\text{-}1)$$

where taken as the reference dimension is the diameter of the cylinder and as the reference temperature that of the free-stream air flow, t_f.

In the case considered

$$\mathrm{Nu}_f = 0.44\,(995)^{0.5} = 13.8$$

and the heat-transfer coefficient

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 13.8\,\frac{2.59\times10^{-2}}{1.5\times10^{-2}} = 23.8\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The admissible current intensity is determined from the energy balance equation

$$\alpha\,(t_w - t_f)\,\pi d l = I^2 R,$$

where

$$R = \frac{\rho l}{\frac{\pi d^2}{4}}\,,\ \mathrm{ohm},$$

whence the following expression for admissible current intensity

$$I = 10^3 \pi d \sqrt{\frac{\alpha \Delta t d}{4\rho}}\,.$$

Substituting known quantities, we obtain:

$$I = 10^3 \times 3.14 \times 1.5 \times 10^{-2} \sqrt{\frac{23.8\times(80-20)\,1.5\times10^{-2}}{4\times0.0175}} = 825\ \mathrm{A}.$$

6-2. How will change the coefficient of heat transfer from the surface of the copper bus bar and the admissible current intensity, if the velocity of the free-stream air flow is reduced by one half and all other conditions remain as in problem 6-1?

Answer

$\alpha = 16.9$ W/m²·°C; $I = 692$ A, i.e. the coefficient of heat transfer from the surface of the bus bar will be reduced $\sqrt{2} \approx 1.4$ times and the admissible current intensity $\sqrt[4]{2} \approx 1.2$ times.

6-3. How shall change the coefficient of heat transfer from the surface of the bus bar and the admissible current carried by it, if the diameter of the bus bar is reduced by one half, leaving all other conditions unchanged, as in problem 6-1.

Answer

$$\alpha = 34 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad I = 348 \text{ A}.$$

6-4. A tubular water calorimeter with an outside diameter $d = 15$ mm is placed in a cross flow of air. The velocity of the air, $w = 2$ m/s, is directed at an angle of 90° to the axis of the calorimeter and the mean air temperature $t_f = 20$°C. Under steady-state conditions the outer surface of the calorimeter acquires a constant mean temperature $t_w = 80$°C. Calculate the coefficient of heat transfer from the tube to the air and the heat transfer rate from unit length of the calorimeter.

Answer

$$\alpha = 36.3 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad q_l = 102 \text{ W/m}.$$

Solution

At a temperature $t_f = 20$°C the physical properties of air are: $\nu_f = 15.06 \times 10^{-6}$ m²/s, $\lambda_f = 2.59 \times 10^{-2}$ W/m·°C.

The Reynolds number

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{2 \times 15 \times 10^{-3}}{15.06 \times 10^{-6}} = 1990.$$

Since $1 \times 10^3 < \text{Re} < 2 \times 10^5$, according to formula (6-1)

$$\text{Nu}_f = 0.22\text{Re}_f^{0.6}.$$

Then

$$\text{Nu}_f = 0.22\,(1.99 \times 10^3)^{0.6} = 21$$

and the heat-transfer coefficient

$$\alpha = \text{Nu}_f \frac{\lambda_f}{d} = \frac{21 \times 2.59 \times 10^{-2}}{1.5 \times 10^{-2}} = 36.3 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The heat transfer rate from unit length

$$q_l = \alpha (t_w - t_f) \pi d = 36.3 (80 - 20) 3.14 \times 1.5 \times \times 10^{-2} = 102 \text{ W/m}.$$

6-5. Under the conditions of problem 6-4 how shall change the heat-transfer coefficient, if air velocity is increased 2 and 4 times?

Answer

The heat-transfer coefficient shall increase respectively 1.51 and 2.3 times.

6-6. Under the conditions of problem 6-4 how shall change the heat transfer coefficient, if air flows past the tube with an angle of attack $\psi = 60°$ (Fig. 6-2), all other conditions of the problem remaining unchanged?

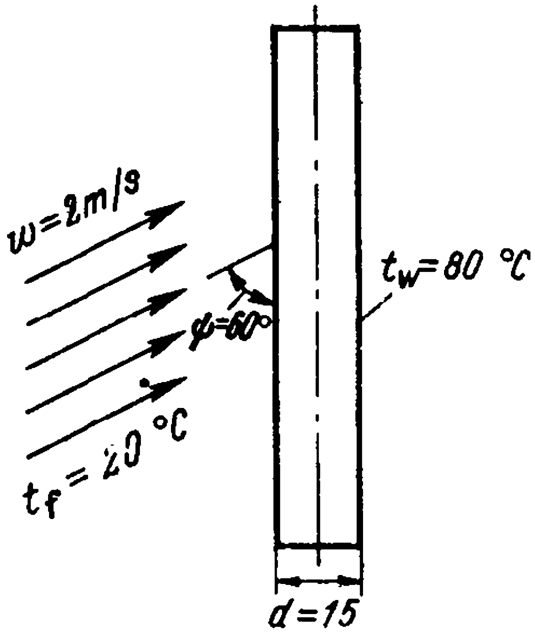

Fig. 6-2. Solution to problem 6-6

Answer

$$\alpha_{\psi=60°} = 33.7 \text{ W/m}^2\cdot°\text{C}.$$

Solution

When a fluid flows past a single cylinder with an angle of attack differing from 90°, then

$$\alpha_\psi = \varepsilon_\psi \alpha, \tag{6-2}$$

where α_ψ and α = respectively heat-transfer coefficients at a given angle of attack and an angle of attack equal 90°;

ε_ψ = correction for the angle of attack ψ the values of which, depending on the angle ψ are given below:

$\psi°$	90	80	70	60	50	40	30
ε_ψ	1	1	0.99	0.93	0.87	0.76	0.66

In the case considered at $\psi = 60°$ $\varepsilon_\psi = 0.93$ and, consequently,

$$\alpha_{\psi=60°} = 0.93 \times 36.3 = 33.7 \text{ W/m}^2\cdot°\text{C}.$$

6-7. A cylindrical tube having a diameter $d = 20$ mm is cooled in a cross flow of water. Flow velocity $w = 1$ m/s. The mean temperature of the cooling water $t_f = 10°\text{C}$ and the surface of the tube is at a temperature $t_w = 50°\text{C}$. Determine the coefficient of heat transfer from the surface of the tube to the cooling water.

Answer

$$\alpha = 7050 \text{ W/m}^2\cdot°\text{C}.$$

Solution

At a water temperature $t_f = 10°\text{C}$

$$\nu_f = 1.3 \times 10^{-6} \text{ m}^2/\text{s}.$$

The Reynolds number

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{1 \times 0.02}{1.3 \times 10^{-6}} = 1.54 \times 10^4.$$

For a single cylinder placed in cross flow of a liquid heat transfer can be calculated with the aid of the following formulae [4]:

$$\left.\begin{aligned} &\text{at } 8 < \text{Re}_f < 1 \times 10^3 \\ &\qquad \text{Nu}_f = 0.50\text{Re}_f^{0.5}\text{Pr}_f^{0.38}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25}; \\ &\text{at } 1 \times 10^3 < \text{Re}_f < 2 \times 10^5 \\ &\qquad \text{Nu}_f = 0.25\text{Re}_f^{0.6}\text{Pr}_f^{0.38}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25}, \end{aligned}\right\} \quad (6\text{-}3)$$

where taken as the reference dimension is the diameter of the cylinder, and the subscripts "f" and "w" indicate that the corresponding physical properties are taken based on the temperature of free-stream flow of liquid t_f and the temperature of the liquid near the wall t_w. In the case considered $1 \times 10^3 < \mathrm{Re}_f < 2 \times 10^5$ and calculations should be done with the second formula.

At $t_f = 10°\mathrm{C}$ $\nu_f = 1.3 \times 10^{-6}$ m²/s, $\lambda_f = 0.574$ W/m·°C, and the Prandtl number $\mathrm{Pr}_f = 9.5$.

At a temperature $t_w = 50°\mathrm{C}$ $\mathrm{Pr}_w = 3.55$. Consequently,

$$\mathrm{Nu}_f = 0.25\,(1.54 \times 10^4)^{0.6}\,(9.5)^{0.38}\left(\frac{9.5}{3.55}\right)^{0.25} = 246$$

and the heat-transfer coefficient

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 246\,\frac{0.574}{0.02} = 7050\ \mathrm{W/m^2\cdot°C}.$$

6-8. Compare the coefficients of heat transfer from the surface of the tube to the cooling air:

(a) with air flowing in a long tube having an inside diameter $d_i = 50$ mm;

(b) with air flowing past a tube having an outside diameter $d_o = 50$ mm.

Carry out the comparison for air velocities $w = 5$, 10, 20, and 50 m/s. Assume a mean air temperature $t_f = 50°\mathrm{C}$ in all cases.

Answer

Calculation results are given below:

w, m/s	5	10	20	50
α_o/α_i	1.8	1.56	1.36	1.14

6-9. A tube having an outside diameter $d = 25$ mm is cooled by placing it in a cross flow of transformer oil. The velocity and mean temperature of the transformer oil are respectively $w = 1$ m/s and $t_f = 20°\mathrm{C}$.

Determine what tube surface temperature should be maintained, to ensure a rate of heat flow $q = 4.5 \times 10^4$ W/m², and calculate the magnitude of the heat-transfer coefficient.

Answer

$$t_w = 70°\mathrm{C};\quad \alpha = 925\ \mathrm{W/m^2\cdot°C}.$$

Solution

Determine the pattern of flow of transformer oil. At $t_f = 20°C$ $\nu_f = 22.5 \times 10^{-6}$ m²/s;

$$\mathrm{Re}_f = \frac{wd}{\lambda_f} = \frac{1 \times 2.5 \times 10^{-2}}{22.5 \times 10^{-6}} = 1.11 \times 10^3.$$

Since the Reynolds number falls within $1 \times 10^3 \leqslant \mathrm{Re}_f \leqslant 2 \times 10^5$, formula (6-3) gives:

$$\mathrm{Nu}_f = 0.25\mathrm{Re}_f^{0.6}\mathrm{Pr}_f^{0.38}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25}.$$

The formula contains the Prandtl number for oil, taken at a wall temperature. The problem must, therefore, be solved either by the method of successive approximations or graphically. Let us use the latter.

For a graphical solution assume three values of a wall temperature: $t_{w1} = 40°C$, $t_{w2} = 60°C$, and $t_{w3} = 80°C$; calculate the rates of heat flow at these temperatures and plot q as a function of t_w, $q = f(t_w)$.

Having assumed $t_{w1} = 40°C$, calculate the heat-transfer coefficient by the above mentioned formula.

At $t_f = 20°C$ $\nu_f = 22.5 \times 10^{-6}$ m²/s; $\lambda_f = 0.1106$ W/m·°C, and the Prandtl number $\mathrm{Pr}_f = 298$.

At a wall temperature $t_{w1} = 40°C$ $\mathrm{Pr}_{w1} = 146$;

$$\mathrm{Nu}_1 = 0.25\,(1.11 \times 10^3)^{0.6}\,(298)^{0.38}\left(\frac{298}{146}\right)^{0.25} = 175.$$

$$\alpha_1 = \mathrm{Nu}_1\frac{\lambda_f}{d} = 175\frac{0.1106}{0.025} = 775 \text{ W/m}^2\cdot°\text{C}.$$

At $t_{w1} = 40°C$ the rate of heat flow

$$q_1 = \alpha_1 \Delta t_1 = 775\,(40 - 20) = 15\,500 \text{ W/m}^2.$$

At a temperature $t_{w2} = 60°C$ the heat-transfer coefficient shall differ from α_1 only due to the change in the Prandtl number, therefore,

$$\alpha_2 = \alpha_1\left(\frac{\mathrm{Pr}_{w1}}{\mathrm{Pr}_{w2}}\right)^{0.25} \quad \text{and} \quad q_2 = q_1\left(\frac{\mathrm{Pr}_{w1}}{\mathrm{Pr}_{w2}}\right)^{0.25}\frac{t_{w2} - t_f}{t_{w1} - t_f}.$$

At $t_{w2} = 60$ °C $\mathrm{Pr}_{w2} = 87.8$ and

$$q_2 = 1.55 \times 10^4\left(\frac{146}{87.8}\right)^{0.25}\frac{60 - 20}{40 - 20} = 3.51 \times 10^4 \text{ W/m}^2.$$

At $t_{w3} = 80\,^\circ\mathrm{C}$ $\mathrm{Pr}_{w3} = 59.3$ and

$$q_3 = 1.55 \times 10^4 \left(\frac{146}{59.3}\right)^{0.25} \frac{80-20}{40-20} = 5.78 \times 10^4\ \mathrm{W/m^2}$$

The calculated values of q are used to plot the graph shown in Fig. 6-3:

$$q = f\,(t_w).$$

From the graph we find that with the given value of the rate of heat flow, $q = 4.5 \times 10^4$ W/m², the wall temperature $t_w = 70$°C.

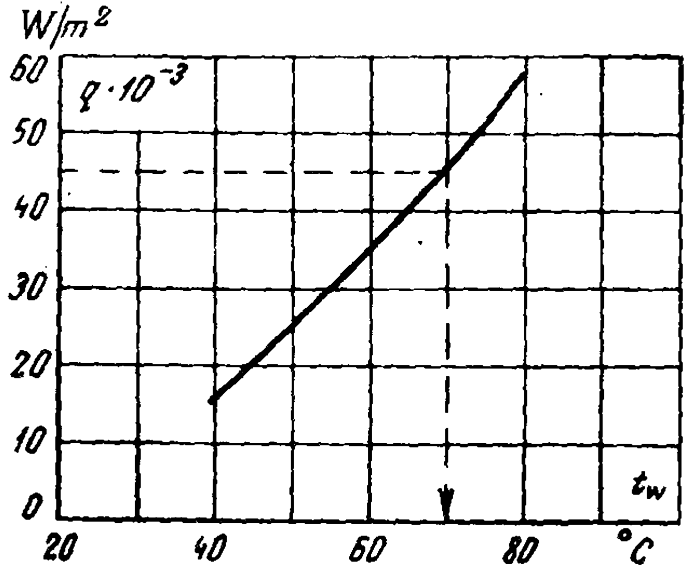

Fig. 6-3. Solution to problem 6-9

At the determined temperature $t_w = 70$°C calculate the heat-transfer coefficient.

At $t_w = 70$°C $\mathrm{Pr}_w = 71.3$. Then,

$$\mathrm{Nu}_f = \mathrm{Nu}_1 \left(\frac{\mathrm{Pr}_{w1}}{\mathrm{Pr}_w}\right)^{0.25} = 175 \left(\frac{146}{71.3}\right)^{0.25} = 209$$

and

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 209\ \frac{0.1106}{0.025} = 925\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

6-10. A tube is being cooled in cross flow of transformer oil under the same conditions as in problem 6.9. Cooling specifications require, however, that the rate of heat flow on the surface of the tube should not exceed 3.5 × 10⁴ W/m².

To ensure this regime of cooling, what should the tube surface temperature and the magnitude of the heat-transfer coefficient be?

Answer

$$t_w = 62°C; \quad \alpha = 890\ W/m^2 \cdot °C.$$

6-11. With a liquid in cross flow past a single cylinder determine the ratio of the heat-transfer coefficients under conditions of heating (α_h) and cooling (α_c) of the liquid.

Comparing the heat-transfer coefficients, assume the same velocities and mean temperatures of the liquid and equal temperature differences.

Answer

$$\frac{\alpha_h}{\alpha_c} = \left(\frac{Pr_{w.c}}{Pr_{w.h}}\right)^{0.25}.$$

6-12. Compare the values of the heat-transfer coefficients for a tube of a diameter $d = 8$ mm placed in cross flow of water and grade MC oil.

Assume the same velocities and mean temperatures of the liquids, equal to respectively $w = 2$ m/s and $t_f = 70°C$, and a tube surface temperature $t_w = 90°C$.

Also determine how shall change the heat-transfer coefficients for water and oil, if with the same mean temperature of the liquids and the same temperature difference the cooling of the liquid involved shall take place ($t_f = 70°C$ and $t_w = 50°C$).

Answer

Liquid	α, W/m²·°C	
	Heating	Cooling
Water	17 900	15 500
Oil	2 070	1 370

6-13. Determine the mean coefficient of heat transfer by convection from cross flow of flue gas of the following composition (by volume):

$$\bar{p}_{H_2O} = 0.11; \quad \bar{p}_{CO_2} = 0.13 \text{ and } \bar{p}_{N_2} = 0.76$$

to a bank of staggered boiler tubes having a diameter $d = 80$ mm. The transverse and longitudinal pitches of the staggered bank of tubes are respectively $s_1 = 2.5d$ and $s_2 = 2d$ The mean velocity of the flue gas in the narrow cross section of the bank of tubes $w = 10$ m/s.

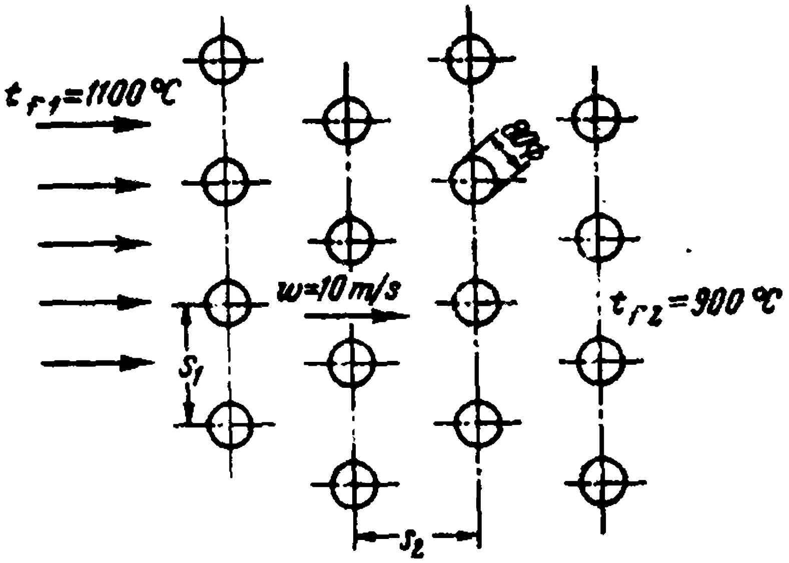

Fig. 6-4. Solution to problem 6-13

In the direction of gas flow, the bank is built-up of four rows of tubes with an equal surface (Fig. 6-4). The temperature of the gas before the bank of tubes is $t_{f1} = 1100$°C and after the bank of tubes, $t_{f2} = 900$°C. The fouling of the surfaces of tubes should be ignored.

Answer

$$\bar{\alpha} = 62\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Solution

With a clean bank of tubes in cross flow of flue gas the rate of heat transfer can be calculated by the following formula [4]:

$$\mathrm{Nu}_f = C\mathrm{Re}_f^n\mathrm{Pr}_f^{0.33}\,\varepsilon_s, \tag{6-4}$$

where: for a bank of tubes of the in-line arrangement $C = 0.26$ and $n = 0.65$; for a bank of tubes of the staggered arrangement $C = 0.41$ and $n = 0.60$; taken as the reference dimension is the tube diameter, and as reference temperature—the mean temperature of the fluid (gas) $t_f = 0.5\,(t_{f1} + t_{f2})$; also taken as reference is the mean velocity in the narrowest cross section in the bank of tubes.

The quantity ε_s is the correction factor, allowing for the effect of the relative pitches.

For an in-line bank of tubes $\varepsilon_s = (s_2/d)^{-0.15}$;

for a staggered bank of tubes:

with $s_1/s_2 < 2$ $\varepsilon_s = (s_1/s_2)^{1/6}$;

with $s_1/s_2 \geqslant 2$ $\varepsilon_s = 1.12$.

The formula is true at $10^3 \leqslant \mathrm{Re}_f \leqslant 10^5$.

The heat-transfer coefficient calculated with the aid of formula (6-4) corresponds to the magnitude of that for the third and all subsequent rows of tubes in the bank.

The heat-transfer coefficient for the first row of the bank of tubes, α_1, is determined as $\alpha_1 = 0.6\alpha_3$.

The heat-transfer coefficient for the second row of an in-line bank of tubes, α_2, is equal to $0.9\alpha_3$, and in a staggered bank of tubes $\alpha_2 = 0.7\alpha_3$.

In the case considered the reference temperature

$$t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(1100 + 900) = 1000\text{°C}.$$

At that temperature the physical properties of flue gas of the given composition are as follows

$$\nu_f = 174.3 \times 10^{-6}\ \mathrm{m^2/s};\ \lambda_f = 0.109\ \mathrm{W/m^2{\cdot}°C};\ \mathrm{Pr}_f = 0.58.$$

The Reynolds number

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{10 \times 8 \times 10^{-2}}{174.3 \times 10^{-6}} = 4.59 \times 10^3.$$

Since $10^3 < \mathrm{Re}_f < 10^5$, formula (6-4) is also applicable to a bank of tubes of a staggered arrangement for which $C = 0.41$, $n = 0.6$.

The ratio $s_1/s_2 = 2.5/2.0 = 1.25$ and the correction factor

$$\varepsilon_s = (1.25)^{1/6} = 1.04.$$

The Nusselt number

$$\mathrm{Nu}_f = 0.41\,(4.59 \times 10^3)^{0.6}\,(0.58)^{0.33} \times 1.04 = 55.$$

The heat-transfer coefficient for the third row of tubes

$$\alpha_3 = \mathrm{Nu}_f \frac{\lambda_f}{d} = 55 \frac{0.109}{8 \times 10^{-2}} = 75\ \mathrm{W/m^2{\cdot}°C}.$$

With equal surfaces of rows of tubes, the mean heat-transfer coefficient

$$\bar{\alpha} = \frac{1}{4}\sum_{i=1}^{4}\alpha_i = \frac{1}{4}(0.6\alpha_3 + 0.7\alpha_3 + 2\alpha_3) =$$

$$= \frac{3.3}{4}75 = 62 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

6-14. How will change the mean heat-transfer coefficient for the bank of tubes considered in problem 6-13, if, under the same conditions, the number of rows of tubes is increased in the direction of gas flow by 2, 3, 4 and 5 times, leaving all other data unchanged?

Answer

With $n > 3$ $\bar{\alpha} = \alpha_3\left(1 - \frac{0.7}{n}\right)$;

with $n = 8$ $\bar{\alpha} = 68.5$ W/m²·°C;
with $n = 12$ $\bar{\alpha} = 70.5$ W/m²·°C;
with $n = 16$ $\bar{\alpha} = 71.8$ W/m²·°C;
with $n = 20$ $\bar{\alpha} = 72.4$ W/m²·°C.

6-15. An air heater, constructed as a staggered bank of tubes, is exposed to cross flow of air. The diameter of the tubes $d = 50$ mm; the transverse pitch of the bank $s_1 = 100$ mm, and the longitudinal pitch $s_2 = 200$ mm. The

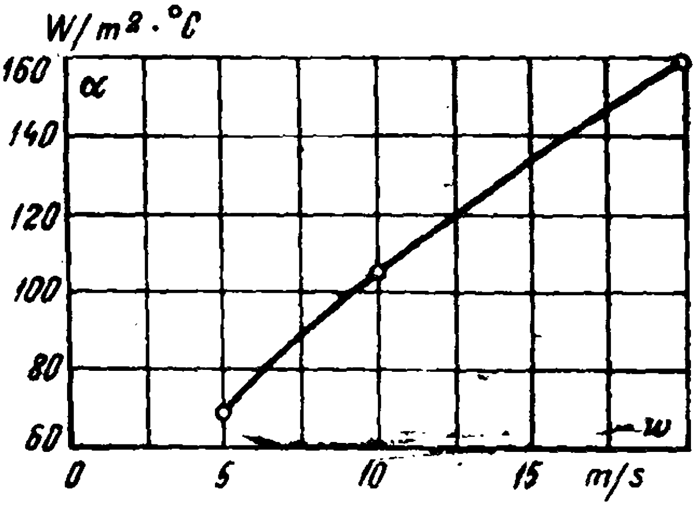

Fig. 6-5. Solution to problem 6-15

mean temperatures of the air and of the outside surface of the tubes are respectively $t_f = 100°C$ and $t_w = 200°C$.

Plot the heat-transfer coefficient as a function of air velocity in the narrowest cross section of the bank of tubes, assuming a range of velocities from 5 to 20 m/s. The number of tubes in the direction of gas flow $n > 20$; the effect produced by the first two rows of tubes on the mean heat-transfer coefficient can be ignored.

Answer

Calculation results are represented below and illustrated graphically in Fig. 6-5.

w, m/s	5	10	20
α, W/m²·°C	69	105	159

6-16. Compare the heat-transfer coefficients for the third row (in the direction of air flow) of tubes of two air heaters, featuring a staggered-tube arrangement. The two banks of tubes are exposed to cross flows of air moving with a velocity $w = 10$ m/s at a mean temperature $t_f = 100°C$. The diameters of tubes in the two air heaters are respectively $d_1 = 50$ mm and $d_2 = 25$ mm. Assume the same ratio of pitches, s_1/s_2, for the two air heaters compared.

Answer

$$\alpha_2 = 2^{0.4} \times \alpha_1 \approx 1.32\alpha_1.$$

6-17. Compare the heat-transfer coefficients for the third row of tubes of the air heaters considered in problem 6-16, under the condition that the tubes of the heaters are not of the staggered but of the in-line arrangement.

Carry out the comparison, assuming: (a) that the relative pitches of the two air heaters, s_1/d and s_2/d are the same; (b) that the transverse and longitudinal pitches are the same, s_1 and s_2

Answer

(a) $\alpha_2 = 2^{0.35} \times \alpha_1 \approx 1.27$;
(b) $\alpha_2 = 2^{0.5} \times \alpha_1 \approx 1.41$.

6-18. The staggered bank of tubes of a heat exchanger is exposed to cross flow of transformer oil. The outside diameter of tubes in the bank $d = 20$ mm. The transverse pitch $s_1 = 2.5d$, the longitudinal pitch $s_2 = 1.5d$. The mean velocity in the narrowest section of the heat exchanger and the mean oil temperature are respectively $w = 0.6$ m/s and $t_f = 40$°C.

Find the coefficient of heat transfer from the surface of tubes to the oil for the third row of tubes under the condition that a tube surface temperature $t_w = 90$°C.

Carry out calculations for two cases:

(a) transformer oil flows past the tubes with an angle of attack $\psi = 90°$;

(b) transformer oil flows past the tubes with an angle of attack $\psi = 60°$.

Answer

$$\alpha_{\psi=90°} = 1130 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad \alpha_{\psi=60°} = 1060 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Solution

With a liquid in cross flow past a bank of tubes the heat-transfer coefficients can be calculated with the aid of formula (6-4) introducing a correction for the change in the physical properties of the liquid over the cross section of the flow, expressed in the form of the ratio $(\text{Pr}_f/\text{Pr}_w)^{0.25}$ [4]. Then,

$$\text{Nu}_f = C\text{Re}_f^n\text{Pr}_f^{0.33}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25}\varepsilon_s, \qquad (6\text{-}5)$$

where Pr_w is the Prandtl number for the liquid taken at the wall temperature, while the constants and other quantities, just as the range of application, are the same as in problem (6-4).

In the case considered, at $t_f = 40$°C, $\nu_f = 10.3 \times \times 10^{-6}$ m²/s, $\lambda_f = 0.109$ W/m·°C, $\text{Pr}_f = 146$; at $t_w = = 90$°C $\text{Pr}_w = 50.5$.

The Reynolds number

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{0.6 \times 0.02}{10.3 \times 10^{-6}}\, 1165 > 10^3$$

and formula (6-5) is applicable.

Since $s_1/s_2 = 2.5/1.5 = 1.66 < 2$, then

$$\varepsilon_s = (s_1/s_1)^{1/6} = (1.66)^{1/6} = 1.08.$$

Then, for the third row of the staggered bank of tubes

$$\mathrm{Nu}_f = 0.41\mathrm{Re}_f^{0.6}\mathrm{Pr}_f^{0.33}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25}\varepsilon_s =$$

$$= 0.41\,(1165)^{0.6}(146)^{0.33}\left(\frac{146}{50.5}\right)^{0.25}1.08 = 208.$$

At an angle of attack $\psi = 90°$ the heat-transfer coefficient

$$\alpha = \mathrm{Nu}_f\frac{\lambda_f}{d} = 208\frac{0.109}{0.02} = 1130\ \mathrm{W/m^2\cdot{}^\circ C}.$$

When liquid flows past the bank of tubes with an angle of attack other than $\psi = 90°$,

$$\alpha_\psi = \varepsilon_\psi\ \alpha,$$

where ε_ψ is the correction for the angle of attack, the values of which depending on ψ are given below:

$\psi°$	90	80	70	60	50	40	30	20	10
ε_ψ	1.0	1.0	0.98	0.94	0.88	0.78	0.67	0.52	0.42

In the case considered, at $\psi = 60°$, the correction $\varepsilon_\psi = 0.94$ and

$$\alpha_{\psi=60°} = 0.94 \times 1130 = 1060\ \mathrm{W/m^2\cdot{}^\circ C}.$$

6-19. Assuming conditions of problem 6-18, how shall change the heat-transfer coefficient for the third row of tubes, if the bank of tubes is exposed to cross flow of water, all other conditions remaining unchanged ($d = 20$ mm; $w = 0.6$ m/s, $t_f = 40°C$; $t_w = 90°C$?

Compare the results assuming the same angles of attack, i.e. $\psi = 90°$ and $\psi = 60°$.

Answer

$$\alpha_{\psi=90°} = 9950\ \mathrm{W/m^2\cdot{}^\circ C};\quad \alpha_{\psi=60°} = 9350\ \mathrm{W/m^2\cdot{}^\circ C}.$$

6-20. How shall change the heat-transfer coefficient for the third row of a staggered bank of tubes placed in cross flow of transformer oil and water under the conditions of problems 6-18 and 6-19, if instead of being heated the liquid is cooled at the same temperature difference as in problem 6-18, i.e. at a mean flow temperature $t_f = 90°C$ and a mean wall temperature $t_w = 40°C$. All other quantities remain unchanged (d = 20 mm; w = 0.6 m/s). Compare the results obtained for an angle of attack $\psi = 90°$.

Answer

When transformer oil is cooled $\alpha = 921$ W/m^2·°C, i.e. the heat-transfer coefficient shall decrease almost by 18 per cent. When water is cooled $\alpha = 8400$ W/m^2·°C, i.e. the heat-transfer coefficient shall diminish about by 15 per cent.

6-21. Compare the heat-transfer coefficients calculated for the third row of an in-line bank of tubes α_{tn} and for a single tube α_t placed in a cross flow of fluid, with the Prandtl number Pr_f varying from 1×10^3 to 1×10^5.

Carry out the comparison at the Prandtl number $\mathrm{Pr} = 1$ for tubes of the same diameter and at equal liquid and tube surface temperatures. Assume a correction factor $\varepsilon_s = 1$.

Answer

Re_f	1×10^3	1×10^4	1×10^5
α_{tn}/α_t	1.32	1.48	1.67

6-22. A tubular air heater is to be constructed from tubes of a diameter d = 38 mm in an in-line arrangement with transverse and longitudinal pitches $s_1 = s_2 = 2.5d$. The number of tubes in a row across the flow is set to be $m = 8$, the number of rows $n = 5$ (Fig. 6-6).

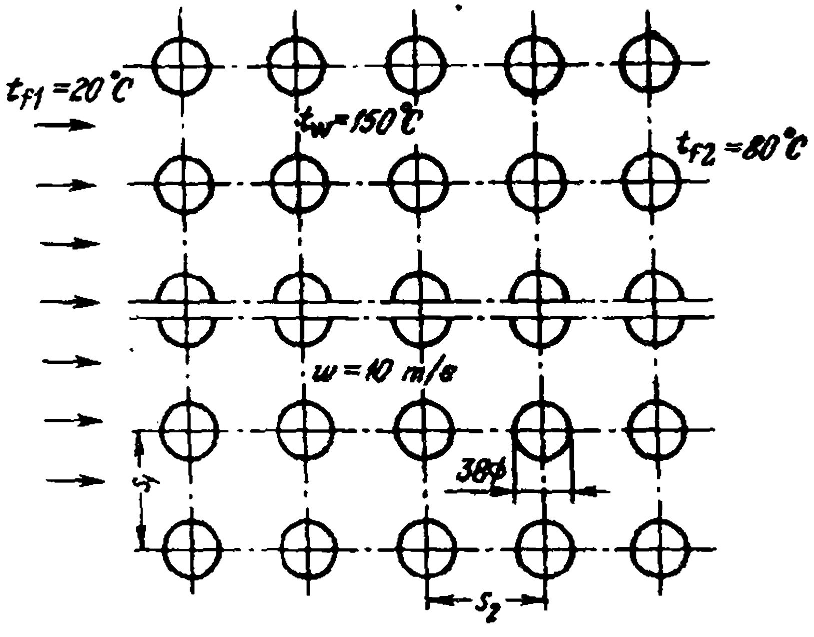

Fig. 6-6. Solution to problem 6-22

The temperature of the air entering the air heater $t_{f1} = 20°C$ and at the outlet of the heater $t_{f2} = 80°C$. The temperature of the outside surface of tubes $t_w = 150°C$. How long should the tubes be, so that at an air velocity $w = 10$ m/s the amount of heat transferred to the air $Q = 125$ kW?

Answer

$$l = 3\text{m}.$$

Solution

The mean temperature of air

$$t_f = 0.5\,(t_{f1} + t_{f2}) = 0.5\,(20 + 80) = 50°\text{C}.$$

At $t_f = 50°C$ $\nu_f = 17.95 \times 10^{-6}$ m²/s;

$$\lambda_f = 2.83 \times 10^{-2}\ \text{W/m·°C}$$

and

$$\text{Re}_f = \frac{wd}{\nu_f} = \frac{10 \times 3.8 \times 10^{-2}}{17.95 \times 10^{-6}} = 2.12 \times 10^4.$$

With $10^3 \leqslant \text{Re}_f \leqslant 10^5$, by formula (6-4), we have for the third row of an in-line bank of tubes

$$\text{Nu}_f = 0.26\,\text{Re}_f^{0.65}\text{Pr}_f^{0.33}\varepsilon_s.$$

For air $\text{Pr}_f \approx 0.70$ and formula (6-4) reduces into

$$\text{Nu}_f = 0.23\text{Re}_f^{0.65}\varepsilon_s,$$

where $\varepsilon_s = (s_2/d)^{-0.15} = (2.5)^{-0.15} = 0.87.$

Substituting the values found, we obtain:

$$\text{Nu}_f = 0.23\,(2.12 \times 10^4)^{0.65}\,0.87 = 130.$$

The heat-transfer coefficient for the third row of tubes

$$\alpha_3 = \text{Nu}_f\,\frac{\lambda_f}{d} = 130\,\frac{2.83 \times 10^{-2}}{3.8 \times 10^{-2}} = 96.8\ \text{W/m}^2\cdot°\text{C}.$$

With $n \geqslant 3$ the mean heat-transfer coefficient of an in-line bank of tubes

$$\bar{\alpha} = \alpha_3\left(1 - \frac{0.5}{n}\right) = 96.8\left(1 - \frac{0.5}{5}\right) = 87.2\ \text{W/m}^2\cdot°\text{C}.$$

The rate of heat flow and the required heating surface:

$$q = \bar{\alpha}\,(t_w - t_f) = 87.2\,(150 - 50) = 8720\ \text{W/m}^2$$

and

$$F = \frac{Q}{q} = \frac{125 \times 10^3}{8.72 \times 10^3} = 14.3 \text{ m}^2.$$

The required length of tubes

$$l = \frac{F}{\pi dmn} = \frac{14.3}{3.14 \times 0.038 \times 8 \times 5} = 3\text{m}.$$

6-23. Under conditions of problem 6-22, how long should the tubes be, if an in-line arrangement is replaced with a staggered one and the velocity in the narrowest section of the bank of tubes is increased to 14 m/s? All other conditions are left unchanged.

Answer

$$l = 2.3 \text{ m}.$$

6-24. In a heat exchanger liquid sodium is in cross flow past a staggered bank of tubes, having an outside diameter $d = 20$ mm. The mean velocity of the free-stream flow of liquid sodium $w = 1$ m/s, and the mean temperature of liquid sodium $t_f = 250°\text{C}$.

Determine the mean coefficient of heat transfer from the tubes to the sodium coolant and the mean rate of heat flow on the surface of the tubes on condition that the mean tube surface temperature $t_w = 256°\text{C}$.

Answer

$$\alpha = 1.34 \times 10^5 \text{ W/m}^2\cdot°\text{C}, \quad q = 8 \times 10^5 \text{ W/m}^2.$$

Solution

With sodium in cross flow past staggered and in-line banks of tubes the mean heat-transfer coefficient can be calculated with the aid of the following formula [16]:

$$\text{Nu}_f = 2\text{Pe}_f^{0.5}, \qquad (6\text{-}6)$$

where, taken as the reference dimension is the tube diameter. Formula (6-6) is valid at $100 \leqslant \text{Pe}_f \leqslant 1000$.

In the case considered, at $t_f = 250°\text{C}$, the physical properties of liquid sodium are respectively:

$\nu_f = 45 \times 10^{-8} \text{ m}^2\text{/s}$; $\lambda_f = 76.1 \text{ W/m}\cdot°\text{C}$; $\text{Pr} = 0.69 \times 10^{-2}$.

The Reynolds and Peclet numbers

$$\mathrm{Re}_f = \frac{wd}{\nu_f} = \frac{1 \times 20 \times 10^{-3}}{45 \times 10^{-8}} = 4.44 \times 10^4;$$

$$\mathrm{Pe}_f = \mathrm{Re}_f \mathrm{Pr}_f = 4.44 \times 10^4 \times 0.69 \times 10^{-2} = 306.$$

The Nusselt number and the heat-transfer coefficient

$$\mathrm{Nu}_f = 2\,(306)^{0.5} = 35;$$

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 35 \frac{76.1}{20 \times 10^{-3}} = 1.34 \times 10^5\ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$q = \alpha\,(t_w - t_f) = 1.34 \times 10^5\,(256 - 250) = 8 \times 10^5\ \mathrm{W/m^2}.$$

6-25. Determine the coefficient of heat transfer from the surface of tubes to liquid sodium in the heat exchanger considered in problem 6-24, if the velocity of the free-stream flow $w = 0.8$ m/s and the mean temperature of sodium $t_f = 300$°C. Also, find the amount of heat transferred to the sodium coolant, if the mean tube surface temperature $t_w = 305$°C and the bank is built up of $n = 56$ tubes of length $l = 1$ m.

Answer

$$\alpha = 1.14 \times 10^5\ \mathrm{W/m^2 \cdot {}^\circ C};\quad Q = 2 \times 10^3\ \mathrm{kW}.$$

Chapter Seven

HEAT TRANSFER BY FREE (NATURAL) CONVECTION

7-1. Calculate the loss of heat per unit time from one square meter of a horizontal cylindrical heat exchanger which is cooled by the ambient air. The outside diameter of the heat-exchanger shell $d = 400$ mm, the shell surface temperature $t_w = 200°C$ and the air temperature in the room $t_f = 30°C$ (Fig. 7-1).

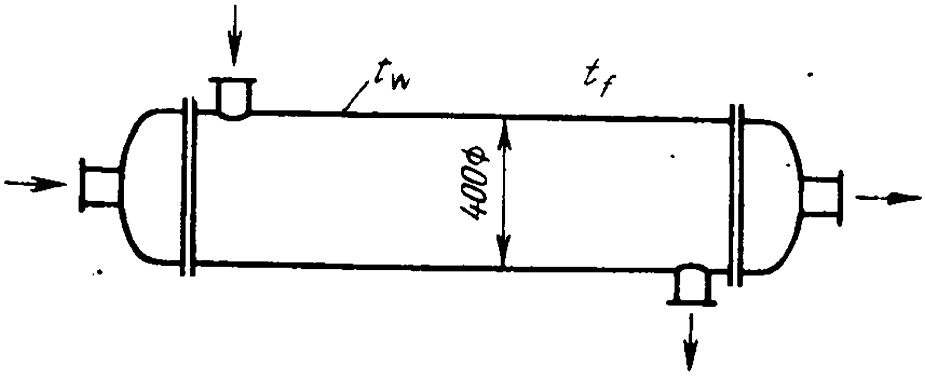

Fig. 7-1. Solution to problem 7-1

Answer

$$\alpha = 5.9 \text{ W/m}^2\cdot°\text{C}; \quad q = 1000 \text{ W/m}^2.$$

Solution

The rate of heat flow on the outside surface of the cylindrical heat exchanger $q = \alpha (t_w - t_f)$, W/m^2.

At the given surface temperature and the ambient air temperature at a distance from the wall the solution of the problem comes to determining the heat-transfer coefficient.

For cases of free convection the mean heat-transfer coefficient can be calculated with the aid of formula [4]

$$\mathrm{Nu}_f = C\,(\mathrm{GrPr})_f^n \left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25}, \qquad (7\text{-}1)^*$$

where the constants C and n depend on the pattern of free flow and the conditions of flow past a surface. They are functions of GrPr and are determined from the following tables:

$(\mathrm{GrPr})_f$	C	n	Conditions of flow
$1\times10^3-1\times10^9$	0.75	0.25	Along a vertical wall
$\geqslant 6\times10^{10}$	0.15	1.3	
$1\times10^3-1\times10^9$	0 50	0.25	On a horizontal tube

The subscripts "f" and "w" in formula (7-1) indicate that the physical properties of the fluid are selected respectively at the fluid temperature t_f, at a distance from the heat-transfer surface, and the wall temperature t_w. In the case of flow along a vertical wall taken as the reference dimension is the height of the heat-transfer surface and for a horizontal cylinder—its outside diameter.

In the case considered the reference temperature $t_f = 30$°C. At that temperature air has the following physical properties:

$$\nu_f = 16.0\times10^{-6}\ \mathrm{m^2/s};\quad \lambda_f = 2.67\times10^{-2}\ \mathrm{W/m\cdot{}^\circ C};$$

$$\beta_f = \frac{1}{t_f+273} = \frac{1}{300}\,\mathrm{K^{-1}};\quad \mathrm{Pr}_f = 0.701.$$

Calculate the magnitude of the dimensionless term

$$(\mathrm{GrPr})_f = \frac{g\beta_f\,\Delta t d^3}{\nu_f^2}\,\mathrm{Pr}_f = \frac{9.81\,(200-30)\,0.4^3}{303\,(16\times10^{-6})^2}\,0.701 = 9.75\times10^8.$$

---

* The correction in formula (7-1), $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}$, is applied only when calculating heat transfer for liquids.

From the table we find that for the calculated values of the dimensionless term $(\mathrm{GrPr})_f$ the constants $C = 0.5$ and $n = 0.25$.

The Nusselt number

$$\mathrm{Nu}_f = 0.50\,(9.75 \times 10^8)^{0.25} = 88.2,$$

whence

$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d} = 88.2\,\frac{2.67 \times 10^{-2}}{0.4} = 5.9\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The loss of heat per unit time from unit surface area of the heat exchanger is

$$q = 5.9\,(200 - 30) = 1000\ \mathrm{W/m^2}.$$

7-2. Under the conditions of problem 7-1, in order to reduce the heat losses, the shell of the heat exchanger is covered with a layer of a heat insulating material.

Find the amount of heat, q, W/m², lost from the surface of the heat exchanger, if after a 50 mm thick layer of heat insulation was applied, the temperature of the outside surface of the heat exchanger got equal to $t_w = 50°\mathrm{C}$, and the ambient air temperature remained as above, i.e. $t_f = 30°\mathrm{C}$.

Answer

$$q = 65\ \mathrm{W/m^2}.$$

7-3. In a boiler house there are two horizontal steam mains with diameters $d_1 = 50$ mm and $d_2 = 150$ mm. The two steam mains have the same surface temperature $t_w = 450°\mathrm{C}$. The ambient air temperature $t_f = 50°\mathrm{C}$. The steam mains are laid at a distance from each other, precluding any mutual heat effect.

Find the ratios of the heat-transfer coefficients α_1/α_2 and of the losses of heat from length of 1 m of steam mains q_{l1}/q_{l2}.

Answer

$$\alpha_1/\alpha_2 = 1.315; \quad q_{l1}/q_{l2} = 0.438.$$

7-4. Solve problem 7-3, assuming that after heat insulation was applied on the steam mains the surface temperature became equal to $t_w = 70°\mathrm{C}$. The outside diameter of the insulating layer of the first pipeline $d_1 = 100$ mm, and of the second, $d_2 = 350$ mm. The ambient air temperature remains as in problem 7-3, $t_f = 50°C$.

Answer

$$\alpha_1/\alpha_2 = 1.37; \quad q_{l1}/q_{l2} = 0.382.$$

7-5. Determine the coefficient of heat transfer from a vertical plate of height $H = 2$ m to the surrounding still air, if it is known that the surface temperature of the plate $t_w = 100°C$, the ambient air temperature at a distance from the plate $t_f = 20°C$.

Answer

$$\alpha = 7.92 \text{ W/m}^2\cdot°\text{C}.$$

Solution

The transfer of heat by free convection from a vertical plate can be determined by formula (7-1):

$$\text{Nu}_f = C\,(\text{GrPr})_f^n \left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25},$$

where taken as the reference dimension is the height of plate H.

At $t_f = 20°C$ the physical properties of air:

$$\lambda_f = 2.59 \times 10^{-2} \text{ W/m}\cdot°\text{C}; \quad \nu_f = 15.06 \times 10^{-6} \text{ m}^2/\text{s};$$

$$\text{Pr}_f = 0.703; \quad \beta_f = \frac{1}{t_f + 273} = \frac{1}{293} \text{ K}^{-1}.$$

Under these conditions the dimensionless term

$$(\text{GrPr})_f = g\beta_f \frac{\Delta t H^3}{\nu_f^2} \text{Pr}_f =$$

$$= 9.81 \frac{1}{293} \frac{80 \times 2^3 \times 10^{12}}{(15.06)^2} 0.703 = 6.64 \times 10^{10}.$$

Using the obtained value of $(\text{GrPr})_f$, find from the table, compiled for formula (7-1), $C = 0.15$ and $n = 1.3$. Then,

$$\text{Nu}_f = 0.15\,(6.64 \times 10^{10})^{1/3} = 610$$

and

$$\alpha = \text{Nu}_f \frac{\lambda_f}{H} = 610 \frac{2.59 \times 10^{-2}}{2} = 7.92 \text{ W/m}^2\cdot°\text{C}.$$

7-6. Under conditions of problem 7-5, how shall change the coefficient of heat transfer from a vertical wall to the surrounding air, if the height of the plate is doubled, with all other conditions remaining unchanged?

Answer

$$\alpha_2/\alpha_1 = 1.$$

7-7. An edge-mounted rectangular bus bar, measuring 100 × 3 mm, is cooled by the surrounding air which is at a temperature of 25°C. The temperature of the bus bar should not exceed 70°C in continuous-load service.

Calculate the heat-transfer coefficient α and the admissible intensity of the current carried by the bus bar under the conditions specified. The resistivity of the material from which the bus bar is made $\rho = 0.13$ Ohm·mm²/m.

Answer

$$\alpha = 9.84 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad I = 450 \text{ A}.$$

7-8. How shall change the heat-transfer coefficient and the admissible current intensity, if the bus bar temperature should remain as in problem 7-7, $t_w = 70^\circ$C, in winter operation when the mean ambient air temperature $t_f = -10^\circ$C?

Answer

$$\alpha = 10.8 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad I = 719 \text{ A}.$$

7-9. Determine the maximum admissible current intensity for a nichrome wire, 0.5 mm in diameter, on condition that the wire temperature should not exceed 300°C. At a temperature $t_w = 300^\circ$C the resistance per 1 m length of the wire $R = 6$ Ohm/m. The temperature of surroundings $t_f = 20^\circ$C.

Carry out calculations for two cases:

(a) the wire is exposed to still air;

(b) the wire is placed in still water under a pressure at which the saturation temperature exceeds 300°C.

Solution hints. For small diameter wires ($d = 0.2$ to 1.0 mm) the dimensionless term GrPr is usually small and a film-type or transient pattern of flow is preserved. For cases where the term GrPr $< 5 \times 10^2$ the following calculation formula [4] is recommended:

$$\overline{\text{Nu}_h} = 1.18\,(\text{GrPr})_h^{1/8}. \qquad (7\text{-}2)$$

When formula (7-2) is being used, taken as the reference temperature is $t_h = 0.5\,(t_w + t_f)$, and as the reference geometrical dimension—the diameter of the wire d.

Answer

$$\text{(a)}\ I_1 = 2.42\ \text{A};\quad \alpha_1 = 80\ \text{W/m}^2\cdot{}^\circ\text{C};$$

$$\text{(b)}\ I_2 = 23.5\ \text{A};\quad \alpha_2 = 7570\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

7-10. Determine the coefficient of heat transfer from a plate, arranged with its heat-transfer surface facing upward and measuring $a \times b = 2 \times 3$ m^2, to the surrounding still air, if the plate surface temperature $t_w = 100°$C and the ambient air temperature, at a distance from the plate, $t_f = 20°$C.

Answer

$$\alpha = 10.3\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

Solution

The rate of heat transfer from horizontal plates can be calculated approximately with the aid of formula (7-1). Then, the smaller side of the plate is taken as the reference dimension. If the heat-transfer surface faces upward, the value of the heat-transfer coefficient, calculated with the aid of formula (7-1) is increased by 30 per cent and, on the contrary, if the heat-transfer surface faces downward, the heat-transfer coefficient is reduced by 30 per cent [13].

In the case considered $t_f = 20°$C and at that temperature, for air, $\nu_f = 15.06 \times 10^{-6}$ m^2/s;

$$\lambda_f = 2.59 \times 10^{-2}\ \text{W/m}\cdot{}^\circ\text{C};\quad \text{Pr}_f = 0.703.$$

The reference dimension is the smaller side of the plate, i.e. $a = 2$ m. Then, the dimensionless term

$$(\text{GrPr})_f = g\beta_f \frac{\Delta t a^3}{\nu_f^2}\,\text{Pr}_f = 9.81\,\frac{80 \times 2^3 \times 0.703}{293\,(15.06 \times 10^{-6})^2} = 6.64 \times 10^{10}.$$

Using the obtained value of (GrPr), find from the table, compiled for formula (7-1), $C = 0.15$ and $n = 1/3$. Then,

$$\text{Nu}_f = 0.15\,(6.64 \times 10^{10})^{1/3} = 610,$$

whence

$$\alpha' = \text{Nu}_f \frac{\lambda_f}{a} = 610\,\frac{2.59 \times 10^{-2}}{2} = 7.9\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\alpha = 1.3\alpha' = 1.3 \times 7.9 = 10.3\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

7-11. Under conditions of problem 7-10, how shall change the heat-transfer coefficient, if the plate is arranged with its heat-transfer surface facing downward, leaving all other conditions unchanged?

Answer

$$\alpha_2 = 5.5\ \mathrm{W/m^2 \cdot {}^\circ C}; \quad \alpha/\alpha_2 = 1.88.$$

7-12. The temperature of oil in a tank is kept constant with the aid of horizontal heating tubes of a diameter $d = 20$ mm.

Determine the coefficient of heat transfer from the surface of tubes to the oil, if the oil temperature $t_f = 60^\circ$C and the tube surface temperature $t_w = 90^\circ$C. The distance between the tubes is relatively great, and the rate of heat transfer can be calculated as for a single cylinder.

Answer

$$\alpha = 96.2\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

7-13. Under conditions of problem 7-12, determine the heat-transfer coefficient, if, with the same oil temperature and temperature difference, heat is transferred from the oil to the tubes with $t_f = 60^\circ$C and $t_w = 30^\circ$C.

Answer

$$\alpha = 47.2\ \mathrm{W/m^2 \cdot {}^\circ C},$$

i.e. the heat-transfer coefficient is about one half of that calculated for the case of heating.

7-14. Determine the equivalent thermal conductivity and the rate of heat flow q, W/m^2, though a vertical slot of thickness $\delta = 20$ mm, filled with air. The temperature of the hot surface, $t_{w1} = 200^\circ$C and of the cold surface, $t_{w2} = 80^\circ$C (Fig. 7-2).

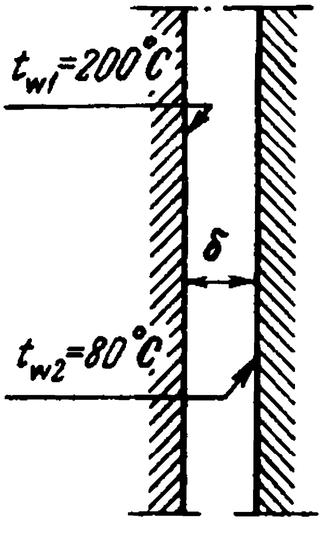

Fig. 7-2. Solution to problem 7-14

Answer

$$\lambda_{eq} = 7.5 \times 10^{-2}\ \mathrm{W/m \cdot {}^\circ C}; \quad q = 448\ \mathrm{W/m^2}.$$

Solution

The equivalent thermal conductivity can be calculated with the aid of the following formula [4];

$$\lambda_{eq} = \lambda \varepsilon_c, \tag{7-3}$$

where λ = real thermal conductivity of fluid;
ε_c = coefficient of convection which is a function of GrPr and can be calculated approximately from formula

$$\varepsilon_c = 0.18\,(\mathrm{GrPr})_{w.h}^{0.25}. \tag{7-4}$$

Here, all physical parameters are taken at the reference temperature $t_{w.h} = 0.5\,(t_{w1} + t_{w2})$.

Taken as the reference dimension is the thickness of the slot δ, and the temperature difference is set at $\Delta t = t_{w1} - t_{w2}$.

In the case considered $t_{w.h} = 0.5\,(200 + 80) = 140°\mathrm{C}$. At that temperature $\nu_{w.h} = 27.8 \times 10^{-6}$ m²/s.

$$\lambda_{w.h} = 0.0349\ \mathrm{W/m{\cdot}°C};\quad \mathrm{Pr}_{w.h} = 0.684.$$

$$\beta_{w.h} = \frac{1}{t_{w.h} + 273} = \frac{1}{413}\ \mathrm{K}^{-1}.$$

Calculate the value of the dimensionless term

$$(\mathrm{GrPr})_{w.h} = g\beta_{w.h}\,\frac{(t_{w1} - t_{w2})\,\delta^3}{\nu_{w.h}^2}\,\mathrm{Pr}_{w.h} =$$

$$= 9.81\,\frac{120\,(2 \times 10^{-2})^3}{413\,(27.8 \times 10^{-6})^2}\,0.684 = 2.02 \times 10^4.$$

The coefficient of convection

$$\varepsilon_c = 0.18\,(2.02 \times 10^4)^{0.25} = 2.14.$$

Then

$$\lambda_{eq} = 3.49 \times 10^{-2} \times 2.14 = 7.47 \times 10^{-2}\ \mathrm{W/m{\cdot}°C}.$$

The rate of heat flow through the layer of air

$$q = \frac{\lambda_{eq}}{\delta}\,(t_{w1} - t_{w2}) = \frac{7.47 \times 10^{-2}}{0.02}\,120 = 448\ \mathrm{W/m^2}.$$

7-15. How shall change the equivalent thermal conductivity and the rate of heat flow under the conditions of problem 7-14, if the slot between flat walls is filled with water under pressure, all other conditions remaining unchanged?

Answer

$$\lambda_{eq} = 15.5\ \text{W/m}\cdot{}^\circ\text{C};\quad q = 92\,600\ \text{W/m}^2.$$

7-16. How shall change the equivalent thermal conductivity, if the thickness of the slot is decreased by one half, all other conditions remaining unchanged, as in problem 7-14?

Answer

λ_{eq} shall decrease by 1.68 times.

7-17. In the service tank of a circuit, used to study hydrodynamics and heat transfer in liquid-metal coolants, a metal coolant is heated with the aid of a horizontal electric heater, of a cylindrical shape and 50 mm in diameter.

Calculate the coefficient of heat transfer from the surface of the heater to the liquid metal, for the case when the circuit is filled with sodium at a temperature $t_f = 200^\circ\text{C}$, and the heater surface temperature $t_w = 400^\circ\text{C}$.

Answer

$$\alpha = 15\,750\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

Solution

With liquid metals in free flow the rate of heat transfer can be calculated by formula [4]:

$$\text{Nu}_h = C\text{Gr}_h^n\text{Pr}_h^{0.4}. \tag{7-5}$$

In Eq. (7-5) the constants C and n depend on the magnitude of the Grashof number:

with $\text{Gr}_h = 10^2$ to 10^9 $C = 0.52$ and $n = 0.25$;

with $\text{Gr}_h = 10^9$ to 10^{13} $C = 0.106$ and $n = 0.33$.

The physical properties are chosen at the temperature $t_h = 0.5\,(t_w + t_f)$.

For the case considered $t_h = 0.5\,(200 + 400) = 300^\circ\text{C}$ and at that temperature the physical properties of sodium are:

$$\nu_h = 39.4 \times 10^{-8}\ \text{m}^2/\text{s};\ \lambda_h = 71\ \text{W/m}\cdot{}^\circ\text{C};\ \text{Pr}_h = 0.63\times10^{-2}$$

$$\beta_h \approx \frac{\rho_f - \rho_w}{\rho_f\,(t_w - t_f)} = \frac{903-854}{903\,(400-200)} = 2.71 \times 10^{-4}\ \text{K}^{-1};$$

$$\text{Gr}_h = g\beta_h\frac{\Delta t d^3}{\nu_h^2} = 9.81 \times 2.71 \times 10^{-4}\,\frac{200\,(5\times10^{-2})^3}{(39.4\times10^{-8})^2} =$$

$$= 4.28 \times 10^8$$

With this value of the Grashof number

$$C = 0.52 \text{ and } n = 0.25.$$

Then

$$\mathrm{Nu}_h = 0.52\,(4.28 \times 10^{8})^{0.25}\,(6.3 \times 10^{-3})^{0.4} = 11.1,$$

whence

$$\alpha = \mathrm{Nu}_h \frac{\lambda_h}{d} = 11.1\,\frac{71}{2 \times 10^{-2}} = 15\,750 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

7-18. How shall change the coefficient of heat transfer from the surface of the heater to the coolant, if the circuit of problem 7-17 is filled with:

(a) lithium Li;

(b) eutectic alloy, 25% Na + 75% K.

The temperatures of the liquid-metal coolants and of the surface of the heater remain as in problem 7-17.

Answer

(a) $\alpha = 10\,500$ W/m²·°C;

(b) $\alpha = 6370$ W/m²·°C.

Chapter Eight

HEAT TRANSFER IN CONDENSING VAPOURS

8-1. Film condensation of dry saturated water vapour (steam) takes place on the surface of a vertical tube of height $H = 3$ m. The steam pressure $p = 2.5 \times 10^5$ Pa. The tube surface temperature $t_w = 123$°C.

Determine the thickness of the forming condensate film δ_x, and the local heat-transfer coefficient α_x, depending on the distance x from the upper end of the tube. Calculate the coefficient for distances x equal to 0.1, 0.2, 0.4, 0.6, 1.0, 1.5, 2.0, and 3 m.

Plot a graph representing δ_x and α_x as a function of the tube height.

Assume the film of condensate to be in laminar flow along the entire height of the tube. Calculate with the aid of approximate Nusselt formulae.

Answer

Calculation results are represented graphically in Fig. 8-1 and tabulated below:

x, m	0.1	0.2	0.4	0.6	1.0	1.5	2.0	3.0
δ_x, mm	0.060	0.0715	0.0845	0.094	0.107	0.118	0.127	0.140
α_x, W/m$^2\cdot$°C	11 430	9620	8150	7320	6530	5880	5410	4900

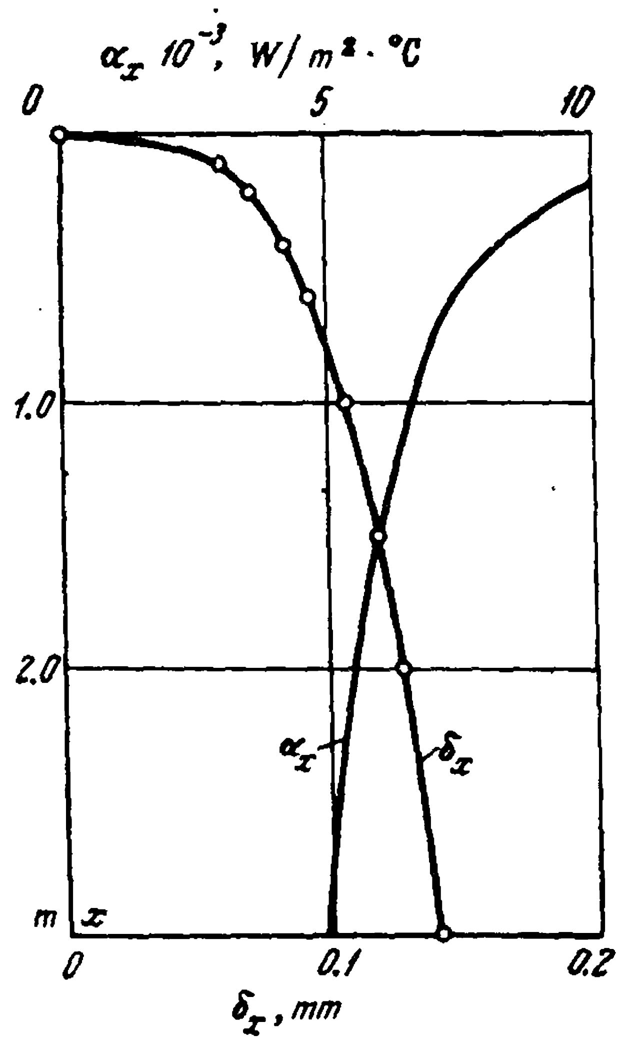

Fig. 8-1. Solution to problem 8-1

Solution

Upon film condensation of dry saturated steam and with the condensate film in laminar flow, the film thickness and the local heat-transfer coefficient can be determined approximately with the aid of the Nusselt formulae [4]:

the film thickness

$$\delta_x = \sqrt[4]{\frac{4\lambda\mu x\Delta t}{\rho^2 g r}}; \tag{8-1}$$

the local heat-transfer coefficient

$$\alpha_x = \frac{\lambda}{\delta_x}, \tag{8-2}$$

where λ, μ, and ρ = thermal conductivity, dynamic-viscosity coefficient and the density of condensate taken at the mean condensate temperature $t_h = 0.5\,(t_s + t_w)$;

r = latent heat of vapourization at the saturation temperature t_s;

$\Delta t = t_s - t_w$ = temperature difference.

In the problem considered, at $p = 2.5 \times 10^5$ Pa, the saturation temperature $t_s \approx 127$°C and the heat of vapourization $r = 2182$ kJ/kg.

Hence,

$\Delta t = 127 - 123 = 4$°C;

$t_h = 0.5\,(127 + 123) = 125$°C and at that temperature the physical properties of water are as follows:

$$\lambda = 0.686\ \text{W/m}\cdot{}^\circ\text{C};\quad \mu = 227 \times 10^{-6}\ \text{Pa}\cdot\text{s};\quad \rho = 939\ \text{kg/m}^3.$$

At a distance $x = 0.1$ m from the top end of the tube the condensate film thickness

$$\delta_{x=0.1} = \sqrt[4]{\frac{4 \times 0.686 \times 227 \times 10^{-6} \times 0.1 \times 4}{939^2 \times 2182 \times 10^3 \times 9.81}} = 0.6 \times 10^{-4}\ \text{m},$$

or $\delta_{x=0.1} = 0.06$ mm.

At a distance $x = 0.1$ m from the top end of the tube the local heat-transfer coefficient

$$\alpha_{x=0.1} = \frac{\lambda}{\delta_{x=0.1}} = \frac{0.686}{0.6 \times 10^{-4}} = 11\,430\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

The thickness of the condensate film shall vary heightwise in direct proportion and the heat-transfer coefficient inversely proportional to the fourth root of the distance from the top end of the tube. Thus,

$$\delta_{x=0.2} = \delta_{x=0.1}\sqrt[4]{2} = 0.06 \times 1.19 = 0.0715\ \text{mm}$$

$$\alpha_{x=0.2} = \frac{\lambda}{\delta_{x=0.2}} = \frac{0.686}{0.715 \times 10^{-4}} = 9620\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

For other values of x, δ_x and α_x are calculated in a similar manner. Calculation results are given in the answer to this problem and represented graphically in Fig. 8-1.

8-2. Under conditions of problem 8-1, how shall change the thickness of the condensate film and the local heat-transfer coefficient, if, at a constant pressure ($p = 2.5 \times 10^5$ Pa) the temperature difference Δt becomes equal to 2, 4, 6, 8 and 10°C?

For calculations assume a distance $x = 2$ m. Plot the following dependences: $\delta_x = f_1(\Delta t)$ and $\alpha_x = f_2(\Delta t)$.

Note. Under the conditions assumed, the mean temperature of the condensate film, t_h, varies but little, and the variation of the physical properties of the condensate with Δt can be ignored.

Answer

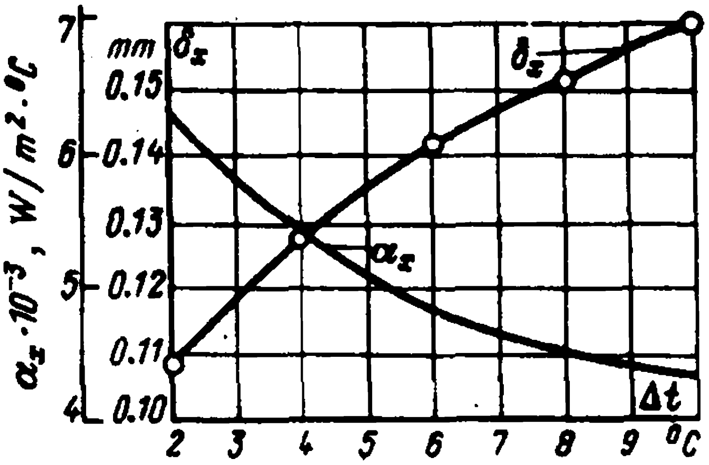

Fig. 8-2. Solution to problem 8-2

Calculation results are shown graphically in Fig. 8-2 and tabulated below:

Δt, °C	2	4	6	8	10
δ_x, mm	0.108	0.127	0.141	0.151	0.160
α_x, W/m²·°C	6360	5410	4870	4540	4300

8-3. Dry saturated steam (water vapour) undergoes film-type condensation on the surface of a vertical tube of height $H = 2$ m. The steam pressure $p = 4$ kPa. The tube surface temperature $t_w = 25$°C.

Determine the local heat-transfer coefficients at distances x, equal to 0.1 and 2 m from the top end of the tube, assuming the film of the condensate to be in laminar flow along the tube.

Compare calculation results with the answer to problem 8-1.

Answer

$$\alpha_{x=0.1} = 7460 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad \alpha_{x=1} = 3530 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Comparing these values with the answer to problem 8-1, we can see that at a pressure $p = 4 \times 10^8$ Pa the heat-transfer coefficient is about 1.5 times smaller than at $p = 2.5 \times 10^5$ Pa, which is primarily due to an increase in condensate density.

8-4. Dry saturated steam condenses on the external wall of a horizontal tube, having a diameter $d = 20$ mm and length $l = 2$ m, at a pressure $p = 1 \times 10^6$ Pa. Tube surface temperature $t_w = 94.5$°C.

Determine the mean coefficient of heat transfer from the condensing steam to the wall and the amount of steam G, kg/h, condensing on the surface of the tube.

Answer

$$\alpha = 15\,600 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad G = 15.9 \text{ kg/h}.$$

Solution

With dry saturated steam in film-type condensation on horizontal tubes, the circumferentially-mean heat-transfer coefficient can be determined by the following formula [10]:

$$\mathrm{Re} = 3.25Z^{0.76}, \tag{8-3}$$

where

$$\mathrm{Re} = \alpha \Delta t \pi R \frac{4}{r\rho\nu};$$

$$Z = \Delta t \pi R \left(\frac{g}{\nu^2}\right)^{1/3} \frac{\lambda}{r\rho\nu}$$

is the reduced length of the tube; $\Delta t = t_s - t_w$—temperature difference; R—tube radius; λ, ν and ρ—thermal conductivity, dynamic viscosity coefficient and density of the condensate at the saturation temperature t_s; r—heat of vapourization at t_s.

The formula holds at $d < 20\,(\sigma/\rho g)^{0.5}$ (σ—surface tension coefficient) and provided the condensate film is in laminar flow which is determined by the condition $Z < 3900$. For commonly encountered practical applications, these two conditions are usually satisfied.

Formula (8-3) can be presented in the following form:

$$\alpha = 3.25\,\frac{A^{0.75}}{B}\,\frac{1}{(\Delta t \pi R)^{0.25}} \qquad (8\text{-}4)$$

where

$$A = \left(\frac{q}{\nu^2}\right)^{1/3} \frac{\lambda}{r\rho\nu}, \ 1/\mathrm{m}\cdot{}^\circ\mathrm{C};$$

$$B = \frac{4}{r\rho\nu}, \ \mathrm{m/W}.$$

The terms A and B depend only on the kind of liquid and saturation temperature. For water these terms are shown as a function of saturation temperature t_s in Table 8-1.

Table 8-1

Values of A and B in Formulae (8-4) and (8-5) for Water

t_s, °C	A, 1/m·°C	$B \times 10^3$, m/W	t_s, °C	A, 1/m·°C	$B \times 10^3$, m/W
20	5.16	1.62	170	136	12.04
30	7.88	2.06	180	150	12.90
40	11.4	2.54	190	167	14.02
50	15.6	3.06	200	182	15.05
60	20.9	3.62	210	197	16.08
70	27.1	4.22	220	218	17.63
80	34.5	4.88	230	227	18.40
90	42.7	5.57	240	246	19.78
100	51.5	6.28	250	264	21.32
110	60.7	6.95	260	278	22.70
120	70.3	7.65	270	296	24.42
130	82.0	8.47	280	312	26.31
140	94.0	9.29	290	336	28.72
150	107	10.15	300	354	31.21
160	122	11.09			

In the problem considered at $p = 1 \times 10^5$ Pa $t_s = 99.6$°C and from table 8-1 we find:

$$A = 51.2\ 1/\mathrm{m}\cdot{}^\circ\mathrm{C}; \quad B = 6.25 \times 10^{-3}\ \mathrm{m/W}.$$

The temperature difference

$$\Delta t = t_s - t_w = 99.6 - 94.5 = 5.1^\circ\text{C}.$$

Substituting the found values into formula (8-4), we obtain:

$$\alpha = 3.25\,\frac{(51.2)^{0.75}}{6.25\times10^{-3}}\,\frac{1}{(3.14\times0.01\times5.1)^{0.25}} =$$

$$= 15\,600\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

The amount of condensing steam is found from the heat balance equation

$$Gr = \alpha\Delta tF,$$

where $F = \pi\, dl$, m^2, is the surface area of the tube.

At $t_s = 99.6^\circ$C the heat of vapourization $r = 2258$ kJ/kg, hence,

$$G = \pi\, dl\,\frac{\alpha\Delta t}{r} = 3.14\times0.02\times2\,\frac{15\,600\times5.1}{2258\times10^3} = 4.43\times10^{-3}\ \text{kg/s}$$

or $G = 4.43 \times 10^{-3} \times 3600 = 15.9$ kg/h.

8-5. Solve problem 8-4, assuming a steam pressure $p = 2\times10^5$ Pa, with all other conditions remaining unchanged. Compare calculation results with the answer to problem 8-4.

Answer

$$\alpha = 10\,800\ \text{W/m}^2\cdot{}^\circ\text{C};\quad G = 57\ \text{kg/h}.$$

8-6. Determine the amount of dry saturated steam G, kg/h, condensing on the surface of a horizontal tube of a diameter $d = 16$ mm and length $l = 1.5$ m, if the steam pressure $p = 1.2$ MPa and tube surface temperature $t_w = 180^\circ$C.

Answer

$$G = 99\ \text{kg/h}.$$

8-7. How shall change the coefficient of heat transfer when dry saturated steam condenses on the surface of the horizontal tube, if the steam pressure rises from $p = 0.04 \times 10^5$ Pa to $p = 4 \times 10^5$ Pa, and the temperature difference $\Delta t = t_s - t_w$ remains unchanged?

Answer

The heat-transfer coefficient shall increase 1.43 times.

8-8. How shall change the heat-transfer coefficient and the amount of dry saturated steam condensing per unit time on the surface of the horizontal tube, if a tube diameter is increased 4 times, and the steam pressure, temperature difference and the length of the tube are left as in problem 8-7?

Answer

The heat-transfer coefficient shall diminish $\sqrt{2} = 1.41$ times; the amount of steam condensing per unit time shall increase $2^{3/2} = 2.84$ times.

8-9. What wall temperature t_s must be maintained to ensure film-type condensation $G = 6.5 \times 10^{-3}$ kg/s of dry saturated steam on the surface of a horizontal tube having a diameter $d = 16$ mm and length $l = 2.4$ m. The steam pressure $p = 5 \times 10^5$ Pa.

Also, determine the heat-transfer coefficient under these conditions.

Answer

$$t_w = 145°\text{C}; \quad \alpha = 16\,600 \text{ W/m}^2\cdot°\text{C}.$$

Solution

The heat balance equations yield

$$\alpha = \frac{Gr}{\Delta t 2\pi R l}, \text{ W/m}^2\cdot°\text{C}.$$

On the other hand, according to formula (8-4), the heat-transfer coefficient

$$\alpha = 3.25 \frac{A^{0.75}}{B} \frac{1}{(\pi R \Delta t)^{0.25}}.$$

Equating the right-hand sides of the two equations, we obtain the expression for the temperature difference:

$$\Delta t^{0.75} = \frac{Gr}{6.5 (\pi R)^{0.75} l} \frac{B}{A^{0.75}}.$$

In the problem considered, at $p = 5 \times 10^5$ Pa, the saturation temperature $t_s = 151.8$°C, at which $r = 2109$ kJ/kg and from Table 8-1 we find $A = 109.7$ 1/m·°C; $B = 10.3 \times 10^{-3}$ m/W. Consequently,

$$\Delta t^{0.75} = \frac{6.5 \times 10^{-3} \times 2109 \times 10^3}{6.5\,(3.14 \times 8 \times 10^{-3})^{0.75} \times 2.4} \frac{10.3 \times 10^{-3}}{(109.7)^{0.75}} = 4.22$$

whence the temperature difference

$$\Delta t = (4.22)^{4/3} = 6.8°\mathrm{C}$$

and the wall temperature required

$$t_w = t_s - \Delta t = 151.8 - 6.8 = 145°\mathrm{C}.$$

The heat-transfer coefficient is found by formula (8-4):

$$\alpha = 3.25\,\frac{(109.7)^{0.75}}{10.3\times 10^{-3}}\,\frac{1}{(3.14\times 8\times 10^{-3}\times 6.8)^{0.25}} =$$
$$= 16\,600\ \mathrm{W/m^2\cdot °C}.$$

8-10. What temperature difference must be maintained to ensure a rate of heat flow, $q = 5.8 \times 10^4$ W/m², upon film-type condensation of dry saturated steam on the surface of a horizontal tube of a diameter $d = 34$ mm? The steam pressure $p = 1 \times 10^5$ Pa.

Also, determine the heat-transfer coefficient under these conditions.

Answer

$$\Delta t = 4°\mathrm{C};\quad \alpha = 14\,500\ \mathrm{W/m^2\cdot °C}.$$

8-11. Dry saturated steam condenses on the surface of a horizontal brass tube, having a diameter $d_2/d_1 = 20/18$ mm, at a pressure $p = 2.4 \times 10^5$ Pa. The tube carries cooling water. The rate of flow and mean water temperature are respectively $G_1 = 400$ kg/h, $t_{f1} = 40°$C.

Determine the amount of steam condensing per 1 h on length of 1 m of the brass tube, G_2, kg/m·h.

Answer

$$G_2 = 20.8\ \mathrm{kg/m\cdot h}.$$

Solution

Since the coefficients of heat transfer from steam to the tube wall and from the latter to the cooling water depend on the temperatures of the outer and inner surfaces of the tube, and these temperatures are not known, the problem should be solved either by the method of successive approximations, assuming respective temperatures, or applying the graphical-analytical method, as shall be done below.

Determine the rates of heat flow, reduced to 1 m length of the tube, from the inner surface of the tube to the water

(q_{l1}, W/m) passing through the wall of the tube (q_{lw}, W/m) and from the condensing steam to the surface of the tube (q_{l2}, W/m), depending on corresponding temperature differences $\Delta t_1 = t_{w1} - t_{f1}$; $\Delta t_w = t_{w2} - t_{w1}$, and $\Delta t_2 = t_s - t_{w2}$.

To determine $q_{l1} = f_1(\Delta t_1)$, let us assume three values of Δt_1: 65, 70 and 75°C, then, $t_{w1} = \Delta t_1 + t_{f1}$ will be equal to 105, 110 and 115°C.

At a cooling water temperature $t_{f1} = 40°\text{C}$ $\mu_{f1} = 653 \times 10^{-6}$ Pa·s, $\lambda_{f1} = 0.635$ W/m·°C, $\text{Pr}_{f1} = 4.31$. The Reynolds number

$$\text{Re}_{f1} = \frac{4G_1}{\pi d_1 \mu_{f1}} = \frac{4 \times 400}{3.14 \times 18 \times 10^{-3} \times 3600 \times 653 \times 10^{-6}} = 1.2 \times 10^4.$$

The cooling water is in turbulent flow, and the heat-transfer coefficient is determined by formula (5-7):

$$\text{Nu}_{f1} = 0.021\text{Re}_{f1}^{0.8}\text{Pr}_{f1}^{0.43}\left(\frac{\text{Pr}_{f1}}{\text{Pr}_w}\right)^{0.25} =$$

$$= 0.021(1.2 \times 10^4)^{0.8}(4.31)^{0.43}\left(\frac{\text{Pr}_{f1}}{\text{Pr}_w}\right)^{0.25} = 73\left(\frac{\text{Pr}_{f1}}{\text{Pr}_w}\right)^{0.25}$$

Whence at $t_{w1} = 105°\text{C}$ ($\text{Pr}_{w1} = 1.67$) we find:

$$\text{Nu}_{f1} = 73\left(\frac{4.31}{1.67}\right)^{0.25} = 92.5$$

The heat-transfer coefficient

$$\alpha_1 = \text{Nu}_{f1}\frac{\lambda_{f1}}{d_1} = 92.5\frac{0.635}{18 \times 10^{-3}} = 3260 \text{ W/m}^2\cdot°\text{C};$$

$$q_{l1} = \alpha_1 \Delta t_1 \pi d_1 = 3620 \times 65 \times 3.14 \times 18 \times 10^{-3} = 12\,000 \text{ W/m}.$$

At $t_{w1} = 110°\text{C}$ we correspondingly obtain:

$\text{Nu}_{f1} = 93.5$, $\alpha_1 = 3300$ W/m²·°C, $q_{l1} = 13\,000$ W/m.

At $t_{w1} = 115°\text{C}$ $\text{Nu}_{f1} = 94.8$, $\alpha_1 = 3340$ W/m²·°C, $q_{l1} = 14\,200$ W/m.

The dependence $q_{l1} = f(\Delta t_1)$ is shown on the chart in Fig. 8-3.

Since the thermal conductivity of brass $\lambda_w \approx 110$ W/m·°C can be assumed to be independent of temperature under conditions of this problem, the function $q_{lw} = f(\Delta t_w)$ is linear:

$$q_{lw} = (t_w - t_{w1}) \frac{2\pi\lambda}{2.3 \log \frac{d^2}{d^1}}\,.$$

At $\Delta t_w = t_{w2} - t_{w1} = 2\,^\circ\mathrm{C}$

$$q_{lw} = 2\,\frac{2 \times 3.14 \times 110}{2.3 \log \frac{20}{18}} = 13\,000 \text{ W/m}.$$

The dependence $q_{lw} = f\,(\Delta t_w)$ is also shown in Fig. 8-3.

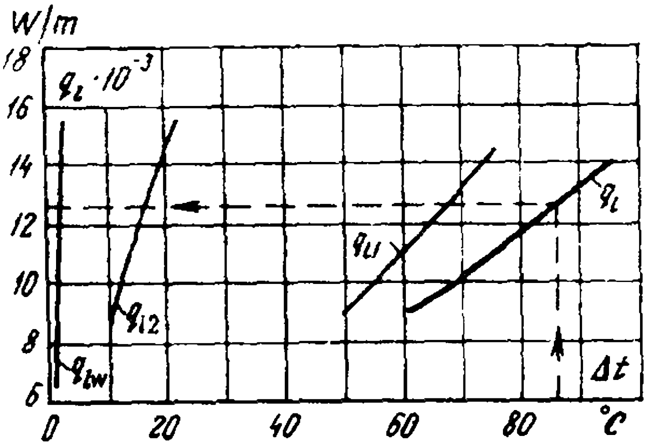

Fig. 8-3. Solution to problem 8-11

The dependence $q_{l2} = f\,(\Delta t_{w2})$ is found, using the formula for the coefficient of heat transfer from the condensing steam to the wall of the brass tube (8-4).

$$\alpha_2 = 3.25\,\frac{A^{0.75}}{B}\,\frac{1}{(\pi R_2 \Delta t_2)^{0.25}}\,;$$

then, the rate of heat flow per length of 1 m shall be

$$q_{l2} = \alpha_2 \pi d_2 \Delta t_2 = 3.25\,\frac{A^{0.75}}{B}\,\frac{2\pi R_2 \Delta t_2}{(\pi R_2 \Delta t_2)^{0.25}}$$

and 58

$$q_{l2} = 6.5\,\frac{(\pi A R_2 \Delta t_2)^{0.75}}{B}\,.$$

At $p = 2.4 \times 10^5$ Pa $t_s = 126.1\,^\circ\mathrm{C}$ and from Table 8-1 we find that $A = 77\ 4$ 1/m·°C, $B = 8.15 \times 10^{-3}$ m/W, hence

$$q_{l2} = 6.5\,\frac{(3.14 \times 77.4 \times 10^{-2})^{0.75}}{8.15 \times 10^{-3}}\,\Delta t_2^{0.75} = 1540 \Delta t_2^{0\ 75}.$$

Having assumed $\Delta t_2 = 10$, 15 and 20°C, we obtain respectively $q_{l2}=8650$, 11 700 and 14 600 W/m. The dependence $q_{l2} = f(\Delta t_2)$ is also plotted on the graph in Fig. 8-3.

To find the dependence of the rate of heat flow on the summary temperature difference, $\Delta t = t_s - t_{w1}$, we add up the three found relationships. The resultant curve, $q_l = f(t_s - t_{w1})$, is represented by the thicker line in Fig. 8-3.

Plotting the given value of the total temperature difference $\Delta t = t_s - t_{f1} = 126.1 - 40 = 86.1°C$ on the abscissa and drawing a vertical line till intersection with the curve $q_l = f(t_s - t_{f1})$, we find the unknown magnitude of the rate of heat flow on the ordinate (Fig. 8-3): $q_l = 12\,600$ W/m.

With $t_s = 126.1°C$ the heat of vapourization $r = 2185$ kJ/kg and, consequently, the rate of condensate flow

$$G_1 = \frac{q_l}{r} = \frac{1.26 \times 10^4}{2185 \times 10^3} \times 3600 = 20.8 \text{ kg/m. h.}$$

8-12. Determine the coefficient of heat transfer α_2, W/m²·°C, from condensing steam to the external surface of a horizontal brass tube with $d_2/d_1 = 18/16$ mm, the temperatures of the external and internal tube surfaces t_{w2} and t_{w1}, and the amount of steam G_2, kg/m·h, condensing on the external surface of the tube.

The pressure of the dry saturated steam $p = 700$ kPa. The tube carries the cooling water flowing with a velocity $w = 1.0$ m/s at a mean temperature $t_{f1} = 30°C$.

Answer

$\alpha_2 = 7600$ W/m²·°C, $t_{w2} \approx 110°C$, $t_{w1} \approx 106°C$,

$G_2 = 41$ kg/m·h.

8-13. Assuming the conditions of problem 8-12, how shall change the amount of the condensing steam G_2, kg/m·h, if the velocity of the cooling water is increased two times (from $w = 1$ m/s to $w = 2$ m/s), leaving all other conditions unchanged?

Answer

The amount of the condensing steam shall increase about by 10 per cent,

$$G_2 = 45 \text{ kg/m·h.}$$

8-14. Dry saturated steam condenses on the external surface of a vertical tube, having a diameter $d = 20$ mm and height $H = 2$m, at a pressure $p = 1 \times 10^5$ Pa (Fig. 8-4). The temperature of the tube external surface $t_w = 94.5°C$.

Determine the heightwise mean coefficient of heat transfer from the condensing steam to the tube and the amount of steam that condenses on the surface of the tube.

Compare calculation results with the answer to problem 8-4 treating heat transfer for a horizontal tube under similar conditions.

Answer

$\alpha = 7840$ W/m^2·C; $G = 8$ kg/h.

Assuming the same conditions for a horizontal tube (of problem 8-4) $\alpha = 15\,600$ W/m^2·°C and $G = 15.9$ kg/h.

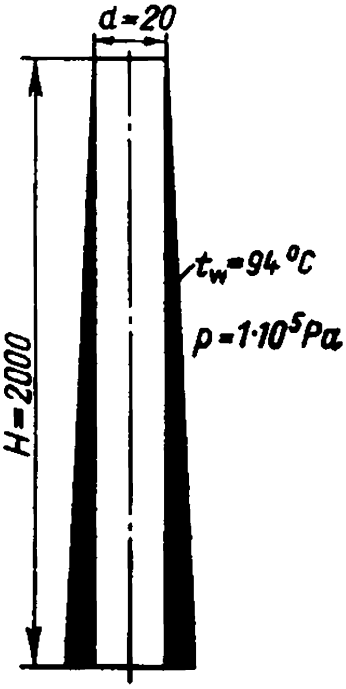

Fig. 8-4. Solution to problem 8-14

Solution

Upon film-type condensation of dry saturated steam and laminar flow of the forming condensate film along vertical surfaces and walls the lengthwise mean heat-transfer coefficient can be calculated by the following formula [10]:

$$\mathrm{Re} = 3.8Z^{0.78}, \tag{8.5}$$

where

$$\mathrm{Re} = \alpha \Delta t H \frac{4}{r\rho\nu};$$

$Z = \Delta t H \left(\frac{g}{\nu^2}\right)^{1/3} \frac{\lambda}{r\rho\nu}$ = reduced length of a tube;

H = height of a vertical surface or tube.

Other notations are as in formula (8-3).

The formula is valid, provided the condensate film is in laminar flow, i.e. when

$$\mathrm{Re} < 1600$$

and, correspondingly,

$$Z < 2300.$$

The magnitudes of the terms, comprising physical properties, present in the expressions for Re and Z, namely

$$\frac{4}{r\rho\nu} = B, \text{ m/W}$$

and

$$\left(\frac{g}{\nu^2}\right)^{\frac{1}{3}} \frac{\lambda}{r\rho\nu} = A, \ 1/\text{m}\cdot{}^\circ\text{C}$$

can be found for condensing steam in Table 8-1, depending on t_s.

In the problem considered at $p = 1 \times 10^5$ Pa $t_s = 99.6$ C from Table 8-1 we find that

$$A = 51.2\ 1/\text{m}\cdot{}^\circ\text{C}; \quad B = 6.25 \times 10^{-3}\ \text{m/W};$$

the temperature difference

$$\Delta t = t_s - t_w = 99\ 6 - 94.5 = 5.1^\circ\text{C};$$

the reduced length of a tube

$$Z = \Delta t H A = 5.1 \times 2 \times 51\ 2 = 522 < 2300.$$

Consequently, the condensate is in laminar flow along the entire height of the tube and heat transfer can be calculated by formula (8-5).

The Reynolds number

$$\text{Re} = 3.8 Z^{0.78} = 3.8(522)^{0.78} = 500.$$

The heat-transfer coefficient

$$\alpha = \frac{\text{Re}}{\Delta t H B} = \frac{500}{5.1 \times 2 \times 6\ 25 \times 10^{-3}} = 7840\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

With $t_s = 99.6^\circ$C the heat of vapourization $r = 2258$ kJ/kg and the amount of steam that condenses on the surface of the tube

$$G = \pi d H \frac{\alpha \Delta t}{r} = 3.14 \times 2 \times 10^{-2} \times 2 \frac{7840 \times 5.1}{2258 \times 10^3} =$$
$$= 2.22 \times 10^{-3}\ \text{kg/s}$$

or

$$G = 2.22 \times 10^{-3} \times 3600 = 8\ \text{kg/h}.$$

Comparing the obtained values of α and G with the answer to problem 8-4, it can be seen that the heat-transfer coefficient and the amount of condensing steam shall be about one half less than for a horizontal tube.

8-15. Dry saturated steam undergoes film condensation on a horizontal tube of a diameter $d = 16$ mm and length $l = 1.2$ m at a pressure $p = 3$ MPa. The tube surface temperature $t_w = 227°$ C.

How shall change the mean coefficient of heat transfer from the condensing steam to the tube, if the tube is vertical, and all other conditions remaining unchanged?

Answer

$$\alpha_{ver} \approx 0.55\alpha_{hor}.$$

8-16. Under the conditions of problem 8-15, determine the number of condensate-removing disks, n, to be arranged on a vertical tube, so that the heat-transfer coefficient for the vertical tube would be equal to that for a horizontally arranged tube ($\alpha_{ver} = \alpha_{hor}$).

Answer

$$n = 15 \text{ disks.}$$

8-17. A steam-to-water heat exchanger is built-up of $n = 218$ vertical tubes having a diameter $d = 16$ mm and height $H = 1.5$ m.

The tubes are cooled with the water flowing through them so that the mean temperature of the tube outside surface $t_w = 173°$C. Dry saturated steam condenses at a pressure $p = 1$ MPa on the external surface of the tubes.

Determine the coefficient of heat transfer from the condensing steam to the surface of the tubes and the amount of heat Q, kW, transferred to the water in the heat exchanger.

Answer

$$\alpha = 8800 \text{ W/m}^2\cdot°\text{C}; \quad Q = 1 \text{ MW}.$$

8-18. Determine the critical height of the tubes H_{cr}, at which, assuming the conditions of problem 8-17, the laminar flow of the condensate film shall turn into turbulent at the bottom end of the tubes.

Answer

$$H_{cr} = 2.2 \text{ m}.$$

Solution

The change from the laminar flow of the condensate film to turbulent flow takes place at the critical magnitude of the reduced length of tubes

$$Z_{cr} = (H\Delta t)_{cr}A = 2300.$$

Under the conditions of problem 8-17 $p = 1$ MPa, $t_w = 173°C$. At the given pressure $t_s = 179.9°C$ and $\Delta t = 179.9 - 173 = 6.9°C$. From Table 8-1, at a saturation temperature $t_s = 179.9°C$, we find that

$$A = 149.9 \ 1/\mathrm{m}\cdot°\mathrm{C};$$

then

$$H_{cr} = \frac{Z_{cr}}{A\Delta t} = \frac{2300}{149.9\times 6.9} = 2.22 \ \mathrm{m}.$$

8-19. Determine the temperature difference at which the film of condensate shall remain in laminar flow along the entire height of the tube, under conditions of problem 8-17.

Answer

$$\Delta t \leqslant 10.2°C$$

8-20. The cooling water, flowing through the tubes of a vertical steam-to-water heat exchanger must remove an amount of heat $Q = 350$ kW.

Dry saturated steam condenses on the external surface of the tubes at a pressure $p = 1.5$ MPa.

Determine the temperature difference required, if the heat exchanger is built-up of $n = 50$ tubes having a diameter $d = 22$ mm and a height $H = 1.5$ m.

Answer

$$\Delta t = 8°C.$$

Hint. Since under the conditions of the problem the temperature difference is unknown, it is impossible to determine directly the reduced length of the tubes Z, and identify the pattern of flow of the condensate film on the external surface of the tubes. In this connection, a preliminary calculation should be carried out, assuming the condensate to be in laminar flow along the entire height of the tubes. Having determined the magnitude of Δt, it is necessary to check the pattern of condensate flow.

With the condensate film in laminar flow according to formula (8-5)

$$\mathrm{Re} = 3.8\,(H\Delta tA)^{0.78},$$

where

$$\mathrm{Re} = \alpha H\Delta tB.$$

Allowing for $\alpha\Delta t = Q/F$, where $F = \pi dHn$, m^2, we obtain the following expression for the temperature difference:

$$\Delta t = \left(\frac{QB}{3.8\pi dn}\right)^{1/0.78}\frac{1}{AH}.$$

8-21. Dry saturated steam condenses on the external surface of a vertical tube. The condensate film is in laminar flow along the entire height of the tube.

Determine the dependence of the rate of heat flow q, W/m^2, and the heat flux Q, W, on the height of the tube.

Answer

$$q \approx H^{-0.22};\quad Q \approx H^{0.78}.$$

8-22. Dry saturated steam condenses on the vertical tube of a water heater at a pressure $p = 8.6$ MPa. The temperature of the tube external surface $t_w = 287$°C and the height of the tube $H = 1.8$ m.

Determine the mean coefficient of heat transfer from the condensing steam to the wall of the tube.

Answer

$$\alpha = 8100\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Solution

At $p = 8.6$ MPa $t_s = 300$°C; from Table 8-1 we find: $A = 354$ 1/m·°C, $B = 31.21 \times 10^{-3}$ m/W.

The temperature difference $\Delta t = t_s - t_w = 300 - 287 = 13$°C, consequently, the reduced length of the tube

$$Z = \Delta tHA = 13 \times 1.8 \times 354 = 8380 > 2300.$$

Since the reduced length exceeds the critical, the condensate film is in turbulent flow along the bottom part of the tube.

With film-type condensation of dry saturated steam and the condensate film in a combined pattern of flow the lengthwise mean heat-transfer coefficient can be determined by formula [10]:

$$\mathrm{Re} = \left[253 + 0.069\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w}\right)^{0.25}\mathrm{Pr}^{0.5}(Z-2300)\right]^{4/3}, \qquad (8\text{-}6)$$

where Pr and Pr_w are Prandtl numbers for condensate respectively at the temperatures t_s and t_w. The remaining notations are as in Formula (8-5).

Formula (8-6) is valid provided $Z \geqslant 2300$.

In the problem considered at $t_s = 300$°C $\mathrm{Pr} = 0.97$; at $t_w = 287$°C $\mathrm{Pr}_w = 0.921$. Formula (8-6) gives:

$$\mathrm{Re} = \left[253 + 0.069\left(\frac{0.97}{0.921}\right)^{0.25} \times 0.97^{0.5}(8380-2300)\right]^{4/3} = \\ = 5930.$$

Allowing for $\mathrm{Re} = \alpha \Delta t H B$, we find:

$$\alpha = \frac{\mathrm{Re}}{\Delta t H B} = \frac{5930}{13 \times 1.8 \times 31.21 \times 10^{-3}} = 8100\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

8-23 Dry saturated steam condenses at a pressure $p = 5.6$ MPa on the tubes of a vertical water heater in which the heated water flows through the tubes, having an external surface temperature $t_w = 260$°C.

Determine the amount of heat transferred to the water, Q, kW, if the water heater is built-up of $n = 112$ tubes of an outside diameter $d = 16$ mm and height $H = 2$ m.

Answer

$Q = 1$ MW.

8-24. Determine the variation of the mean heat-transfer coefficient along a vertical tube on which dry saturated steam condenses at a pressure $p = 6$ MPa. Tube surface temperature $t_w = 265$°C.

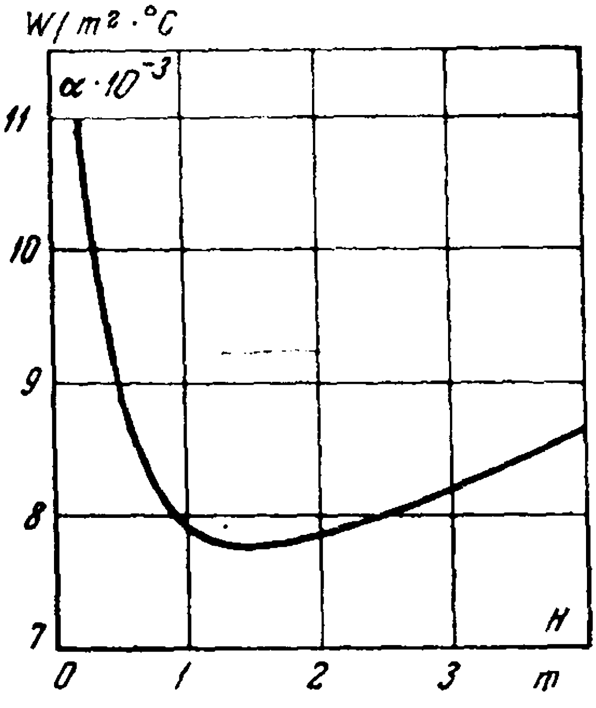

Fig. 8-5. Solution to problem 8-24

Calculate the mean heat-transfer coefficient for tubes of height H, equal to 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 1.5, 1.75, 2.0, 2.5, 3, and 4 m and plot α as a function of height, $\alpha = f(H)$.

Answer

Calculation results are shown graphically in Fig. 8-5 and also tabulated below:

H, m	0.2	0.4	0.6	0.8	1.0	1.25
α, W/m$^2\cdot$°C	11 000	9440	8600	8150	7880	7780
H, m	1.5	1.75	2.0	2.5	3.0	4.0
α, W/m$^2\cdot$°C	7760	7800	7840	8000	8200	8630

8-25. Under conditions of problem 8-24 determine the variation of the amount of heat Q, kW, transferred from the steam to the surface of the vertical tube, depending on its height, assuming a tube diameter $d = 22$ mm.

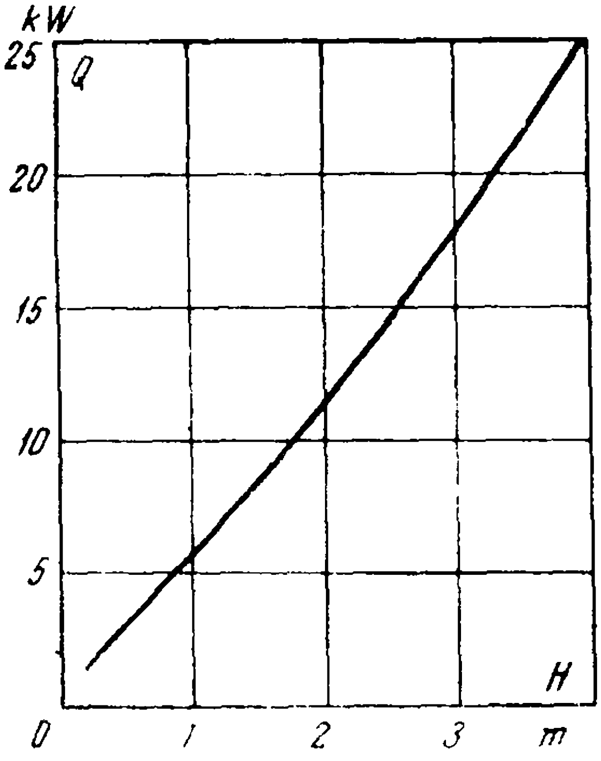

Fig. 8-6. Solution to problem 8-25

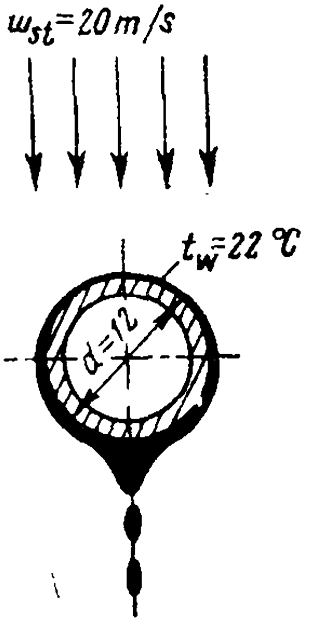

Fig. 8-7. Solution to problem 8-26

Answer

Calculation results are presented graphically in Fig. 8-6 and also tabulated below:

H, m	0.2	0.4	0.6	0.8	1.00	1.25
Q, kW	1.61	2.76	3.76	4.76	5.76	7.10
H, m	1.5	1.75	2.0	2.5	3.0	4.0
Q, kW	8.53	10.0	11.4	14.6	17.9	25.2

8-26. Determine the coefficient of heat transfer from steam to a tube of the upper row of the horizontal bank of tubes of a turbine condenser. The outside diameter of the tube $d = 18$ mm and the tube surface temperature $t_w = 22°C$.

The dry saturated steam, condensing under a pressure $p = 5$ kPa on the surface of the tube, moves downward with a velocity $w_v = 20$ m/s (Fig. 8-7).

Compare the result obtained with the heat-transfer coefficient for stagnant (still) steam.

Answer

$$\alpha = 13\,700\ \text{W/m}^2\cdot{}^\circ\text{C};\quad \alpha/\alpha_{st} = 1.34.$$

Solution

For the case of dry saturated steam, condensing on horizontal tubes with $w_{st}^2\rho'' < 1$, the heat-transfer coefficient can be calculated with no account taken of the effect of steam velocity on heat transfer, i.e. by means of formula (8-4).

With $w_{st}^2\rho'' > 1$ account should be taken of the effect of steam velocity on the intensity of heat transfer. The heat-transfer coefficient then can be calculated by formula [26]:

$$\frac{\alpha}{\alpha_{st}} = 28.3\Pi^{0.08}\text{Nu}_{st}^{-0.58}, \tag{8-7}$$

where α_{st} = heat-transfer coefficient calculated for stagnant steam with the aid of formula (8-4):

$$\text{Nu}_{st} = \frac{\alpha_{st}d}{\lambda};\quad \Pi = \frac{w_{st}^2\rho''\alpha_{st}}{g\rho'\lambda}.$$

where w_{st} = velocity of free-stream steam flow;
ρ'' = steam density at saturation temperature t_s;
ρ' and λ = density and thermal conductivity of condensate at saturation temperature t_s.

Formula (8-7) is applicable at steam pressures ranging from 5 to 100 kPa, temperature differences, $\Delta t = t_s - t_w$, from 2 to 20°C, and $\Pi \leqslant 800$.

In the case considered, at $p = 5$ kPa, $t_s = 32.9$°C and at that temperature $\rho'' = 0.0354$ kg/m^3, $\rho' = 995$ kg/m^3, $\lambda = 0.623$ W/m·°C, and $w_{st}^2\rho'' = 20^2 \times 0.0354 = 14.2$ kg/m·s^2. Since $w_{st}^2\rho'' > 1$, calculation formula (8-7) is applied.

With stagnant steam condensing on a tube the heat-transfer coefficient is determined by formula (8-4).

At $t_s = 32.9$°C we find from Table 8-1 that $A = 8.9$ 1/m·°C, $B = 2.20 \times 10^{-3}$ m/W, the temperature difference $\Delta t = t_s - t_w \doteq 32.9 - 22 = 10.9$°C, and the heat-transfer coefficient for stagnant steam

$$\alpha_{st} = 3.25\frac{A^{0.75}}{B}\frac{1}{(\pi R\Delta t)^{0.25}} =$$

$$= 3.25\frac{(8.9)^{0.75}}{2.2 \times 10^{-3}} \times \frac{1}{(3.14 \times 9 \times 10^{-3} \times 10.9)^{0.25}} =$$

$$= 10\,200 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The dimensionless terms Nu_{st} and Π:

$$\mathrm{Nu}_{st} = \frac{\alpha_{st}d}{\lambda} = \frac{1.02 \times 10^4 \times 18 \times 10^{-3}}{0.623} = 295;$$

$$\Pi = \frac{w_{st}^2\rho''\alpha_{st}}{g\rho'\lambda} = \frac{20^2 \times 0.0354 \times 1.02 \times 10^4}{9.81 \times 995 \times 0.623} = 23.7.$$

The heat-transfer coefficient

$$\frac{\alpha}{\alpha_{st}} = 28.3\Pi^{0.08}\mathrm{Nu}_{st}^{-0.58} = 28.3\,(23.7)^{0.08}\,(295)^{-0.58} = 1.34;$$

$$\alpha = 10\,200 \times 1.34 = 13\,700 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

8-27. Under conditions of problem 8-26, how shall change the coefficient of heat transfer from steam to the tube of the condenser, if steam velocity is changed from 10 to 40 m/s. Plot a graph presenting α as a function of w_v.

Answer

At $p = \text{const}$ $\alpha \approx w_v^{0.16}$ and, consequently, the heat-transfer coefficient shall increase by 25 per cent. Calculation results are presented graphically in Fig. 8-8.

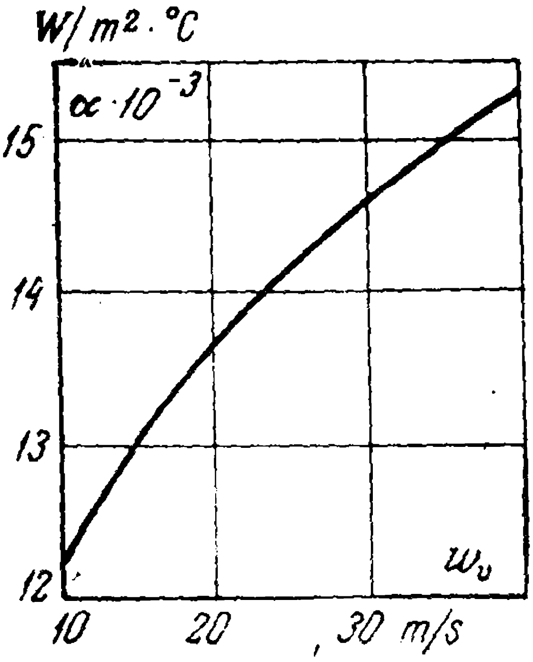

Fig. 8-8. Solution to problem 8-27

8-28. Under conditions of problem 8-26, how shall change the coefficient of heat transfer from steam to the tube of the condenser, if steam pressure is changed from 0.05×10^5 to 0.5×10^5 Pa with the temperature difference $\Delta t = 10.9$°C and all other data remaining unchanged?

Answer

The heat-transfer coefficient shall increase by 37 per cent and at a pressure $p = 0.5 \times 10^5$ Pa $\alpha = 18\ 000$ W/m²·°C.

8-29. Determine the mean coefficient of heat transfer from steam to the tubes of a condenser which is constructed as a horizontal in-line bank of tubes comprising $n = 14$ rows of tubes across the direction of steam flow in the condenser.

The outside tube diameter $d = 16$ mm. The transverse pitch of the tubes are arranged horizontally with a pitch $s = 1.25d$ (Fig. 8-9). The heat-transfer surface is the same for all rows of tubes.

The dry saturated steam, condensing on the surface of the horizontal tubes at a pressure $p = 9.8$ kPa, moves downward. Steam velocity upstream of the first (top) row of tubes $w_1 = 35$ m/s. The surface temperature of all tubes $t_w = 35$°C.

The steam pressure and the temperature difference are assumed to be constant over the height of the bank of tubes.

Answer

$$\alpha = 13\ 700 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Solution

With downward flowing pure steam, condensing on horizontal banks of tubes, the heat-transfer coefficients for individual rows of tubes of a bank of tubes can be determined with the aid of the following approximate calculation method [26]:

1. The amount of steam ΔG_n, condensing on the tubes of each row, the rate of steam flow G_n, and the coefficient of heat transfer for each row of tubes α'_n are calculated. In so doing,

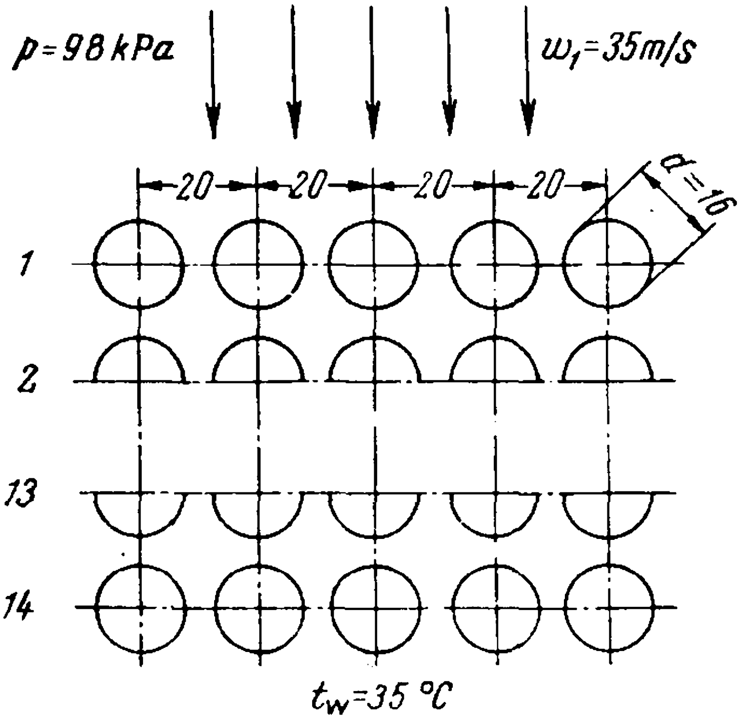

Fig. 8-9. Solution to problem 8-29

$$\Delta G_n = \frac{\alpha'_n F_n \Delta t}{r}, \text{ kg/s};$$

$$G_{n+1} = G_n - \Delta G_n, \text{ kg/s},$$

where, according to formula (8-7),

$$\frac{\alpha'_n}{\alpha_1} = \left(\frac{w_n}{w_1}\right)^{0.16}; \qquad (8\text{-}8)$$

F_n = heat-transfer surface for the row of tubes considered;

w_1 and w_n = steam velocities in front of the first and nth rows.

2. The corrections accounting for the downward flow of condensate are determined by formula

$$\varepsilon_n = \left(\frac{\sum_{i=1}^{n} \Delta G_i}{\Delta G_n} \right)^{-0.07}, \tag{8-9}$$

where $\sum_{i=1}^{n} \Delta G_i$ = the total amount of condensate flowing down the nth tube;

ΔG_n = the amount of condensate forming on the nth tube ($i = n$).

3. The heat-transfer coefficient for the nth row of tubes is determined, with account taken of steam velocity and the downward flow of condensate, by formula

$$\alpha_n = \alpha'_n \varepsilon_n. \tag{8-10}$$

In the problem considered, first the heat-transfer coefficient for stagnant steam is determined with the aid of formula (8-4).

At $p = 9.8$ kPa $t_s = 45.5$ °C.

From Table 8-1 we find that

$$A = 13.7\ 1/\text{m}\cdot{}^\circ\text{C};\quad B = 2.83 \times 10^{-3}\ \text{m/W}.$$

The temperature difference $\Delta t = t_s - t_w = 45.5 - 35 = 10.5$ °C;

$$\alpha_{st} = 3.25 \frac{A^{0.75}}{B} \frac{1}{(\pi R \Delta t)^{0.25}} =$$

$$= 3.25 \frac{(13.7)^{0.75}}{2.83 \times 10^{-3}} \frac{1}{(3.14 \times 0.008 \times 10.5)^{0.25}} =$$

$$= 11\,300\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

With account taken of steam velocity, the heat-transfer coefficient for the first row of tubes is calculated by formula (8-7).

At $t_s = 45.5\,°C$ $\rho'' = 0.0668$ kg/m^3, $\rho' = 991$ kg/m^3, $\lambda = 0.645$ W/m·°C, $r = 2393$ kJ/kg.

$$Nu_{st} = \frac{\alpha_{st} d}{\lambda} = \frac{1.13\times10^4\times1.6\times10^{2}}{0.645} = 281;$$

$$\Pi = \frac{w_1^2\rho''\alpha_{st}}{g\rho'\lambda} = \frac{35^2\times0.0668\times1.13\times10^4}{9.81\times991\times0.645} = 147;$$

$$\frac{\alpha_1}{\alpha_{st}} = 28.3\Pi^{0.08}Nu_{st}^{-0.58} = 28.3\,(147)^{0.08}\,(281)^{-0.58} = 1.6,$$

whence

$$\alpha_1 = 1.6\alpha_{st} = 1.6\times 11\,300 = 18\,000 \text{ W/m}^2\text{·°C}.$$

The amount of steam condensing on length of 1 m of a tube of the first row

$$\Delta G_1 = \frac{\alpha_1\,\Delta t\,\pi d}{r} = \frac{1.81\times10^4\times10.5\times3.14\times1.6\times10^{-2}}{2393\times10^3} =$$

$$= 3.98\times10^{-3} \text{ kg/m·s}$$

or

$$\Delta G_1 = 3.98\times10^{-3}\times3600 = 14.3 \text{ kg/m·h}.$$

The rate of steam flow per 1 m length of a tube of the first row $G_1 = w_1\rho''s\times3600 = 35\times0.0668\times1.25\times\times0.016\times3600 = 168$ kg/m·h.

The amount of steam condensing on a tube of the second row:

$$G_2 = G_1 - \Delta G_1 = 168 - 14.3 = 153.7 \text{ kg/m·h}.$$

Since, according to the conditions of the problem, the geometric characteristics of all rows of tubes are the same, then

$$\frac{w_2}{w_1} = \frac{G_2}{G_1} = \frac{153.7}{168} = 0.915 \text{ and } w_2 = 0.915\times35 = 32 \text{ m/s}.$$

In accordance with formula (8-8)

$$\frac{\alpha_2'}{\alpha_1} = \left(\frac{w_2}{w_1}\right)^{0.16} = (0.915)^{0.16} \approx 0.985$$

and, since p, Δt, r and d are constant for all rows of tubes, then

$$\Delta G_2 = \Delta G_1\frac{\alpha_2'}{\alpha_1} = 14.3\times0.985 = 14.1 \text{ kg/m·h}.$$

The values of ΔG_i, w_i and α'_i/α_1 for all rows of tubes are calculated in a similar manner.

Calculation results are tabulated below:

Row, No.	Q, kg/m·h	w, m/s	$\frac{\alpha'_n}{\alpha_1}$	ΔG, kg/m·h	ε	α, W/m²·°C
1	168	35	1.00	14.3	1.0	18 100
2	153.7	32	0.985	14.1	0.954	17 000
3	139.6	29	0.970	13.9	0.927	16 300
4	125.7	26.1	0.953	13.6	0.910	15 700
5	112.1	23.4	0.936	13.4	0.893	15 100
6	98.7	20.6	0.918	13.1	0.880	14 500
7	85.6	17.8	0.900	12.9	0.870	14 150
8	72.7	15.1	0.875	12.5	0.861	13 550
9	60.2	12.5	0.850	12.1	0.853	13 100
10	48.1	10.0	0.820	11.7	0.845	12 500
11	36.4	7.60	0.785	11.2	0.837	11 850
12	25.2	5.25	0.738	10.5	0.830	11 050
13	14.7	3.06	0.680	9.7	0.822	10 100
14	5.0	1.04	0.625	5.0	0.814	9 200

A correction is introduced into the values of the heat-transfer coefficients, accounting for their lessening, due to downward flow of the forming condensate:

$$\varepsilon_2=\left(\frac{\Delta G_1+\Delta G_2}{\Delta G_2}\right)^{-0.07}=\left(\frac{14.3+14.1}{14.1}\right)^{-0.07}=0.954;$$

$$\alpha_2=\alpha'_2\varepsilon_2=\alpha_1\frac{\alpha'_2}{\alpha_1}\varepsilon_2=18\,100\times0.985\times0.954=$$

$$=17\,000\ \text{W/m}^2\cdot{}^\circ\text{C};$$

$$\varepsilon_3=\left(\frac{\Delta G_1+\Delta G_2+\Delta G_3}{\Delta G_3}\right)^{-0.07}=$$

$$=\left(\frac{14.3+14.1+13.9}{13.9}\right)^{-0.07}=0.927$$

and

$$\alpha_3=18\,100\times0.97\times0.927=16\,300\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

The results of a successive calculation of the heat-transfer coefficients for the rows of tubes are tabulated above.

For the entire bank of tubes the mean heat-transfer coefficient

$$\bar{\alpha}=\frac{1}{F}\sum_{i=1}^{n}\alpha_i F_i,$$

where F_i = heat-transfer surface of individual rows of tubes;

F = total heat-transfer surface.

Since the heat-transfer surfaces of all rows of tubes are assumed to be the same, then

$$\bar{\alpha}=\frac{1}{n}\sum_{i=1}^{n}\alpha_i.$$

Summing up the values of α_i, given in the last column of the table, and dividing this sum by the number of rows $n = 14$, we find

$$\bar{\alpha} = 13\,700 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

8-30. Determine the heat-transfer coefficient for a tube of the fifth (counting from the top) row of the condenser considered in problem 8.29, if the velocity of steam, upstream of the upper row of tubes, is reduced by one half, and all other conditions remain unchanged. Also, determine the amount of steam condensing on length of 1 m of the tube, ΔG, kg/m·h.

Compare calculation results with the data obtained in solving prolem 8-29.

Answer

With $w_1 = 35$ m/s $\alpha_5 = 15\,100$ W/m²·°C and $\Delta G_5 = 13.4$ kg/m·h.

With $w_1 = 17.5$ m/s $\alpha_5 = 12\,450$ W/m²·°C and $\Delta G_5 = 11.1$ kg/m·h.

Chapter Nine

HEAT TRANSFER
IN BOILING LIQUIDS

9-1. Determine the coefficient of heat transfer from the external surface of the tube of an evaporator to boiling water, if the rate of heat flow through the heat-transfer surface $q = 2 \times 10^5$ W/m^2, the liquid is in nucleate boiling under a pressure $p = 2 \times 10^5$ Pa.

Answer

$$\alpha = 18\,400 \ \text{W/m}^2 \cdot {}^\circ\text{C}.$$

Solution

With liquid in pool nucleate boiling the heat-transfer coefficient can be calculated by formula [11]:

with $\text{Re}_* \geqslant 10^{-2}$

$$\text{Nu}_* = 0.125\text{Re}_*^{0.65}\text{Pr}^{1/3}; \qquad (9\text{-}1a)$$

with $\text{Re}_* \leqslant 10^{-2}$

$$\text{Nu}_* = 0.0625\text{Re}_*^{0.5}\text{Pr}^{1/3}. \qquad (9\text{-}1b)$$

where

$$\text{Re}_* = \frac{q l_*}{r \rho'' \nu};$$

$$\text{Nu}_* = \frac{\alpha l_*}{\lambda};$$

$$\text{Pr} = \frac{\nu}{a};$$

$$l_* = \frac{c_p \rho' \sigma T_s}{(r \rho'')^2}, \ \text{m};$$

ν, c_p, r, λ, a and σ = respectively the dynamic viscosity coefficient, heat capacity, evaporation, thermal conductivity, thermal diffusivity, surface tension coefficient at the saturation temperature t_s;

ρ' and ρ'' = respectively densities of liquid and vapour at saturation temperature t_s;

T_s = saturation temperature, K.

Formulae (9-1a) and (9-1b) are valid at $0.86 \leqslant \mathrm{Pr} \leqslant 7.6$, $10^{-5} \leqslant \mathrm{Re}_* \leqslant 10^4$, and a pressure from 45×10^3 to 175×10^5 Pa.

For water the values of l_* and $l_*/r\rho''\nu$, depending on temperature, are given in Table 9-1.

Table 9-1

Values of l_*, $\frac{l_*}{r\rho''\nu}$ and $\frac{\lambda}{r\rho''\nu}$ in Formulae (9-1a) and (9-1b)

t_s, °C	$l_* \times 10^6$, m	$\frac{l_*}{r\rho''\nu} 10^6$, m²/W	$\frac{\lambda}{r\rho''\nu} 10^2$, 1/°C	t_s, °C	$l_* \times 10^6$, m	$\frac{l_*}{r\rho''\nu} 10^6$, m²/W	$\frac{\lambda}{r\rho''\nu} 10^2$, 1/°C
30	16 450	276 870	1040	190	0.450	0.216	32.2
40	5 950	73 345	782	200	0.296	0.123	27.5
50	2 305	20 894	587	210	0.200	0.0718	23.5
60	960	6 543	450	220	0.136	0.0426	20.2
70	423	2 201	347	230	0.0938	0.0254	17.3
80	197	798	273	240	0.0646	0.0155	15.1
90	96.0	304	216	250	0.0451	0.00989	13.6
100	48.7	122.4	172	260	0.0318	0.00593	11.4
110	25.9	51.8	138	270	0.0224	0.00373	9.80
120	14.2	22.8	110	280	0.0158	0.00243	8.80
130	8.05	10.7	96.0	290	0.0114	0.00153	7.47
140	4.70	5.13	75.0	300	0.00800	0.000911	6.16
150	2.82	2.58	60.5	310	0.00565	0.000609	5.64
160	1.73	1.33	52.6	320	0.00398	0.000388	4.93
170	1.08	0.710	44.5	330	0.00278	0.000249	4.34
180	0.715	0.396	37.5	340	0.00192	0.000158	3.77
				350	0.00126	0.0000989	3.36

In the case considered, at a pressure $p = 2 \times 10^5$ Pa, saturation temperature $t_s = 120.2$ °C, $\lambda = 0.686$ W/m·°C.

Pr = 1.47. From Table 9-1 we find:

$l_* = 14.08 \times 10^{-6}$ m and $l_*/r\rho''\nu = 22.56 \times 10^{-6}$ m^2/W.

The Reynolds number

$$\mathrm{Re}_* = \frac{ql_*}{r\rho''\nu} = 2 \times 10^5 \times 22.56 \times 10^{-6} = 4.51.$$

Since $\mathrm{Re}_* > 10^{-2}$, calculations are carried out with the aid of formula (9-1a). Substituting the values of Re_* and Pr into that formula, we find that:

$$\mathrm{Nu}_* = 0.125\,(4.51)^{0.65}\,(1.47)^{1/3} = 0.378.$$

The heat-transfer coefficient

$$\alpha = \mathrm{Nu}_* \frac{\lambda}{l_*} = 0.378 \frac{0.686}{14.08 \times 10^{-6}} = 18\,400 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

9-2. Solve problem 9-1 on condition that water is under a pressure p equal to 1, 2.5 and 5.0 MPa. Also, determine the difference between the temperatures of the heating surface and boiling water $\Delta t = t_w - t_s$ at these pressures.

Answer

With $p = 1.0$ MPa $\alpha = 22\,600$ W/m$^2\cdot$°C and $\Delta t \approx 8.9$ °C;
with $p = 2.5$ MPa $\alpha = 27\,400$ W/m$^2\cdot$°C and $\Delta t \approx 7.3$ °C;
with $p = 5$ MPa $\alpha = 40\,400$ W/m$^2\cdot$°C and $\Delta t \approx 5$ °C.

9-3. Determine the coefficient of heat transfer from the outside surface of the tube of the evaporator, considered in problem 9-1, under the condition that the rate of heat flow $q = 3 \times 10^5$ and 4×10^5 W/m^2, all other conditions remaining unchanged.

Answer

With $q = 3 \times 10^5$ W/m^2 $\alpha = 24\,200$ W/m$^2\cdot$°C;
with $q = 4 \times 10^5$ W/m^2 $\alpha = 29\,000$ W/m$^2\cdot$°C.

9-4. Derive the formulae for heat transfer in nucleate pool boiling, in which the Nusselt number Nu_* would be a function of the temperature difference.

Hint. To obtain the indicated relationships the rate of heat flow q should be replaced in formulae (9-1a) and (9-1b) with $\alpha\Delta T$. For instance, presenting formula (9-1b) in the form

$$\frac{\alpha l_*}{\lambda}=0.0625\left(\frac{ql_*}{r\rho''\nu}\right)^{0.5}\mathrm{Pr}^{1/3}$$

and, making use of the equality $q=\alpha\Delta t$, we obtain:

$$\frac{\alpha l_*}{\lambda}=0.0625\left(\frac{\alpha\,\Delta t\,l_*}{r\rho''\nu}\right)^{0.5}\mathrm{Pr}^{1/3},$$

whence

$$(\alpha l_*)^{0.5}=0.0625\lambda\left(\frac{\Delta t}{r\rho''\nu}\right)^{0.5}\mathrm{Pr}^{1/3}$$

and

$$\frac{\alpha l_*}{\lambda}=(0.0625)^2\left(\frac{\lambda\,\Delta t}{r\rho''\nu}\right)\mathrm{Pr}^{2/3}.$$

Hence, formula (9-1b) acquires the appearance:

$$\mathrm{Nu}_*=3.91\times10^{-3}\left(\frac{\lambda\Delta t}{r\rho''\nu}\right)\mathrm{Pr}^{2/3}.$$

Proceeding in a similar way, we can transform formula (9-1a) and find the applicability range for these formulae.

Answer

With $\frac{\lambda\Delta t}{r\rho''\nu}\mathrm{Pr}^{1/3}>1.6$

$$\mathrm{Nu}_*=2.63\times10^{-3}\left(\frac{\lambda\Delta t}{r\rho''\nu}\right)^{1.86}\mathrm{Pr}^{0.952};\qquad(9\text{-}2a)$$

with $\frac{\lambda\Delta t}{r\rho''\nu}\mathrm{Pr}^{1/3}\leqslant1.6$

$$\mathrm{Nu}_*=3.91\times10^{-3}\left(\frac{\lambda\Delta t}{r\rho''\nu}\right)\mathrm{Pr}^{2/3}.\qquad(9\text{-}2b)$$

Formulae (9-2a) and (9-2b) are applicable within the same limits of Prandtl and Reynolds numbers and pressures as formulae (9-1a) and (9-1b), and on condition that

$$0.05\leqslant\frac{\lambda\Delta t}{r\rho''\nu}\mathrm{Pr}^{1/3}\leqslant200.$$

For water the values of the term $\lambda/r\rho''\nu$, 1/°C, depending on temperature, are given in Table 9-1.

9-5. Determine the rate of heat flow through the heating surface of a steam generator (boiler unit) with water in nucleate pool boiling, if the water is under a pressure $p = 6.2 \times 10^5$ Pa and the temperature of the heating surface $t_w = 175$ °C.

Answer

$$q = 790 \text{ kW/m}^2.$$

Solution

With $p = 6.2 \times 10^5$ Pa $t_s = 160$ °C, $\Pr = 1.1$, $\lambda = 0.683$ W/m·°C; from Table 9-1 we find:

$$\lambda/r\rho''\nu = 0.526 \text{ 1/°C}; \; l_* = 1.73 \times 10^{-6} \text{ m}.$$

The temperature difference $\Delta t = t_w - t_s = 175 - 160 = 15$ °C.

Then,

$$\frac{\lambda \Delta t}{r\rho''\nu} = 0.526 \times 15 = 7.9$$

and

$$\frac{\lambda \Delta t}{r\rho''\nu} \Pr^{1/3} = 7.9\,(1.1)^{1/3} = 8.15.$$

Since

$$\frac{\lambda \Delta t}{r\rho''\nu} \Pr^{1/3} > 1.6,$$

calculation formula (9-2a) is used:

$$\mathrm{Nu}_* = 2.63 \times 10^{-3}\,(7.9)^{1.86}\,(1.1)^{0.952} = 0.134.$$

The heat-transfer coefficient and the rate of heat flow:

$$\alpha = \mathrm{Nu}_* \frac{\lambda}{l_*} = 0.134 \frac{0.683}{1.73 \times 10^{-6}} = 52\,800 \text{ W/m}^2\cdot\text{°C}$$

and

$$q = \alpha \Delta t = 52\,800 \times 15 = 7.9 \times 10^5 \text{ W/m}^2.$$

9-6. Determine the coefficient of heat transfer from the outer surface of the tube of a steam generator to water, boiling under a pressure $p = 4.7$ MPa with tube surface temperatures, t_w, equal to 265, 270 and 275 °C. Also, determine the rates of heat flow at respective tube surface temperatures.

Answer

With $t_w = 265$ °C $\alpha = 38\,400$ W/m²·°C $q = 192$ kW/m²;
with $t_w = 270$ °C $\alpha = 76\,800$ W/m²·°C $q = 768$ kW/m²;
with $t_w = 275$ °C $\alpha = 123\,000$ W/m²·°C $q = 1850$ kW/m².

9-7. Determine the critical heat flux with water in pool boiling at a pressure $p = 1 \times 10^5$ Pa.

Answer

$$q_{cr} = 1.4 \times 10^6 \text{ W/m}^2.$$

Solution

With water in pool boiling the critical heat flux can be calculated with the aid of formula [11]

$$\mathrm{Re}_{*cr} = 68\mathrm{Ar}_*^{4/9}\mathrm{Pr}^{-1/3}, \tag{9-3}$$

where

$$\mathrm{Re}_{*cr} = \frac{q_{cr} l_*}{r\rho'' \nu};$$

$$\mathrm{Ar}_* = g\frac{l_*^3}{\nu^2}\frac{\rho' - \rho''}{\rho'}.$$

The notations of all quantities are the same as in formulae (9-1) and (9-2). The formula is applicable at $0.86 \leqslant \mathrm{Pr} \leqslant 13.1$ and pressures falling within $1 \times 10^5 \leqslant p \leqslant 185 \times 10^5$ Pa.

In the case considered with $p = 1 \times 10^5$ Pa $t_s = 99.6$ °C, $\nu = 0.296 \times 10^{-6}$ m²/s, $\mathrm{Pr} = 1.76$, $\rho' = 960$ kg/m³, $\rho'' = 0.59$ kg/m³.

From Table 9-1 we find:

$$l_* = 50.6 \times 10^{-6} \text{ m}; \quad \frac{l_*}{r\rho''\nu} = 130 \times 10^{-6} \text{ m}^2/\text{W}.$$

The Archimedes number

$$\mathrm{Ar}_* = g\frac{l_*^3}{\nu^2}\frac{\rho' - \rho''}{\rho'} = 9.81\frac{(5.06 \times 10^{-5})^3}{(2.96 \times 10^{-3})^2}\frac{960 - 0.59}{960} = 14.4.$$

With the aid of formula (9-3) we find:

$$\mathrm{Re}_{*cr} = 68\mathrm{Ar}_*^{4/9}\mathrm{Pr}^{-1/3} = 68\,(14.4)^{4/9}\,(1.76)^{-1/3} = 184$$

and

$$q_{cr} = \mathrm{Re}_{*cr}\frac{r\rho''\nu}{l_*} = 184\,\frac{1}{130 \times 10^{-6}} = 1.41 \times 10^6 \text{ W/m}^2.$$

9-8. Determine the critical rate of heat flow, or heat flux, with water in pool boiling under a pressure $p = 7.5$ and 15 MPa. Compare calculation results with the answer to problem 9-7.

Answer

With $p = 1 \times 10^5$ Pa $q_{cr} = 1.4 \times 10^6$ W/m²; with $p = 75 \times 10^5$ Pa $q_{cr} = 4.1 \times 10^6$ W/m²; with $p = 150 \times 10^5$ Pa $q_{cr} = 3 \times 10^6$ W/m².

9-9. Calculate (approximately) the maximum heat-transfer coefficient for water in nucleate pool boiling, if water pressure is 1×10^5 and 75×10^5 Pa.

Also, determine the temperatures of the heating surface at which film-type boiling sets in at the indicated pressures.

Note. In determining the critical temperature difference $\Delta t_{cr} = (t_w - t_s)_{cr}$, use can be made of the answers to problems 9-7 and 9-8. The magnitude of α_{cr} can be evaluated with the aid of formulae (9-1a) and (9-1b).

Answer

At $p = 1 \times 10^5$ Pa $\alpha_{cr} \approx 60\,000$ W/m²·°C, $\Delta t_{cr} \approx 23$ °C, and surface temperature $t_w \approx 123$ °C.

At $p = 75 \times 10^5$ Pa $\alpha_{cr} \approx 2.4 \times 10^5$ W/m²·°C, $\Delta t_{cr} \approx 17$ °C, and $t_w \approx 307$ °C.

9-10. Water boils at a pressure $p = 4.9 \times 10^5$ Pa on the surface of a tube having a diameter $d = 38$ mm and length $l = 0.5$ m. The inside surface of the tube is heated with the aid of an electric heater. The power consumed for heating $W = 7$ kW.

Determine the outside surface temperature of the tube.

Answer

$$t_w = 159 \text{ °C}.$$

9-11. Determine the heating surface required for a steam boiler generating $G = 4$ t/h of steam at a pressure $p = 15.7 \times 10^5$. The temperature difference would be

$$\Delta t = t_w - t_s = 10 \text{ °C}.$$

Answer

$$F = 6 \text{ m}^2.$$

9-12. Assuming the conditions of problem 9-11, what temperature difference should be ensured, to increase the capacity of the steam generator 2.5 times with the same heating surface $F = 6$ m²?

Answer

$\Delta t = 13.8$ °C. An increase of the temperature difference from 10 to 13.8 °C results in an increase in the steaming capacity of the boiler unit from 4 to 10 t/h.

9-13. Water boils on the outside surface of a tube at a pressure $p = 3.3$ MPa. The rate of heat flow on the surface of the tube $q = 1.75 \times 10^6$ W/m^2.

Determine the surface temperature of the tube: (a) if the surface is clean; (b) if the surface of the tube is covered with an oxide film, whose resistance to heat flow $R = 7.75$ °C·m^2/W. Solving the problem, assume that due to the roughness of the oxide film, the heat-transfer coefficient increases on its surface 2.5 times, compared with boiling on a clean surface [8].

Answer

When boiling takes place on a clean surface $t_w = 245$ °C. In the presence of an oxide film $t_w \approx 255$ °C. Respective temperature distribution curves are shown in Fig. 9-1.

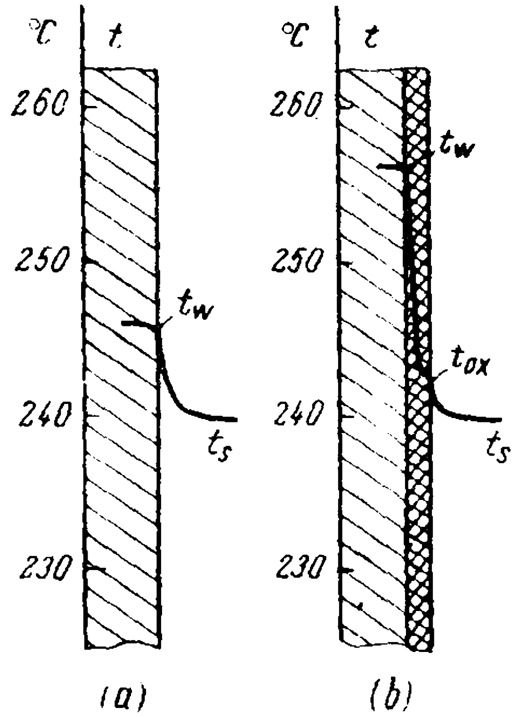

Fig. 9-1. Solution to problem 9-13

Solution

(a) If the surface of the tube is clean, the difference between the temperatures of the wall and boiling liquid, $t_w - t_s = q/\alpha$, and the heat-transfer coefficient in the above relationship is determined by means of formula (9-1a) or (9-1b).

With $p = 3.3$ MPa $t_s = 239.2$ °C and $l_* = 0.0699 \times 10^{-6}$ m, $l_*/r\rho''\nu = 0.0163 \times 10^{-6}$ m^2/W, $\lambda = 0.629$ W/m·°C, Pr $= 0.87$.

The Reynolds number

$$\mathrm{Re}_* = \frac{ql_*}{r\rho''\nu} = 1.75 \times 10^6 \times 0.0163 \times 10^{-6} = 2.86 \times 10^{-3}.$$

Since $Re_* < 10^{-2}$, the Nusselt number Nu_* is determined from formula (9-1b):

$$Nu_* = 0.0625 Re_*^{0.5} Pr^{1/3} = 0.0625\,(2.86\times10^{-3})^{0.5}\times$$
$$\times\,(0.87)^{1/3} = 3.19\times10^{-3}.$$

The heat-transfer coefficient

$$\alpha = Nu_*\frac{\lambda}{l_*} = 3.19\times10^{-3}\,\frac{0.629}{0.0669\times10^{-6}} = 3\times10^{4}\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and the unknown temperature difference

$$t_w - t_s = \frac{q}{\alpha} = \frac{1.75\times10^{5}}{3\times10^{4}} = 5.83\ {}^\circ\text{C},$$

whence

$$t_w = 5.83 + 239.2 \approx 245\ {}^\circ\text{C}.$$

(b) With account taken of the additional thermal resistance to heat flow offered by the oxide film $t_w - t_s = q/k$, where, assuming approximately as for a plane wall,

$$k = \frac{1}{\dfrac{1}{\alpha'} + R},$$

and using the conditions of the problem

$$\alpha' = 2.5\alpha = 2.5\times3.10^{4} = 7.5\times10^{4},\ \text{W/m}^2\cdot{}^\circ\text{C},$$

we obtain:

$$k = \frac{1}{\dfrac{1}{7.5\times10^{4}} + 7.75\times10^{-5}} = 1.1\times10^{4}\ \text{W/m}^2\cdot{}^\circ\text{C},$$

$$t_w - t_s = \frac{q}{k} = \frac{1.75\times10^{5}}{1.1\times10^{4}} = 15.9\ {}^\circ\text{C}$$

and

$$t_w = 15.9 + 239.2 \approx 255\ {}^\circ\text{C}.$$

9-14. Solve problem 9-13 on conditions that the rate of heat flow on the tube increased 2 times ($q = 3.5\times10^{5}$ W/m^2), and all other conditions remained unchanged.

Answer

With liquid boiling on a clean surface the temperature difference shall increase $\sqrt{2}$ times and $t_w \approx 247.4$ °C.

In the presence of an oxide film $t_w \approx 270$ °C.

9-15. A tube having an inside diameter $d = 18$ mm carries boiling water flowing at a velocity $w = 1$ m/s The boiling water is under a pressure $p = 8 \times 10^5$ Pa.

Determine the coefficient of heat transfer from the wall to the boiling water, if the inside surface temperature of the tube $t_w = 173$ °C.

Answer

$$\alpha = \alpha_w = 8040 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

Solution

With a boiling liquid in forced flow in tubes on condition that the liquid is heated to the saturation temperature, the heat-transfer coefficient can be calculated by the following formulae [11]:

with $\frac{\alpha_b}{\alpha_w} \leqslant 0.5$

$$\alpha \doteq \alpha_w; \qquad (9\text{-}4a)$$

with $\frac{\alpha_b}{\alpha_w} \geqslant 2$

$$\alpha = \alpha_b; \qquad (9\text{-}4b)$$

with $0.5 \leqslant \frac{\alpha_b}{\alpha_w} \leqslant 2$

$$\frac{\alpha}{\alpha_w} = \frac{4\alpha_w + \alpha_b}{5\alpha_w - \alpha_b} \qquad (9\text{-}4c)$$

where α = heat-transfer coefficient for boiling liquid in forced flow in tubes;

α_b = heat-transfer coefficient for liquid in nucleate pool boiling, determined by means of formulae (9-1) or (9-2);

α_w = heat-transfer coefficient for a single-phase liquid in turbulent flow in tubes, determined by formula (5-7).

Formulae (9-4a) through (9-4c) hold for water at pressures ranging from 1×10^5 to 86×10^5 Pa with velocities from 0.2 to 6.7 m/s and a volume vapour fraction $\beta < 70\%$.

Determine the heat transfer coefficient for a single-phase liquid in forced flow in tubes, α_w. With $p = 8 \times 10^5$ Pa $t_s = 170.4$ °C, $\nu_f = 0.181 \times 10^{-6}$ m²/s, $\lambda_f = 0.679$ W/m·°C, $\mathrm{Pr}_f = 1.05$.

With $t_w = 173$ °C $\mathrm{Pr}_w = 1.04$.

The Reynolds number

$$\mathrm{Re}_f=\frac{wd}{\nu_f}=\frac{1\times 18\times 10^{-3}}{0.181\times 10^{-6}}=99\,400.$$

By formula (5-7) we find:

$$\mathrm{Nu}_f=0.021\,\mathrm{Re}_f^{0.8}\mathrm{Pr}_f^{0.43}\,(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}=$$

$$=0.021\,(9.94\times 10^4)^{0.8}\,(1.05)^{0.43}\left(\frac{1.05}{1.04}\right)^{0.25}=213;$$

hence, the heat-transfer coefficient

$$\alpha_w=\mathrm{Nu}_f\,\frac{\lambda_f}{d}=213\,\frac{0.679}{18\times 10^{-3}}=8040\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Determine the heat-transfer coefficient for liquid in nucleate pool boiling, α_b

With t_s = 170.4 °C from Table 9-1 we find:

$$l_* = 1.07\times 10^{-6}\ \mathrm{m};$$

$$\frac{\lambda}{r\rho''\nu}=44.2\times 10^{-2}\,1/{}^\circ\mathrm{C};$$

$$\Delta t = t_w - t_s = 173 - 170.4 = 2.6\ {}^\circ\mathrm{C};$$

$$\frac{\lambda\,\Delta t}{r\rho''\nu}=44.2\times 10^{-2}\times 2.6=1.15;$$

$$\frac{\lambda\,\Delta t}{r\rho''\nu}\,\mathrm{Pr}^{1/3}=1.15\,(1.05)^{1/3}=1.17<1.6$$

and calculation formula (9-2b) is used

$$\mathrm{Nu}_*=3.91\times 10^{-3}\left(\frac{\lambda\,\Delta t}{r\rho''\nu}\right)\mathrm{Pr}^{2/3}=3.91\times 10^{-3}\times$$

$$\times 1.15\,(1.05)^{2/3}=4.67\times 10^{-3}.$$

The heat-transfer coefficient

$$\alpha_b=\mathrm{Nu}_*\,\frac{\lambda}{l_*}=4.67\times 10^{-3}\,\frac{0.679}{1.07\times 10^{-6}}=2960\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Determine the ratio of the heat-transfer coefficients α_b/α_w:

$$\frac{\alpha_b}{\alpha_w}=\frac{2960}{8040}=0.368.$$

Since $\alpha_b/\alpha_w < 0.5$, then, according to formula (9-4a) the intensity of heat transfer is characterized by forced flow and $\alpha = \alpha_w = 8040$ W/m²·°C.

9-16. Under conditions of problem 9-15 determine the coefficient of heat transfer for boiling water in forced flow in a tube, if the temperature of the tube inner surface is $t_w = 175$ and 180 °C.

Answer

With $t_w = 175$ °C $\alpha = 9150$ W/m²·°C;
with $t_w = 180$ °C $\alpha = 25\,200$ W/m²·°C.

Solution

(a) $t_w = 175$ °C. With the wall at different temperatures the heat-transfer coefficient for a moving single-phase liquid changes only due to the variation of the physical properties of the liquid over the cross section of flow, which is accounted for by the correction $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25}$. With $t_w = 175$ °C $\mathrm{Pr}_w = 1.03$ and, just as it was assumed in problem 9-15, $(\mathrm{Pr}_f/\mathrm{Pr}_w)^{0.25} \approx 1$. It follows from the solution of problem 9-15 that

$$\alpha_w = 8040 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Determine the heat-transfer coefficient for liquid in nucleate pool boiling:

$$\Delta t = t_w - t_s = 175 - 170.4 = 4.6 \text{ °C};$$

$$\frac{\lambda\,\Delta t}{r\rho''\nu} = 4.6 \times 44.2 \times 10^{-2} = 2.03.$$

Since $\frac{\lambda\,\Delta t}{r\rho''\nu} > 1.6$, calculation formula (9-2a) is used:

$$\mathrm{Nu}_* = 2.63 \times 10^{-3}\left(\frac{\lambda\,\Delta t}{r\rho''\nu}\right)^{1.86}\mathrm{Pr}^{0.952} =$$

$$= 2.63 \times 10^{-3}\,(2.03)^{1.86}\,(1.05)^{0.952} = 1.017 \times 10^{-2};$$

$$\alpha_b = \mathrm{Nu}_*\,\frac{\lambda}{l_*} = 1.017 \times 10^{-2}\,\frac{0.679}{1.07 \times 10^{-6}} = 6450 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The ratio of heat-transfer coefficients

$$\frac{\alpha_b}{\alpha_w} = \frac{6450}{8040} = 0.8.$$

Since $0.5 < \alpha_b/\alpha_w < 2$, then, according to formula (9-4c), the intensity of heat transfer is determined both by forced flow of the liquid and by the process of boiling, and

$$\frac{\alpha}{\alpha_w}=\frac{4\alpha_w+\alpha_b}{5\alpha_w-\alpha_b}=\frac{4+0.8}{5-0.8}=1.14,$$

consequently, $\alpha = 1.14\alpha_w = 1.14 \times 8040 = 9150$ W/m²·°C.

(b) With $t_w = 180$ °C $\mathrm{Pr}_w = 1.00$ and, just as in case (a), it may be assumed that

$$\alpha_w = 8040 \ \mathrm{W/m^2\cdot{}^\circ C},$$

Determine the heat-transfer coefficient for liquid in nucleate pool boiling

$$\Delta t = t_w - t_s = 180 - 170.4 = 9.6 \ {}^\circ\mathrm{C}$$

and, just as in case (a),

$$\frac{\lambda\,\Delta t}{r\rho''\nu}\mathrm{Pr}^{1/3} > 1.6,$$

therefore,

$$\alpha_b=\alpha_{b,\mathrm{a}}\left(\frac{\Delta t_{\mathrm{b}}}{\Delta t_{\mathrm{a}}}\right)^{1.86}=6450\left(\frac{9.6}{4.6}\right)^{1.86}=25\,200 \ \mathrm{W/m^2\cdot{}^\circ C}.$$

The ratio of the heat transfer coefficients

$$\frac{\alpha_b}{\alpha_w}=\frac{25\,200}{8040}\approx 3.14 > 2.$$

In accordance with formula (9-4b), the intensity of heat transfer is determined then only by the process of boiling and

$$\alpha = \alpha_b = 25\,200 \ \mathrm{W/m^2\cdot{}^\circ C}.$$

9-17. A tube having an inside diameter $d = 38$ mm carries boiling water moving with a velocity $w = 1$ m/s, under a pressure $p = 2.8$ MPa.

Determine the rate of heat flow q, W/m², and the coefficient of heat transfer from the wall of the tube to the boiling water, if the temperature of the tube inside surface $t_w = 236.9$ °C.

Answer

$$q = 2 \times 10^5 \ \mathrm{W/m^2}, \quad \alpha = \alpha_b = 2.9 \times 10^4 \ \mathrm{W/m^2\cdot{}^\circ C}.$$

9-18. Determine the inside surface temperature of a tube t_w, carrying boiling water, if the rate of heat flow on the surface $q = 4.5 \times 10^5$ W/m², the velocity of water $w = 4$ m/s and the pressure $p = 1.57$ MPa. The tube inside diameter $d = 12$ mm.

Answer

$$\Delta t = 10\ ^\circ\text{C};\quad t_w = 210.4\ ^\circ\text{C}.$$

9-19. Determine the heat-transfer coefficient and the temperature of the inner surface of a tube having a diameter $d = 38$ mm and carrying boiling water, assuming a rate of heat flow $q = 2 \times 10^5$ W/m², water pressure $p = 2.8$ MPa and velocity $w = 1$ m/s.

Answer

$$\alpha = 29\,000\ \text{W/m}^2\cdot{}^\circ\text{C};\quad t_w = 237\ ^\circ\text{C}.$$

9-20. How shall change the coefficient of heat transfer with water boiling in a tube having a diameter $d = 20$ mm, if the rate of heat flow from the heating surface is increased from $q = 5 \times 10^4$ W/m² to $q = 1 \times 10^5$ W/m², assuming a water flow velocity $w = 5$ m/s and a pressure $p = 2 \times 10^5$ Pa?

Answer

The heat-transfer coefficient shall not change. In both cases $\alpha = 25\,600$ W/m²·°C.

9-21. Under conditions of problem 9-20, determine the magnitude of the rate of heat flow at which boiling of the liquid shall begin to affect the intensity (rate) of heat transfer.

Answer

$$q = 1.15 \times 10^5\ \text{W/m}^2.$$

9-22. How shall change the heat-transfer coefficient with water boiling in a tube having a diameter $d = 38$ mm, raising the velocity of water flow from $w = 0.3$ m/s to $w = 3$ m/s, if the rate of heat flow from the heating surface $q = 2.5 \times 10^5$ W/m² and water pressure $p = 7.5$ MPa?

Answer

The heat-transfer coefficient shall not change. In both cases $\alpha = 60\,000$ W/m^2·°C.

9-23. Determine the velocity of water flow at which, under conditions of problem 9-22, forced flow shall begin to tell on the intensity (rate) of heat transfer.

Answer

$$w = 6 \text{ m/s}.$$

Chapter Ten

RADIATION HEAT TRANSFER BETWEEN SOLIDS SEPARATED BY A TRANSPARENT (DIATHERMANOUS) PARTICIPATING MEDIUM

10-1. Determine the emissive ability of the surface of the Sun, if its temperature is known to be 5700 °C, and the conditions of emission are close to these of a black body. Also, calculate the wavelength at which a maximum spectral intensity shall be observed and the total amount of radiant energy emitted by the Sun per unit time, if its diameter can be assumed equal to 1.391×10^9 m.

Answer

$$E_0 = 72.2 \times 10^6 \ \mathrm{W/m^2}; \quad \lambda_{max} = 0.485 \ \mu\mathrm{m};$$
$$Q = 4.38 \times 10^{26} \ \mathrm{W}.$$

10-2. The surface of a steel product has a temperature $t_w = 727$ °C and its emissivity is $\varepsilon_w = 0.7$. The emitting surface can be considered to be a grey one.

Calculate the density of emitted radiation and the wavelength corresponding to the maximum spectral intensity of radiation.

Answer

$$E = 3.97 \times 10^4 \ \mathrm{W/m^2}; \quad \lambda_{max} = 2.898 \ \mu\mathrm{m}.$$

10-3. Assuming the conditions of problems 10-1 and 10-2, find the maximum spectral intensities of radiation.

Answer

$$J_{0\lambda max} = 9.94 \times 10^{13} \ \mathrm{W/m^3}; \quad J_{\lambda max} = 9.15 \times 10^9 \ \mathrm{W/m^3}.$$

10-4. Determine the fraction of black body radiation that shall be reflected by the surface of polished aluminium at the temperature $t = 250$ °C, if it is known that at that temperature the emissive ability of the surface $E = 170$ W/m². The temperature of the black body emitter is equal to the surface temperature of the aluminium.

Answer

$$R = 0.96.$$

10-5. The temperature of a surface that can be assumed to be grey is equal to 827 °C. At that temperature the maximum spectral intensity of radiation

$$J_{\lambda max} = 1.37 \times 10^{10} \text{ W/m}^3.$$

Determine the emissivity of the body and the wavelength at which the spectral intensity of radiation shall be maximum.

Answer

$$\varepsilon = 0.65; \quad \lambda_{max} = 2.634 \ \mu\text{m}.$$

10-6. An instrument used to measure high temperature—an optical pyrometer is based on comparing the radiosity of a body investigated with that of an incandescent filament. The instrument is calibrated based on the emission of a black body and, therefore, it measures the temperature which the black body would have at a radiosity equal to that of the investigated body. The pyrometer incorporates a red light filter ($\lambda = 0.65$ μm).

What is the true temperature of the body, if the pyrometer records the temperature of 1400°C, and the emissivity of the body at $\lambda = 0.65$ μm is equal to 0.6?

Answer

$$t = 1467 \text{ °C}.$$

Solution

The radiosity of the investigated body

$$B_\lambda = \frac{J_\lambda}{\pi} = \frac{1}{\pi} \frac{\varepsilon_\lambda c_1 \lambda^{-5}}{e^{c_2/\lambda T} - 1},$$

where T is the absolute temperature of the investigated body.

The radiosity of a black body

$$B_{0\lambda} = \frac{J_{0\lambda}}{\pi} = \frac{1}{\pi} \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T_0} - 1},$$

where T_0 is the absolute temperature of the black body, and at $B_\lambda = B_{0\lambda}$ it will be the temperature indicated by the pyrometer.

Since in the case considered $c_2/\lambda T_0 = 13.2$, then $e^{c_2/\lambda T_0}$ is considerably greater than unity. That is why the digit 1 in the denominator of the formulae can be ignored compared with $e^{c_2/\lambda T}$.

It follows from the condition $B_\lambda = B_{0\lambda}$ that

$$\frac{1}{T}=\frac{1}{T_0}-\frac{\lambda}{c^2}\ln\frac{1}{\varepsilon_\lambda},$$

whence

$$T=\frac{1}{\frac{1}{T_0}-\frac{\lambda}{c_2}\ln\frac{1}{\varepsilon_\lambda}}=$$

$$=\frac{1}{\frac{1}{1673}-\frac{0.65\times10^{-6}}{1.439\times10^{-2}}2.3\log\frac{1}{0.6}}=1740\ \text{K}.$$

The temperature of the body

$$t=1740-273=1467^\circ\text{C}.$$

10-7. An optical pyrometer with a red light filter (see problem 10-6) records a temperature $t_0 = 1600\ ^\circ\text{C}$.

Find the emissivity of the investigated body at $\lambda = 65\ \mu\text{m}$, if it is known that its true temperature $t = 1700\ ^\circ\text{C}$.

Answer

$$\varepsilon_\lambda = 0.55.$$

10-8. The temperature of a body is measured with the aid of two optical pyrometers fitted with different light filters. The first pyrometer mounts a red light filter ($\lambda_1 = 0.65\ \mu\text{m}$) and the second—a green one ($\lambda_2 = 0.50\ \mu\text{m}$). The temperature readings of the two pyrometers are respectively $t_{01} = 1400\ ^\circ\text{C}$ and $t_{02} = 1420\ ^\circ\text{C}$.

Find the true temperature of the body and its emissivity, assuming the body to be grey.

Answer

$$t = 1492\ ^\circ\text{C};\quad \varepsilon = 0.71.$$

Solution

The formulae derived in solving problem 10-6 permit us to write:

$$\left.\begin{aligned}\frac{1}{T_{01}}-\frac{1}{T}&=\frac{\lambda_1}{c_2}\ln\frac{1}{\varepsilon_{\lambda 1}};\\ \frac{1}{T_{02}}-\frac{1}{T}&=\frac{\lambda_2}{c_2}\ln\frac{1}{\varepsilon_{\lambda 2}}.\end{aligned}\right\}\tag{1}$$

For a grey body $\varepsilon_{\lambda_1}=\varepsilon_{\lambda_2}=\varepsilon$. From the system of equations (1) we obtain the following expressions for T and ε:

$$T=\frac{\frac{\lambda_1}{\lambda_2}-1}{\frac{\lambda_1}{\lambda_2}\frac{1}{T_{02}}-\frac{1}{T_{01}}};\tag{2}$$

$$\ln\varepsilon=\frac{c_2(T_{01}-T_{02})}{T_{01}T_{02}(\lambda_1-\lambda_2)}.\tag{3}$$

Substituting the found values into relationship (2) and (3), we obtain:

$$T=1765\ \text{K};\quad t=1492\ ^\circ\text{C};\quad \varepsilon=0.71.$$

10-9. Find the ratio of the relative hemispherical and directional emissive abilities for the surface of oxidized copper at 130 °C, if it is known that:

(a) within the limits of the angle $0<\varphi<60°$ the emission of oxidized copper obeys Lambert's cosine law, with the emissivity of directional radiation within this interval being $\varepsilon_\varphi=0.8$;

(b) within the limits of the angle $60°<\varphi<90°$ 67 per cent of the radiation coming in these directions from a black body which has the same temperature as the surface of oxidized copper is absorbed.

Note. φ is the angle between an arbitrary direction and the normal to the surface.

Answer

$$\varepsilon/\varepsilon_{\varphi=0}=0.96.$$

Solution

By definition, the relative hemispherical emissive ability (emissivity)

$$\varepsilon=\frac{E}{E_0}.$$

For oxidized copper the hemispherical emissive ability can be expressed in terms of the intensity of radiation:

$$E = \int\limits_{(2\pi)} J_\varphi \, d\omega = \int\limits_{(\omega_1)} J_\varphi \, d\omega + \int\limits_{(2\pi-\omega_1)} J_\varphi \, d\omega,$$

where ω_1 is the solid angle within which the emissivity of directional radiation is constant.

Allowing for $J_\varphi = \varepsilon_\varphi J_{0\varphi}$ and $J_{0\varphi} = B_0 \cos\varphi$ (B_0 is the radiosity of black body radiation), the first integral can be presented as follows:

$$\int\limits_{(\omega_1)} J_\varphi \, d\omega = \varepsilon_\varphi B_0 \int\limits_{(\omega_1)} \cos\varphi \, d\omega.$$

According to Kirchhoff's law, the second integral

$$\int\limits_{(2\pi-\omega_1)} J_\varphi \, d\omega = A' \int\limits_{(2\pi-\omega_1)} J_{0\varphi} \, d\omega = A' B_0 \int\limits_{(2\pi-\omega_1)} \cos\varphi \, d\omega,$$

where A' is the absorptivity relating to the entire black body radiation that is incident within the solid angle $2\pi - \omega_1$.

Since $d\omega = \sin\varphi d\varphi d\vartheta$, then

$$E = \varepsilon_\varphi B_0 \int\limits_0^{2\pi} d\vartheta \int\limits_0^{\pi/3} \sin\varphi \cos\varphi \, d\varphi +$$

$$+ A' B_0 \int\limits_0^{2\pi} d\vartheta \int\limits_{\pi/3}^{\pi/2} \sin\varphi \, \mathrm{oos}\, \varphi \, d\varphi = \varepsilon_\varphi B_0 \frac{3}{4} \pi + A' B_0 \frac{1}{4} \pi.$$

According to Lambert's law, $E_0 = B_0 \pi$. Finally we obtain:

$$\varepsilon = \frac{\varepsilon_\varphi B_0 \frac{3}{4} \pi + A' B_0 \frac{1}{4} \pi}{B_0 \pi} = \varepsilon_\varphi \frac{3}{4} + A' \frac{1}{4} =$$

$$= 0.8 \frac{3}{4} + 0.67 \frac{1}{4} = 0.7675.$$

The normal relative emissive ability $\varepsilon_{\varphi=0} = \varepsilon_\varphi = 0.8$. The ratio to be determined

$$\varepsilon / \varepsilon_{\varphi=0} = 0.7675/0.8 = 0.96.$$

10-10. A surface covered with a layer of lamp (jet) black emits in the normal direction per unit solid angle the radiant energy $J_{\varphi=0} = 1.87 \times 10^3$ W/m²·°C. For black body radiation the absorptivity of lamp black is equal to 0.96. Determine the temperature of this surface, assuming that Lambert's cosine law is valid for lamp black.

Answer

$$t = 300\ ^\circ\mathrm{C}.$$

10-11. Determine the intensity of solar radiation that is incident upon a plane normal to sun rays and arranged outside the Earth's atmosphere. It is known that solar radiation is close to that of a black body at a temperature $t_0 = 5700$ °C. The diameter of the Sun $D = 1.391 \times 10^6$ km, the distance from the Earth to the Sun $l = 149.5 \times 10^6$ km.

Answer

$$E_{in} = 1550\ \mathrm{W/m^2}.$$

Solution

The density of incident solar radiation is determined by formula:

$$E_{in} = B d\omega,$$

where B = radiosity of solar radiation;

$d\omega$ = solid angle at which a unit area "sees" the Sun.

The radiosity of solar radiation

$$B = \frac{E_0}{\pi} = \frac{C_0 (T_0/100)^4}{\pi}.$$

The solid angle

$$d\omega = \frac{\pi \frac{D^2}{4}}{l^2}.$$

With account taken of these relationships

$$E_{in} = \frac{C_0 \left(\frac{T_0}{100}\right)^4 \pi D^2}{\pi 4 l^2} = \frac{5.67\,(59.73)^4 \times 1.391^2}{4 \times 149.4^2} = 1550\ \mathrm{W/m^2}.$$

10-12. An artificial spherical satellite (Sputnik) flies round the Earth being launched on its day's light side. The absorptivity of the Sputnik's surface in respect to incident solar radiation is A, and the emissivity of this surface is ε.

Determine the temperature of the Sputnik's surface, assuming that there are no inner heat sources inside the Sputnik and that the surface temperature is the same all over it. The solar radiation reflected from the Earth and the radiation emitted from the Earth should be ignored.

Answer

$$T = 288 \sqrt[4]{\frac{A}{\varepsilon}}, K; \qquad t = 288 \sqrt[4]{\frac{A}{\varepsilon}} - 273 \text{ °C}.$$

Solution

Under steady-state conditions the amount of radiant energy absorbed by the Sputnik and the amount of energy emitted by it into space are equal to each other, i.e.

$$AE_{in}F_N = \varepsilon C_0 E \left(\frac{T}{100}\right)^4,$$

where F_N = projection of the Sputnik's irradiated surface on the plane normal to incident radiation;

F = surface area of the Sputnik.

For a sphere

$$\frac{F_N}{F} = \frac{\pi d^2}{4\pi d^2} = \frac{1}{4}.$$

The density of incident radiation E_{in} = 1550 W/m^2 (see problem 10-11). Finally, the temperature of the Sputnik

$$T = 100 \sqrt[4]{\frac{AE_{in}F_N}{\varepsilon C_0 F}} = 100 \sqrt[4]{\frac{A \times 1550}{\varepsilon 5.67 \times 4}} = 288 \sqrt[4]{\frac{A}{\varepsilon}}.$$

10-13. Solve problem 10-12, assuming that the surface is made from metal for which $A = 0.2$ and $\varepsilon = 0.1$.

Answer

$$t = 70 \text{ °C}.$$

10-14. Under the conditions of problem 10-12 find the surface temperature for the Sputnik, assuming the surface to be grey.

Answer

$$t = 15 \text{ °C}.$$

10-15. Under the conditions of problem 10-12 find the ratio of the surface absorptivity of the Sputnik in respect to incident radiation to the emissivity at which the surface temperature will be equal to 30 °C.

Answer

$$A/\varepsilon = 1.225.$$

10-16. A spaceship launched from the Earth runs to Venus. The distance from Venus to the Sun is 108.1×10^6 km and from the Earth to the Sun, 149.5×10^6 km. The temperature of the spaceship surface near the Earth was equal to t_1 °C.

How shall change the temperature of the spaceship surface when it approaches the Venus, assuming that the emissivity of the surface does not vary with spaceship temperature?

Answer

$$t_2 = (1.18t_1 + 48),\ ^\circ\mathrm{C}.$$

Solution

The surface temperatures of the spaceship near the Earth and Venus are determined from the equations:

$$\varepsilon C_0 F \left(\frac{T_1}{100}\right)^4 = A E_{in1} F_N;$$

$$\varepsilon C_0 F \left(\frac{T_2}{100}\right)^4 = A E_{in2} F_N;$$

whence

$$\left(\frac{T_2}{T_1}\right)^4 = \frac{E_{in2}}{E_{in1}} = \left(\frac{l_1}{l_2}\right)^2.$$

Consequently,

$$\frac{T_2}{T_1} = \sqrt{\frac{149.5}{108.1}} = 1.18;$$

$$t_2 + 273 = (t_1 + 273)\,1.18;$$

$$t_2 = (1.18t_1 + 48),\ ^\circ\mathrm{C}.$$

10-17. The setting of a boiler furnace is laid from fireclay (chamotte) brick and the outside lagging—from plate steel. The distance between the lagging and setting brick is equal to 30 mm and it may be assumed small, compared with the size of the furnace.

Under steady-state conditions calculate the loss of heat to surroundings by radiation from unit surface area per unit time between the lagging and setting. The temperature of the outer side of the brick setting $t_1 = 127$ °C and the temperature of the steel lagging $t_2 = 50$ °C. The emissivity of fireclay $\varepsilon_f = 0.8$ and of plate steel $\varepsilon_s = 0.6$.

Answer

$$E_{r1} = q_{1,2} \doteq 435 \text{ W/m}^2.$$

Solution

The lagging and setting can be considered as two infinite flat parallel surfaces separated by a transparent participating medium. For a system of this kind net radiation is calculated by formula

$$E_{r1} = q_{1,2} = \varepsilon_{red} C_0 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right], \qquad (10\text{-}1)$$

where the reduced emissivity

$$\varepsilon_{red} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.6} - 1} = 0.522.$$

Then,

$$E_{r1} = 0.522 \times 5.67 \left[\left(\frac{127+237}{100} \right)^4 - \left(\frac{50+237}{100} \right)^4 \right] = \doteq 435 \text{ W/m}^2.$$

10-18. Calculate the magnitudes of the emitted, outgoing, reflected and incident radiation fluxes for the surfaces of the firebrick setting and steel lagging, assuming the conditions of problem 10-17.

Answer

$E_{em1} = 1161$ W/m², $E_{em2} = 370$ W/m²;

$E_{out1} = 1342$ W/m², $E_{out2} = 907$ W/m²;

$E_{ref1} = 181$ W/m², $E_{ref2} = 537$ W/m²;

$E_{in1} = 907$ W/m², $E_{in2} = 1342$ W/m².

Solution

The emitted radiation is calculated with the aid of Stefan-Boltzmann's law:

$$E_{em} = \varepsilon C_0 \left(\frac{T}{100} \right)^4.$$

For the wall laid from fireclay brick

$$E_{em1} = 0.8 \times 5.67 \left(\frac{127+237}{100} \right)^4 = 1161 \text{ W/m}^2,$$

and for the steel lagging

$$E_{em2}=0.6\times 5.67\left(\frac{50+273}{100}\right)^4=370\ \text{W/m}^2.$$

The outgoing radiation, or the radiation leaving the surface,

$$E_{out}=\frac{1}{A}\left[E_{em}-(1-A)\,E_r\right].$$

For a steady-state process $E_{r1}=-E_{r2}$. From the solution of problem 10-17 we know that $E_{r1}=435$ W/m². Then,

$$E_{out1}=\frac{E_{em1}}{\varepsilon_1}-\left(\frac{1}{\varepsilon_1}-1\right)E_{r1}=$$

$$=\frac{1161}{0.8}-\left(\frac{1}{0.8}-1\right)435=1340\ \text{W/m}^2;$$

$$E_{out2}=\frac{E_{em2}}{\varepsilon_2}+\left(\frac{1}{\varepsilon_2}-1\right)E_{r1}=$$

$$=\frac{370}{0.6}+\left(\frac{1}{0.6}-1\right)435=907\ \text{W/m}^2.$$

For the system considered

$$E_{in1}=E_{out2}=907\ \text{W/m}^2;$$

$$E_{in2}=E_{out1}=1342\ \text{W/m}^2.$$

The reflected radiation

$$E_{ref}=(1-A)\,E_{in}.$$

Substituting in the above formula corresponding values, we get:

$$E_{ref1}=(1-A_1)\,E_{in1}=(1-0.8)\,907=181\ \text{W/m}^2;$$

$$E_{ref2}=(1-A_2)\,E_{in2}=(1-0.6)\,1340=537\ \text{W/m}^2.$$

10-19. How shall change the loss of heat by radiation to surroundings, g_{rad1}, W/m², and the outgoing radiant flux, E_{out1}, W/m², if the brick setting of the furnace and its lagging, considered in problem 10-17, are separated by a steel screen having an emissivity $\varepsilon_{sc}=0.6$?

Answer

$$E_{r1}=q_{rad1}=196\ \text{W/m}^2.$$

$$E_{out1}=1400\ \text{W/m}^2.$$

Solution

In the presence of n screens, intended to diminish the intensity of thermal radiation, the heat flux

$$q_{rad}=\frac{C_0\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{A_1}+2\sum\limits_{i=1}^{n}\frac{1}{A_{sci}}-\frac{1}{A_2}-(n+1)},\ \mathrm{W/m^2}. \qquad (10\text{-}2)$$

In the presence of one screen ($n = 1$)

$$q_{rad}=\frac{C_0\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{A_1}+\frac{1}{A_2}+2\left(\frac{1}{A_{sc}}-1\right)},\ \mathrm{W/m^2}.$$

Substituting into the above formula the data based on conditions of the problem, and assuming $A = \varepsilon$, we obtain

$$q_{rad}=\frac{5.67\left(\frac{127+273}{100}\right)^4-\left(\frac{50+273}{100}\right)^4}{\frac{1}{0.8}+\frac{1}{0.6}+2\left(\frac{1}{0.6}-1\right)}=196\ \mathrm{W/m^2}.$$

The outgoing radiation can be found from the condition $q_{rad1} = E_{r1}$ (as given in the problem),

$$E_{out1}=\frac{1}{A_1}[E_{em}-(1-A_1)E_{r1}]=$$

$$=\frac{1161}{0{,}8}-\left(\frac{1}{0.8}-1\right)196=1400\ \mathrm{W/m^2}.$$

10-20. What should be the emissivity of the screen so that the loss of heat to surroundings due to radiation would not exceed 60 W/m² in the presence of a protective screen between the furnace setting and steel lagging? All other conditions remain as in problem 10-17.

Answer

$$\varepsilon_{sc} = 0.143.$$

10-21. A steel ingot is heated in an electric muffle furnace the walls of which are at a temperature $t_2 = 1000$ °C. The emissivity of ingot surfaces $\varepsilon_1 = 0.8$ (average for the heating period), and the emissivity of the fireclay-brick wall of the muffle furnace $\varepsilon_2 = 0.8$. The surface of the furnace participating in radiation heat exchange, F_2, is considerably larger than the surface of the heated ingot, F_1, i.e. $F_1 \ll F_2$.

Calculate the magnitude of the radiant flux, depending on ingot temperature in the course of heating, and plot the radiant flux as a function of ingot temperature.

Calculations should be carried out for the following temperatures:

$$t_1 = 20,\ 100,\ 300,\ 500 \text{ and } 700\ ^\circ\text{C}.$$

Answer

t_1, °C	20	100	300	500	700
q_{rad}, kW/m²	118.8	118.2	114.2	102.9	78.5

The curve representing the heat flux as a function of temperature, $q_{rad} = f(t_1)$, is shown in Fig. 10-1.

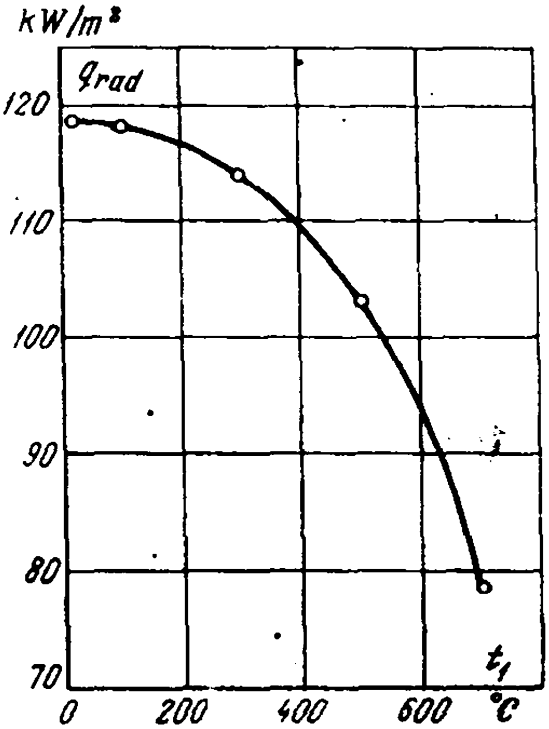

Fig. 10-1. Solution to problem 10-21

Solution

For an enclosure built-up of two bodies, one of which (with a surface F_1) is enveloped by the other (with a surface F_2), the heat flux for the first body is determined by formula

$$q_{rad}=\frac{C_0\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{\varepsilon_1}+\frac{F_1}{F_2}\left(\frac{1}{\varepsilon_2}-1\right)},\ \mathrm{W/m^2}. \qquad (10\text{-}3)$$

For the case considered, when $F_1/F_2 = 0$, we obtain:

$$q_{rad}=\varepsilon_1 C_0\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right],\ \mathrm{W/m^2}. \qquad (10\text{-}4)$$

Substituting the values of ε_1, C_0 and T_1 into formula (10-4), we obtain the expression for the absolute value of the density of thermal radiation in the following form:

$$q_{rad}=0.8\times 5.67\left[\left(\frac{1273}{100}\right)^4-\left(\frac{T_1}{100}\right)^4\right]=$$

$$=4.536\left[26\,260-\left(\frac{T_1}{100}\right)^4\right],\ \mathrm{W/m^2}.$$

Other calculation results are given in the answer to this problem.

10-22. Under the conditions in problem 10-21 calculate the intensity of radiation from the walls of a muffle furnace incident upon the surfaces of a steel ingot, if the ratio of the surfaces, participating in radiation heat exchange, $F_1/F_2 = 1/5$.

Answer

t_1, °C	20	100	300	500	700
q_{rad}, kW/m²	114.2	113.6	109.8	98.9	75.5

10-23. The emissivity of a tungsten wire, determined at a temperature of 2000 °C, $\varepsilon = 0.3$.

Determine the magnitude of the coefficient of radiation heat transfer on the surface of the wire at $t = 2000$ °C, if the surface of the enclosure was at a temperature of 20 °C. The surface of the wire is small, compared with that of the enclosure.

Answer

$$\alpha_{rad} = 230\ \mathrm{W/m^2\cdot {}^\circ C}.$$

10-24. The surfaces of a double-walled cylindrical vessel, used for storing liquid oxygen, are covered with a layer of silver, the absorption coefficient of which $A_1 = A_2 = 0.02$. The temperature of the outside surface of the inside wall $t_1 = -183$ °C and the temperature of the inside surface of the outside wall $t_2 = 20$ °C. The distance between the walls is small and surface F_1 can be considered equal to surface F_2.

Calculate the rate of radiation heat transfer through the walls into the vessel, if the heat-transfer surface $F = 0.157$ m^2.

Answer

$$Q_{rad} = 0.66 \text{ W}.$$

10-25. The surface temperature of the outlet header of a high-pressure steam superheater $t_w = 500$ °C.

Calculate the loss of heat by radiation from the length of 1 m of the non-insulated header, if its outside diameter $d = 275$ mm, absorption coefficient $A_w = 0.8$, and enclosure temperature $t_2 = 30$ °C.

Answer

$$q_{radl} = 13.7 \text{ kW/m}.$$

10-26. Calculate the loss of heat from unit length of the header considered in problem 10-25, on condition that its surface is enveloped with a steel screen having a diameter $d_{sc} = 325$ mm and an absorption coefficient $A_{sc} = 0.7$. Heat is being transferred between the surface of the screen and external enclosure both by thermal radiation and free convection. The transfer of heat by convection and conduction from the surface of the header to that of the screen can be ignored.

On the surface of the screen the coefficient of heat transfer by convection $\alpha = 29$ W/m$^2\cdot$°C. Compare the results obtained with the answer to problem 10-25.

Answer

$$q_{radl}^{sc} = 8.75 \text{ kW/m}; \quad q_{radl}/q_{radl}^{sc} = 1.565.$$

Solution

The temperature of the screen is found from the energy balance equation

$$\frac{\pi d C_0\left[\left(\frac{T_w}{100}\right)^4-\left(\frac{T_{sc}}{100}\right)^4\right]}{\frac{1}{A_w}+\frac{F_w}{F_{sc}}\left(\frac{1}{A_{sc}}-1\right)}=$$

$$=\pi d_{sc}A_{sc}C_0\left[\left(\frac{T_{sc}}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]+\pi d_{sc}\alpha\,(t_{sc}-t_2).$$

The reduced absorption coefficient for the system "header screen"

$$A_{red}=\frac{1}{\frac{1}{A_w}+\frac{F_w}{F_{sc}}\left(\frac{1}{A_{sc}}-1\right)}=$$

$$=\frac{1}{\frac{1}{0.8}+\frac{0.275}{0.325}\left(\frac{1}{0.7}-1\right)}=0.62.$$

Substituting the numerical values of the known quantities into the energy balance equation, we obtain:

$$0.275\times0.62\times5.67\left[\left(\frac{773}{100}\right)^4-\left(\frac{T_{sc}}{100}\right)^4\right]=$$

$$=0.325\times0.7\times5.67\times$$

$$\times\left[\left(\frac{T_{sc}}{100}\right)^4-\left(\frac{303}{100}\right)^4\right]+0.325\times29\,(t_{sc}-30),$$

whence

$$3842.4=2.26\left(\frac{T_{sc}}{100}\right)^4+9.42t_{sc}.$$

It is most convenient to solve equations of this kind graphically. Denote $3842.4-9.42t_{sc}=Y_1$ and

$$2.26\left(\frac{T_{sc}}{100}\right)^4=Y_2.$$

Calculate the screen temperatures $t_{sc}=100$, 200, 300 °C. Calculation results are tabulated below:

Y	t_{sc}, °C		
	100	200	300
$Y_1 = 3842.4 - 9.42 t_{sc}$	2905.6	1963	1020
$Y_2 = 2.26 (T_{sc}/100)^4$	437	1129	2436

and represented on the chart in Fig. 10-2. From the graph we find that t_{sc} = 240 °C.

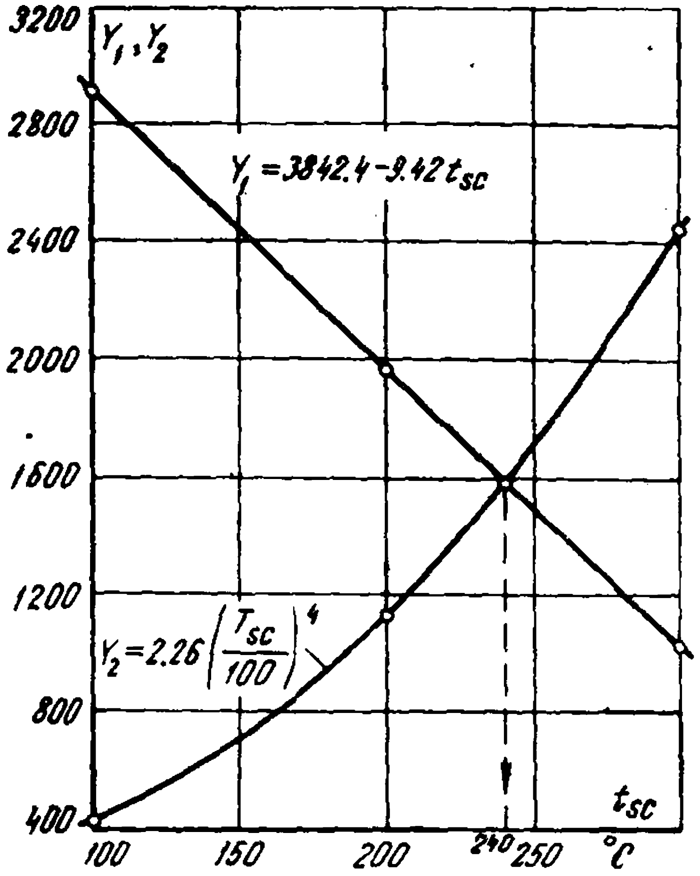

Fig. 10-2. Solution to problem 10-26

The radiant heat flux from unit length of the header

$$q^{sc}_{radl} = \pi d A_{red} C_0 \left[\left(\frac{T_w}{100}\right)^4 - \left(\frac{T_{sc}}{100}\right)^4 \right] =$$

$$= 3.14 \times 0.275 \times 0.62 \times 5.67 \times$$

$$\times \left[\left(\frac{773}{100}\right)^4 - \left(\frac{513}{100}\right)^4 \right] = 8.75 \text{ kW/m}.$$

Compare the loss of heat by radiation from a bared header and from a header surrounded by a screen:

$$\frac{q_{radl}}{q^{sc}_{radl}}=\frac{13.7}{8.75}\approx 1.565.$$

The screen reduced heat losses more than 1.5 times.

10-27. How shall change the loss of heat from length of 1 m of the header, due to radiation, if, assuming the conditions of problem 10-26, the steel screen is replaced with a screen made from aluminum foil of the same diameter and with an absorption coefficient A_{sc} = 0.05? All other conditions are left as in problem 10-26. Compare the results obtained with the loss of heat from a bared superheater header (problem 10-25).

Answer

$$q^{sc}_{radl}=0.975 \text{ kW/m}; \quad q_{radl}/q^{sc}_{radl}=14.1.$$

10-28. A steam main having an outside diameter d = 200 mm runs in a large room in which the air temperature t_f = 30 °C. The surface temperature of the steam main t_{w1} = 400 °C. Determine the loss of heat to surroundings from length of 1 m of the steam main both due to thermal radiation and convection.

The emissivity of the steam main surface ε = 0.8. The temperature of room walls may be assumed equal to air temperature, i.e., t_{w2} = 30 °C. Also, find the losses of heat, by radiation and convection, assuming a steam main surface temperature t_{w1} = 200 °C.

Answer

At $t_{w1}=400$ °C $q_l^{rad}=5600$ W/m, $q_l^c=1970$ W/m, $q_l^{rad}/q_l^c=2.84$.

At $t_{w1}=200$ °C $q_l^{rad}=1185$ W/m, $q_l^c=750$ W/m, $q_l^{rad}/q_l^c=1.58$.

Solution

The loss of heat by thermal radiation

$$q_l^{rad}=\varepsilon C_0\pi d\left[\left(\frac{T_{w1}}{100}\right)^4-\left(\frac{T_{w2}}{100}\right)^4\right].$$

With $t_{w1} = 400\ ^\circ\mathrm{C}$

$$q_l^{rad} = 0.8 \times 5.67 \times 3.14 \times 0.2\,(6.73^4 - 3.03^4) = \\ = 5.6 \times 10^3\ \mathrm{W/m}.$$

With $t_{w1} = 200\ ^\circ\mathrm{C}$

$$q_l^{rad} = 2.85\,(4.73^4 - 3.03^4) = 1.185 \times 10^3\ \mathrm{W/m}.$$

To determine the coefficient of heat transfer by convection, use formula (7-1) derived for a horizontal tube:

$$\mathrm{Nu}_f = 0.50\,(\mathrm{GrPr})_f^{1/4}.$$

In the first case

$$(\mathrm{GrPr})_f = \frac{g\beta_f \Delta t\, d^3}{\nu_f^2}\,\mathrm{Pr}_f = \frac{9.81 \times 370 \times 8 \times 10^{-3}}{303 \times 2.56 \times 10^{-10}} = 2.62 \times 10^8.$$

In the second case

$$(\mathrm{GrPr})_f = 2.62 \times 10^8\,\frac{170}{370} = 1.2 \times 10^8.$$

The Nusselt numbers and the heat-transfer coefficients

$$\mathrm{Nu}_f = 0.50\,(2.62 \times 10^8)^{1/4} = 63.5;$$

$$\mathrm{Nu}_{f2} = 0.50\,(1.2 \times 10^8)^{1/4} = 52.3;$$

$$\alpha_1 = \mathrm{Nu}_{f1}\,\frac{\lambda_f}{d} = 63.5\,\frac{2.68 \times 10^{-2}}{0.2} = 8.5\ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$\alpha_2 = 7.0\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The heat lost by convection:
with $t_{w1} = 400\ ^\circ\mathrm{C}$

$$q_l^c = \alpha_1 \pi\,(t_w - t_f) = 8.5 \times 3.14 \times 0.2 \times 370 = 1970\ \mathrm{W/m};$$

with $t_{w1} = 200\ ^\circ\mathrm{C}$

$$q_l^c = 7.0 \times 3.14 \times 0.2 \times 170 = 750\ \mathrm{W/m}.$$

10-29. In a room the temperature of air is measured with the aid of a mercury-in-glass thermometer, indicating 27 °C. The temperature of the walls in the room is 25 °C.

Evaluate the reading error for the thermometer, which is due to radiation heat exchange between the thermometer and walls of the room, and determine the actual air temperature, assuming a glass emissivity equal to 0.94, and the coefficient of heat transfer from the air to the surface of the thermometer $\alpha = 5$ W/m$^2\cdot$°C.

Answer

The error amounts to 3 °C; $t_f \doteq 20$ °C.

10-30. The temperatures of two plates, held in an evacuated space, are equal to 127 and 327 °C. The plates have the same emissivity, equal to 0.8. A screen with an emissivity of 0.05 is placed between the two plates arranged parallel to each other.

Calculate the heat flux passing through unit area of the screen, and also the temperature of the screen.

Answer

$$q = 146 \text{ W/m}^2; \quad t \doteq 254 \text{ °C}.$$

10-31. Find the number of screens to be placed between two flat parallel surfaces, so that the net radiation from one surface to the other would be reduced 79 times.

Assume that the temperatures of the surfaces do not change after the screens are placed between the plates.

The emissivity of the screens is 0.05 and that of the surfaces—0.8.

Answer

3 screens.

10-32. To diminish the loss of heat to surroundings by radiation, a heating furnace was surrounded with a steel screen. The dimensions of the furnace are large compared with the distance from its external surface to the screen.

Measurements have shown the temperature of the external surface of the furnace brickwork to be 107 °C and that of the steel screen—57 °C.

Find the density of net radiation from the surface of the furnace brickwork to the screen, assuming that the emissivities of the brickwork and screen are equal respectively to 0.85 and 0.75.

Answer

$$q_{rad} = 342 \text{ W/m}^2.$$

10-33. What should be the emissivity of the protective screen of a superheater header, to ensure a loss of heat from the surface of that superheater header by radiation not exceeding 580 W/m^2 and a screen surface temperature not higher than 70 °C? The diameter of the protective screen is 325 mm, the coefficient of heat transfer from the external surface of the screen by convection $\alpha = 11.6$ W/m^2·°C, and the temperature of surroundings and enclosures $t_2 = 30$ °C.

Answer

$$\varepsilon_{sc} = 0.34.$$

10-34. In a pilot plant used to determine the emissivity of various substances 20 W of electric power is consumed to maintain a tungsten wire, having a diameter $d = 3$ mm and length $l = 200$ mm, at a constant temperature $t_1 = 800$ °C. The surface of the vacuum (evacuated) chamber, accommodating the wire, is great compared with that of the wire. During the tests the walls of the vacuum chamber are kept at a constant temperature $t_2 = 20$ °C.

Determine the emissivity of the tungsten wire at a temperature of 800 °C.

Answer

$$\varepsilon_1 = 0.132.$$

10-35. Calculate the emissivity of the tungsten wire at temperatures 1000, 1500 and 2000 °C, if the amounts of power consumed to maintain the temperatures indicated were equal to respectively 45, 234 and 834 W. All other conditions are as in problem 10-34.

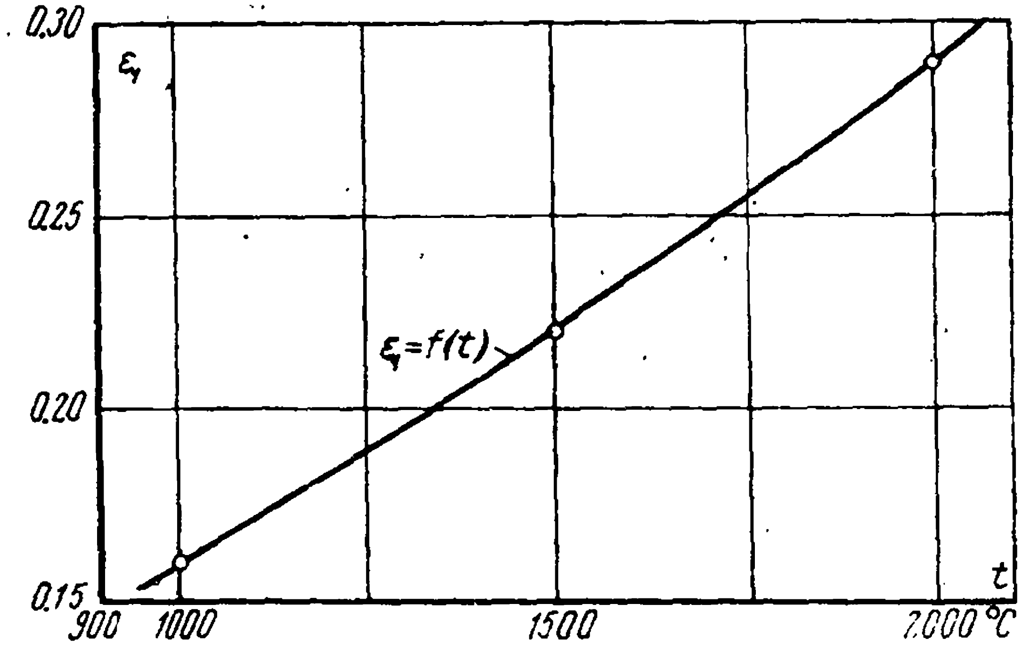

Fig. 10-3. Solution to problem 10-35

Plot emissivity as a function of temperature.

Answer

$$\varepsilon_1' = 0.16; \quad \varepsilon_1'' = 0.22; \quad \varepsilon_1''' = 0.29.$$

The dependence $\varepsilon_1 = f(t)$ is presented in Fig. 10-3.

10-36. In a channel carrying a hot gas the temperature of the gas is measured with the aid of a thermocouple (Fig. 10-4). Under steady-state conditions the thermocouple indicates a temperature $t_1 = 300$ °C and the wall temperature $t_2 = 200$ °C.

Calculate the error admitted in measuring the gas temperature, due to radiation heat transfer between the bead of the thermocouple and the wall of the channel, and also determine the real temperature of the gas. Assume for the thermocouple bead an emissivity $\varepsilon_1 = 0.8$, and a coefficient of heat transfer from the gas to the surface of the bead $\alpha = 58$ W/m²·°C.

Answer

The error amounts to 45.5 °C; $t_f = 345.5$ °C.

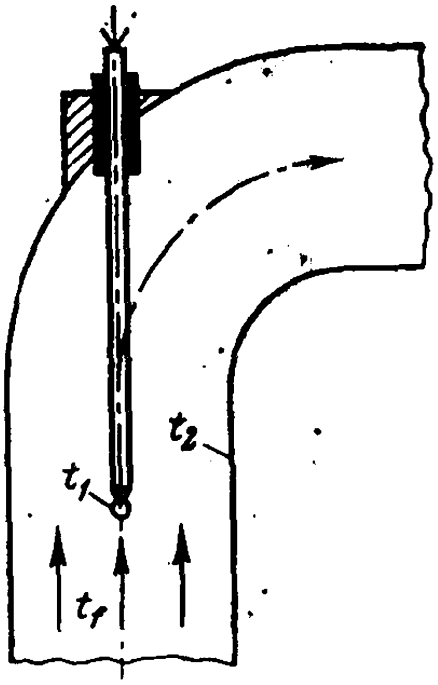

Fig. 10-4. Solution to problem 10-36

Solution

Let us compose a heat balance equation for the thermocouple bead.

The thermocouple loses heat by radiation

$$Q = F_1\varepsilon_1 C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

and heat is transferred to it by convection, in the amount

$$Q = F_1\alpha(t_f - t_1).$$

Under steady-state conditions

$$\alpha(t_f - t_1) = \varepsilon_1 C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right].$$

Substituting into the above equation the known quantities, we obtain:

$$t_f - t_1 = \frac{0.8\times 5.67}{58}\left[\left(\frac{300+273}{100}\right)^4 - \left(\frac{200+273}{100}\right)^4\right] = 45.5°\text{C}.$$

The actual temperature of the gas

$$t_f = 300 + 45.5 = 345.5\ °\text{C}.$$

10-37. What shall be the error admitted in measuring the temperature of a moving gas, if the temperature of the inside wall of the gas conduit is reduced to $t_2 = 250$ °C by insulating thoroughly the outside surfaces of the conduit? All other conditions are as in problem 10-36.

Answer

The error is equal to 23.5 °C; $t_f = 323.5$ °C.

10-38. In a gas conduit having a diameter $D = 500$ mm the temperature of a hot gas was measured with the aid of a resistance thermometer of a diameter $d_1 = 5$ mm surrounded with a cylindrical screen having a diameter $d_2 = 10$ mm (Fig. 10-5). The resistance thermometer indicates a temperature $t_1 = 300$ °C, the temperature of the inside surface of the gas conduit is $t_2 = 200$ °C, and the emissivity of the surface of the thermometer and screen $\varepsilon_1 = \varepsilon_{sc} = 0.8$.

Calculate the measurement error and the actual temperature of the gas, if the coefficient of heat transfer to the surface of the resistance thermometer and to the surface of the screen $\alpha = 58$ W/m²·°C. Compare the obtained results with the answer to problem 10-36.

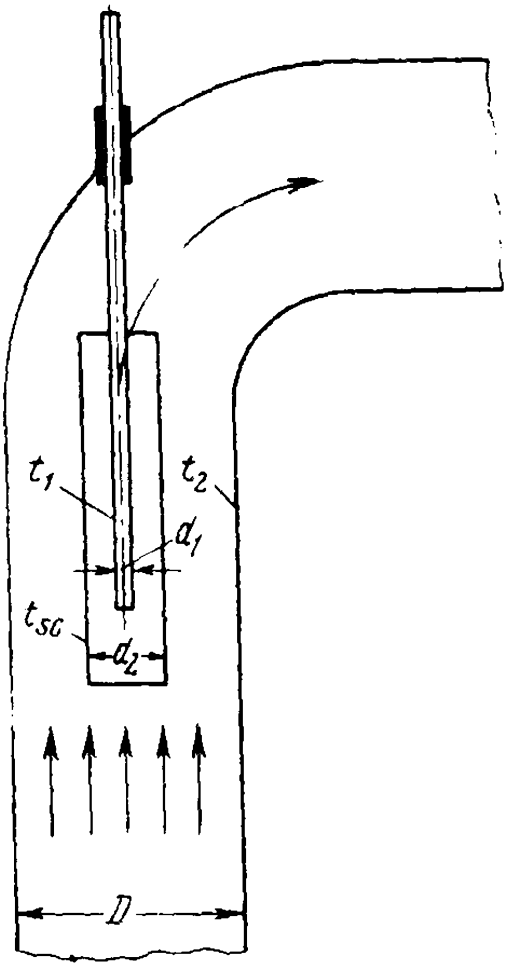

Fig. 10-5. Solution to problem 10-38

Answer

$t_f = 309$ °C; the measurement error diminished by about 5 times and amounts to 9 °C.

Solution

Let us compose the heat balance equation:

(a) for the resistance thermometer

$$\alpha \pi d_1 (t_f - t_1) = \frac{\pi d_1 C_0 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_{sc}}{100} \right)^4 \right]}{\frac{1}{\varepsilon_1} + \frac{d_1}{d_2} \left(\frac{1}{\varepsilon_{sc}} - 1 \right)}; \tag{a}$$

(b) for the screen

$$\alpha \pi d_1 (t_f - t_1) + 2\alpha \pi d_2 (t_f - t_{sc}) = \\ = \pi d_2 \varepsilon_{sc} C_0 \left[\left(\frac{T_{sc}}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]. \tag{b}$$

In equation (b) account is taken of the fact that the surface of the screen is small compared with the surface of the gas conduit surrounding it, since $d_2 \ll D$.

From equations (a) and (b) we find that

$$t_f = \frac{C_0}{\alpha} \frac{\left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_{sc}}{100} \right)^4 \right]}{\frac{1}{\varepsilon_1} + \frac{d_1}{d_2} \left(\frac{1}{\varepsilon_{sc}} - 1 \right)} + t_1$$

and

$$t_f = \frac{2\alpha d_2 t_{sc} + \alpha d_1 t_1 + d_2 \varepsilon_{sc} C_0 \left(\frac{T_{sc}}{100} \right)^4 - d_2 \varepsilon_{sc} C_0 \left(\frac{T_2}{100} \right)^4}{2 d_2 \alpha + \alpha d_1}.$$

Substituting the known quantities into the above equation, we get:

$$t_f = 377 - 0.071 \left(\frac{T_{sc}}{100} \right)^4$$

and

$$t_f = 44.3 + 0.0314 \left(\frac{T_{sc}}{100} \right)^4 + 0.8 t_{sc}.$$

The values of $t_f = f_1 (t_{sc})$ and $t_f = f_2 (t_{sc})$, calculated for various screen temperatures, are represented in Fig. 10-6

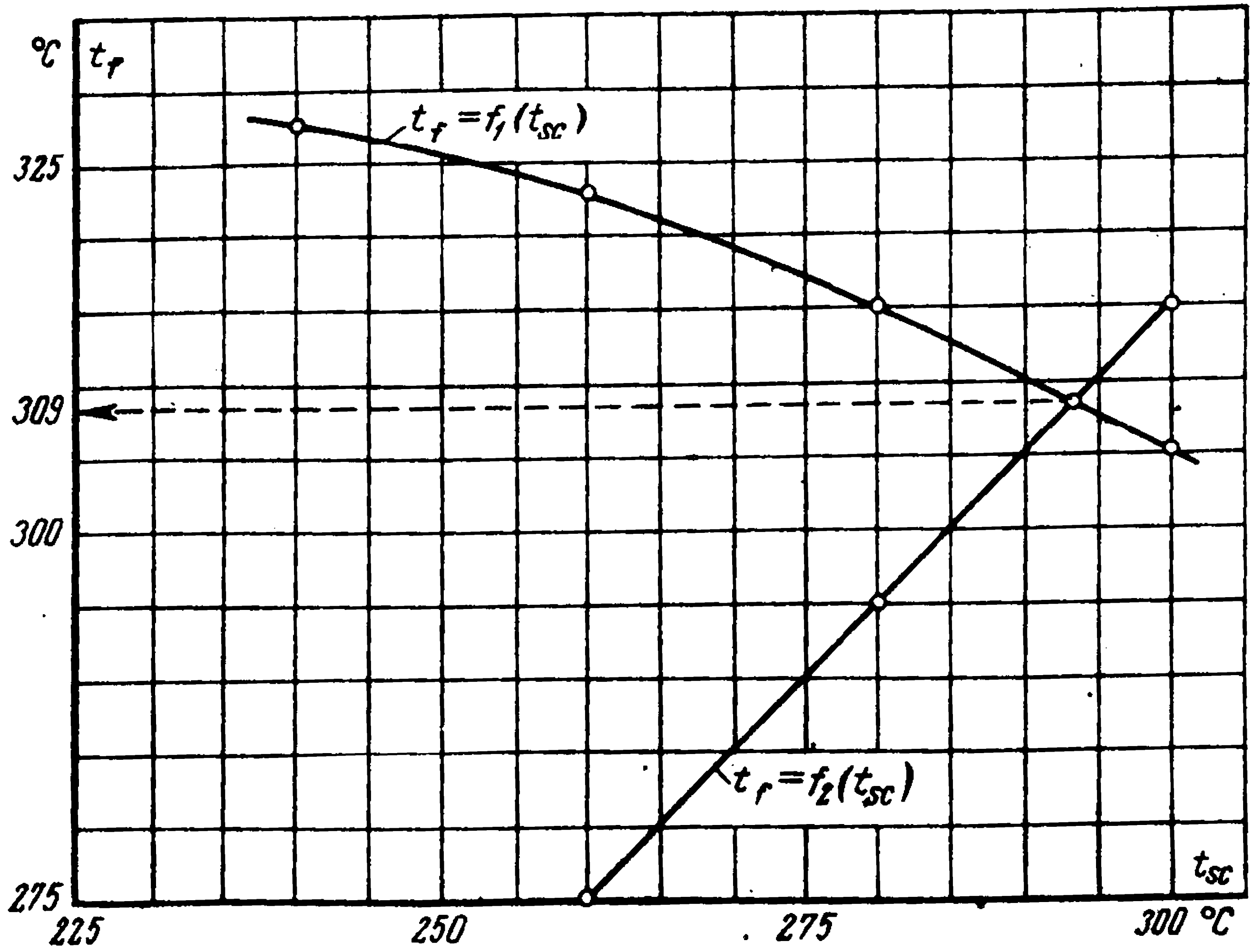

Fig. 10-6. Solution to problem 10-38

and tabulated below:

t_f	t_{sc}, °C			
	240	260	280	300
$t_f = f_1(t_{sc})$	327.6	323.6	315.1	305.8
$t_f = f_2(t_{sc})$	258.0	275.8	295.6	315.4

From the chart in Fig. (10-6) we find that the actual temperature of the gas $t_f = 309$ °C.

The error admitted in measuring the temperature of the gas is equal to $309 - 300 = 9$ °C. Comparing this result with the answer to problem 10-36, we see that under the same conditions, but in the presence of one screen the gas temperature measurement error diminished 5 times.

10-39. Calculate the view (configuration) factor and the rate of heat flow by radiation between two parallel strips the distance between which $h = 3$ m. The strips are of an equal width, $a_1 = a_2 = 2$ m, and their length is great compared with the width (Fig. 10-7). The emissivity of the strips $\varepsilon_1 = \varepsilon_2 = 0.8$ and the temperatures of their surface $t_1 = 500$ °C and $t_2 = 200$ °C.

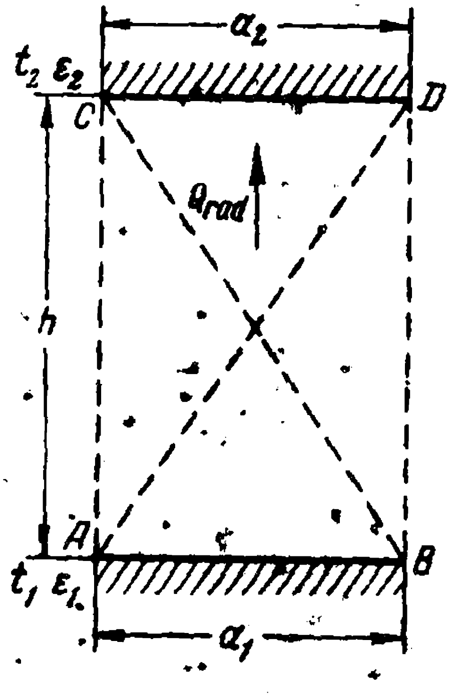

Fig. 10-7. Solution to problem 10-39

Answer

$\bar{\varphi}_{1,2} = \bar{\varphi}_{2,1} = 0.3$; $Q_{rad} = 9050$ W/m.

Solution

For two parallel strips of equal width the view factors $\bar{\varphi}_{1,2} = \bar{\varphi}_{2,1}$ are determined from equation [4]

$$\bar{\varphi}_{1,2} = \sqrt{1 + \left(\frac{h}{a}\right)^2} - \frac{h}{a}.$$

Substituting the numerical values of the quantities, we get:

$$\bar{\varphi}_{1,2}=\sqrt{1+\left(\frac{3}{2}\right)^2}-\frac{3}{2}=0.3.$$

The rate of heat flow, reduced to unit length of the strip, is determined from equation [4]:

$$Q_{rad}=\varepsilon_{red}C_0\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]\bar{H},$$

where

$$\varepsilon_{red}=\frac{1}{1+\bar{\varphi}_{1,2}\left(\frac{1}{\varepsilon_1}-1\right)+\bar{\varphi}_{2,1}\left(\frac{1}{\varepsilon_2}-1\right)}=$$

$$=\frac{1}{1+2\times0.3\left(\frac{1}{0,8}-1\right)}=0.87;$$

H is the surface-surface direct exchange area per 1 m length of a strip;

$$\bar{H}_1=\bar{H}_2=\bar{\varphi}_{1,2}F_1=0.3\times2\times1=0.6\ \text{m}^2/\text{m}.$$

Then

$$Q_{rad}=0.87\times5.67\left[\left(\frac{500+273}{100}\right)^4-\right.$$

$$\left.-\left(\frac{200+273}{100}\right)^4\right]0.6=9050\ \text{W/m}.$$

10-40. How shall change the view factor, the reduced emissivity and the rate of heat flow, if the strips, considered in problem 10-39, are arranged with distances between them h_1 = 1.5 m and h_2 = 6 m?

Plot a graph presenting the quantities $\bar{\varphi}_{1,2}$, ε_{red} and Q_{rad} as functions of the distance between the strips.

Answer

With $h_1=1.5$ m $\bar{\varphi}_{1,2}=0.5$ m, $\varepsilon_{red}=0.8$, $Q_{rad}=13.9$ kW/m.

with $h_2=6$ m $\bar{\varphi}_{1,2}=0.16$, $\varepsilon_{red}=0.926$, $Q_{rad}=5.15$ kW/m.

The dependence of corresponding quantities on h is represented graphically in Fig. 10-8.

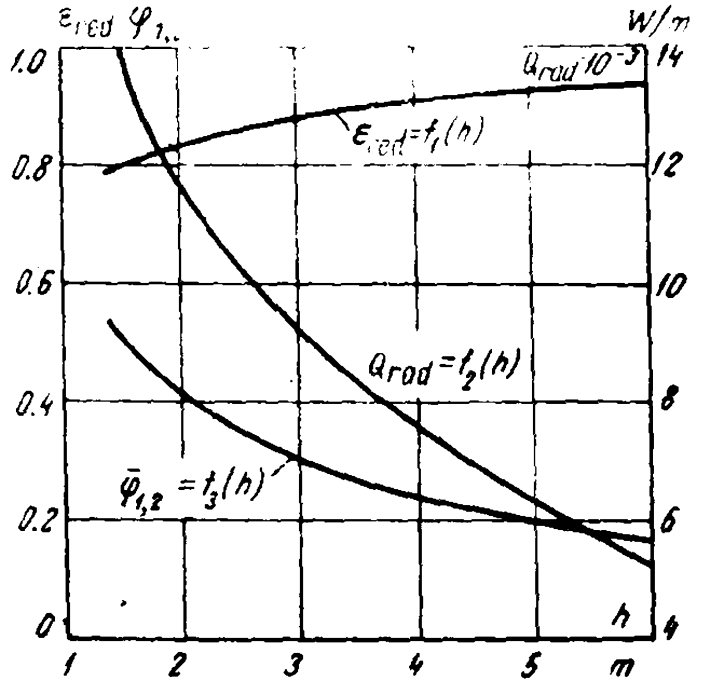

Fig. 10-8. Solution to problem 10-40

10-41. Determine the view factors and the surface-surface direct exchange area for the walls of a channel the cross section of which is an equilateral triangle with sides $a = b = c = 2$ m.

Answer

$$\bar{\varphi}_{1,2} = \bar{\varphi}_{2,1} = \bar{\varphi}_{2,3} = \bar{\varphi}_{3,2} = \bar{\varphi}_{1,3} = \bar{\varphi}_{3,1} = \frac{1}{2};$$

$$\overline{H}_{1,2} = \overline{H}_{2,1} = \overline{H}_{2,3} = \overline{H}_{3,2} = \overline{H}_{1,3} = \overline{H}_{3,1} = 1 \text{ m}^2/\text{m}.$$

10-42. Determine the view factors and the surface-surface direct exchange area for the walls of a channel the cross section of which is an isosceles triangle with sides:

$$\text{(a)} \quad b = c = 2a$$

$$\text{(b)} \quad b = c = 3a.$$

In both cases $a = 2$ m.

Answer

(a)

$$\bar{\varphi}_{ab} = \bar{\varphi}_{ac} = \frac{1}{2};$$

$$\bar{\varphi}_{ba} = \bar{\varphi}_{ca} = \frac{1}{4};$$

$$\bar{\varphi}_{be} = \bar{\varphi}_{cb} = \frac{3}{4}.$$

$$\bar{H}_{ab} = \bar{H}_{ba} = \bar{H}_{ac} = \bar{H}_{ca} = 1\ \text{m}^2/\text{m};\quad \bar{H}_{bc} = \bar{H}_{cb} = 3\ \text{m}^2/\text{m}.$$

(b) $$\bar{\varphi}_{ab} = \bar{\varphi}_{ac} = \frac{1}{2};\ \bar{\varphi}_{ba} = \bar{\varphi}_{ca} = \frac{1}{6};\ \bar{\varphi}_{bc} = \bar{\varphi}_{cb} = \frac{5}{6};$$

$$\bar{H}_{ab} = \bar{H}_{ba} = \bar{H}_{ac} = \bar{H}_{ca} = 1\ \text{m}^2/\text{m};$$

$$\bar{H}_{bc} = \bar{H}_{cb} = 5\ \text{m}^2/\text{m}.$$

10-43. Calculate the radiant heat flux between two black disks arranged in two parallel planes, opposite each other. The temperature of the first disk $t_1 = 500°\text{C}$ and of the second disk $t_2 = 200°\text{C}$. The disks are of the same size, $d_1 = d_2 = 200$ mm, and they are arranged at a distance $h = 400$ mm from each other.

Answer

$$\bar{\varphi}_{1,2} = \bar{\varphi}_{2,1} = 0.055;\quad Q_{rad} = 30\ \text{W}.$$

Solution

The view factor for the system of bodies concerned is calculated from equation [24]:

$$\bar{\varphi}_{1,2} = 1 + 2\left(\frac{h}{d}\right)^2 - 2\frac{h}{d}\sqrt{1 + \left(\frac{h^2}{d}\right)}.$$

Substituting into the above formula the numerical values of the quantities present in it, we obtain:

$$\bar{\varphi}_{1,2} = 1 + 2\left(\frac{400}{200}\right)^2 - 2\,\frac{400}{200}\sqrt{1 + \left(\frac{400}{200}\right)^2} = 0.055.$$

Since the disks are of the same size, then $\bar{\varphi}_{1,2} = \bar{\varphi}_{2,1}$. The surface-surface direct exchange area is found to be

$$\bar{H}_{1,2} = \varphi_{1,2}F_1 = 0.055 \times 0.785\ (0.2)^2 = 1.73 \times 10^{-3}\ \text{m}^2.$$

The heat transferred by radiation from the first disk to the second

$$Q_{rad} = C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T^2}{100}\right)^4\right]\bar{H}_{1,2} =$$

$$= 5.67\left[\left(\frac{500+273}{100}\right)^4 - \left(\frac{200+273}{100}\right)^4\right]1.73 \times 10^{-3} = 30\ \text{W}.$$

10-44. How shall change the view factors and the radiant heat fluxes between the two disks considered in problem 10-43, if the distance between them decreased 2 and 4 times?

Plot a graph presenting the view factor and heat flux as functions of the distance between the disks.

Answer

(*a*) $\bar{\varphi}_{1,2} = 0.17$; $Q_{rad} = 92.6$ W;

(*b*) $\bar{\varphi}_{1,2} = 0.382$; $Q_{rad} = 208.0$ W.

10-45. Calculate the mean view (configuration) factors $\bar{\varphi}_{1,2}$ and $\bar{\varphi}_{2,1}$ and the net radiation for the case when the diameter of the disk which is at a lower temperature is increased two times and all other conditions remain as in problems 10-43.

Answer

$\bar{\varphi}_{1,2} = 0.194$; $\bar{\varphi}_{2,1} = 0.0484$; $Q_{rad\ 1,2} = 105.9$ W.

Solution

For the case considered, when $d_1 < d_2$, the mean view factor is calculated by the following formula [24]:

$$\bar{\varphi}_{1,2} = \frac{1}{d_1^2}\left[\sqrt{\left(\frac{d_1+d_2}{2}\right)^2 + h^2} - \sqrt{\left(\frac{d_2-d_1}{2}\right)^2 + h^2}\,\right].$$

Substituting into the above formula the numerical values of the quantities, we obtain:

$$\bar{\varphi}_{1,2} = \frac{1}{0.2^2}\left[\sqrt{\left(\frac{0.2+0.4}{2}\right)^2 + 0.4^2} - \sqrt{\left(\frac{0.4-0.2}{2}\right)^2 + 0.4^2}\,\right] = 0.1935.$$

$$\bar{\varphi}_{2,1} = \bar{\varphi}_{1,2}\frac{F_1}{F_2} = 0.1935\frac{d_1^2}{d_2^2} = 0.0484.$$

The surface-surface direct exchange area

$$\bar{H}_{1,2} = \bar{\varphi}_{2,1}F_1 = 0.1935 \times 0.785 \times 0.2^2 = 0.00608 \text{ m}^2.$$

The net radiant flux

$$Q_{rad\ 1,2} = C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]\bar{H}_{1,2} =$$

$$= 5.67\left[\left(\frac{500+273}{100}\right)^4 - \left(\frac{200+273}{100}\right)^4\right]0.00608 = 105.9 \text{ W}.$$

10-46. Two long strips of equal width form in the cross section an angle α (Fig. 10-9).

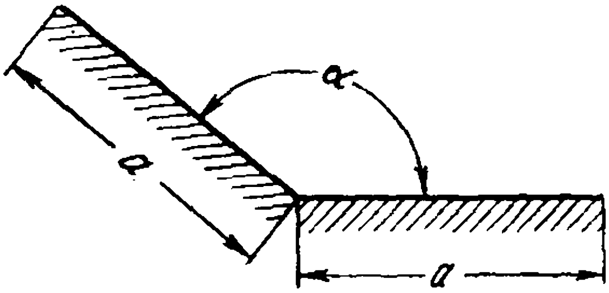

Fig. 10-9. Solution to problem 10-46

Find the view factor between the two strips.

Answer

$$\bar{\varphi}_{1,2} = 1 - \sin\frac{\alpha}{2}.$$

10-47. Two long strips form in the cross section a straight angle (Fig. 10-10). One of the strips is two times wider than the other.

Find the view factor between the smaller and larger strips.

Answer

$$\bar{\varphi}_{1,2} = 0.38.$$

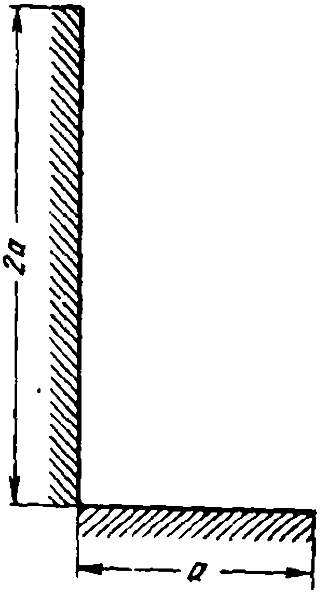

Fig. 10-10. Solution to problem 10-47

10-48. Calculate the view factor between a round disk with a radius r_0 and an elementary area dF_1, the normal to which coincides with the normal to the disk drawn from its centre. The distance between the disk and the elementary area is equal to h. Solve the problem with the aid of the method of projections ratio (unit-sphere method).

Answer

$$\varphi_{1,2} = \frac{r_0^2}{r_0^2 + h^2}.$$

10-49. The walls of a boiler furnace are covered with one row of waterwall tubes having a diameter $d = 100$ mm and arranged with a pitch $s = 120$ mm (Fig. 10-11).

The size of the walls and the length of the waterwall tubes are sufficiently large, and the distance between the wall and tubes is immaterial for radiation heat transfer.

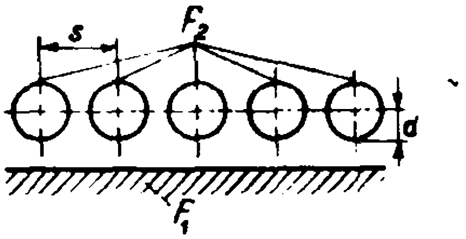

Fig. 10-11. Solution to problem 10-49

Calculate the mean view factors $\bar{\varphi}_{1,2}$ and $\bar{\varphi}_{2,1}$ and the surface-surface direct exchange areas for this system of bodies.

Answer

$$\bar{\varphi}_{1,2} = 0.934; \quad \bar{\varphi}_{2,1} = 0.357;$$
$$\bar{H}_{1,2} = \bar{H}_{2,1} = 0.112 \text{ m}^2/\text{m}.$$

Solution

For the system of bodies indicated the view factor is calculated with the aid of formula [4]

$$\bar{\varphi}_{1,2} = 1 - \sqrt{1 - \left(\frac{d}{s}\right)^2} + \frac{d}{s} \arctan \sqrt{\left(\frac{s}{d}\right)^2 - 1}.$$

Substituting into the above formula the values of the quantities assumed in the problem, we obtain:

$$\bar{\varphi}_{1,2} = 1 - \sqrt{1 - \left(\frac{100}{120}\right)^2} + \frac{100}{120} \arctan \sqrt{\left(\frac{120}{100}\right)^2 - 1} = 0.934.$$

The view factor $\bar{\varphi}_{2,1}$ is found from the reciprocity property:

$$\bar{\varphi}_{2,1} F_2 = \bar{\varphi}_{1,2} F_1,$$

whence

$$\bar{\varphi}_{2,1} = \bar{\varphi}_{1,2} \frac{F_1}{F_2} = 0.934 \frac{s}{\pi d} = 0.357.$$

The surface-surface direct exchange areas

$$\overline{H}_{1,2} = \overline{H}_{2,1} = \overline{\varphi}_{1,2}F_1 = 0.934 \times 0.12 = 0.112 \text{ m}^2/\text{m}.$$

10-50. How shall change the mean view (configuration) factors and the surface-surface direct exchange areas, if the distance between the axes of the waterwall tubes, considered in problem 10-49, is increased 2 and 3 times, leaving all other conditions of the problem unchanged?

Plot a graph presenting the mean view factors and the surface-surface direct exchange areas as functions of the distance between the waterwall tubes, making use, in doing so, of the calculation results for problem 10-49.

Answer

(a) $\overline{\varphi}_{1,2} = 0.566$; $\overline{\varphi}_{2,1} = 0.432$; $\overline{H}_{1,2} = \overline{H}_{2,1} = 0.1358 \text{ m}^2/\text{m}$;

(b) $\overline{\varphi}_{1,2} = 0.401$; $\overline{\varphi}_{2,1} = 0.460$; $\overline{H}_{1,2} = \overline{H}_{2,1} = 0.1445 \text{ m}^2/\text{m}$.

10-51. The walls of a boiler furnace are covered with two rows of waterwall tubes having an outside diameter

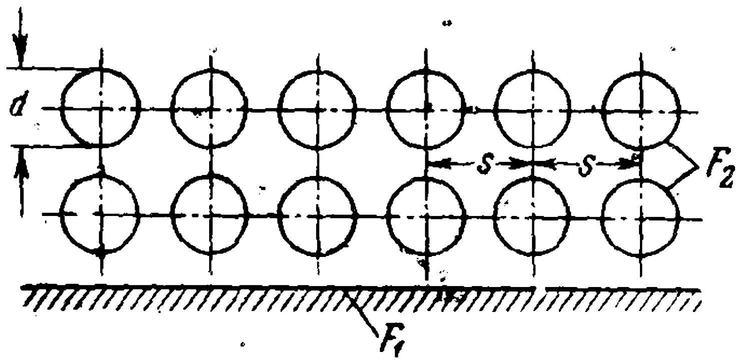

Fig. 10-12. Solution to problem 10-51

$d = 80$ mm. The waterwall tubes are arranged in the two rows with an equal pitch (in the plane parallel to the wall) $s = 400$ mm (Fig. 10-12).

Calculate the mean view factor between the surface of the furnace wall and the waterwall tubes.

Answer

$$\overline{\varphi}'_{1,2} = 0.502.$$

Solution

The view factor between the wall and one row of water-wall tubes is calculated by formula [24]

$$\bar{\varphi}_{1,2}=1-\sqrt{1-\left(\frac{d}{s}\right)^2}+\frac{d}{s}\arctan\sqrt{\left(\frac{s}{d}\right)^2-1}=$$
$$=1-\sqrt{1-\left(\frac{80}{400}\right)^2}+\frac{80}{400}\arctan\sqrt{\left(\frac{400}{80}\right)^2-1}=0.294.$$

For a multirow bank of tubes

$$\bar{\varphi}'_{1,2}=1-(1-\bar{\varphi}_{1,2})^n=1-(1-0.294)^2=0.502,$$

where n is the number of rows in the bank and in our case $n = 2$.

10-52. Calculate the view factors between a flat surface and a bank of tubes, if the number of the rows of tubes, in the direction normal to the surface of the wall, $n = 3, 4, 5$ and 6, and all other conditions are as in problem 10-51.

Plot a graph presenting the view factor $\varphi'_{1,2}$ as a function of the number of rows n, making use of the calculation results in problem 10-51.

Answer

$n = 3$, $\bar{\varphi}'_{1,2} = 0.6485$;
$n = 4$, $\bar{\varphi}'_{1,2} = 0.752$;
$n = 5$, $\bar{\varphi}'_{1,2} = 0.825$;
$n = 6$, $\bar{\varphi}'_{1,2} = 0.8754$.

Chapter Eleven

RADIATION HEAT TRANSFER IN AN ABSORBING MEDIUM

11-1. Determine the attenuation coefficient for a ray passing through a layer of carbon dioxide 30 mm thick, if it is known that after passing through that layer the spectral intensity of this ray diminished by 90 per cent.

Answer

$$\varkappa_\lambda = 76.7\ 1/\mathrm{m}.$$

Solution

In an absorbing medium the ray attenuation factor $\varkappa_\lambda$ can be found from the Bouguer's law:

$$J_{\lambda,x} = J_{\lambda,x=0}\, e^{-\varkappa_\lambda x}, \tag{11-1}$$

whence

$$\varkappa_\lambda = -\frac{1}{x}\ln\frac{J_{\lambda,x}}{J_{\lambda,x=0}}. \tag{11-2}$$

The conditions of the problem give

$$\frac{J_{\lambda,x}}{J_{\lambda,x=0}} = 0.1.$$

Substituting the numerical values of the quantities given in the problem into equation (11-2), we obtain:

$$\varkappa_\lambda = -\frac{1}{3\times 10^{-2}}\,2.3\log 0.1 = 76.6\ 1/\mathrm{m}.$$

11-2. The absorptivity of a layer of gas of thickness l_1 at a partial pressure p_1 is $A_{\lambda 1}$.

Determine the absorptivity of the gas after a simultaneous change of the thickness of the layer and of the partial pressure respectively to l_2 and p_2. Bouguer's law is assumed to be valid for the given gas, and the gas temperature is the same in both cases.

Answer

$$A_{\lambda 2} = 1 - (1 - A_{\lambda 1})^{\frac{p_2 l_2}{p_1 l_1}}.$$

Solution

In accordance with Bouguer's law, the absorptivity of gases at a constant temperature is a function of the quantity pl:

$$A_\lambda = 1 - e^{-k(pl)}.$$

Let us write down the above equality as applied to the conditions of the problem considered:

$$A_{\lambda 1} = 1 - e^{-k(p_1 l_1)};$$

$$A_{\lambda 2} = 1 - e^{-k(p_2 l_2)}.$$

Eliminating k from the equations, we obtain:

$$A_{\lambda 2} = 1 - (1 - A_{\lambda 1})^{p_2 l_2 / p_1 l_1}.$$

11-3. A tube 200 mm in diameter and 1 m long has closed ends, and contains a mixture of dry air and carbon dioxide. The total pressure and temperature of the mixture are equal respectively to 98.1 kPa and 800°C. The partial pressure of the carbon dioxide gas is equal to 9 kPa.

Find the emissivity of the mixture of gases in the tube.

Answer

$$\varepsilon = 0.06.$$

11-4. In a heating furnace the temperature of hot gases is constant over the entire volume and is equal to 1200°C. Furnace volume $V = 12$ m^3 and the total surface of the enclosure $F = 28$ m^2.

The total pressure of the products of combustion $p = 98.1$ kPa, the partial pressure of water vapour $p_{H_2O} = 8$ kPa, and of carbon dioxide $p_{CO_2} = 12$ kPa.

Calculate the emissivity of the emitting gaseous mixture and the radiation flux emitted by the combustion products.

Answer

$$\varepsilon_g = 0.215; \quad E_{em.g} = 57\,400 \text{ W/m}^2.$$

Solution

For the gas layer the mean path of a ray in the furnace is calculated from formula [28]:

$$l = 3.6\frac{V}{F} = 3.6\frac{12}{28} = 1.54 \text{ m}.$$

The product of the partial pressure of carbon dioxide

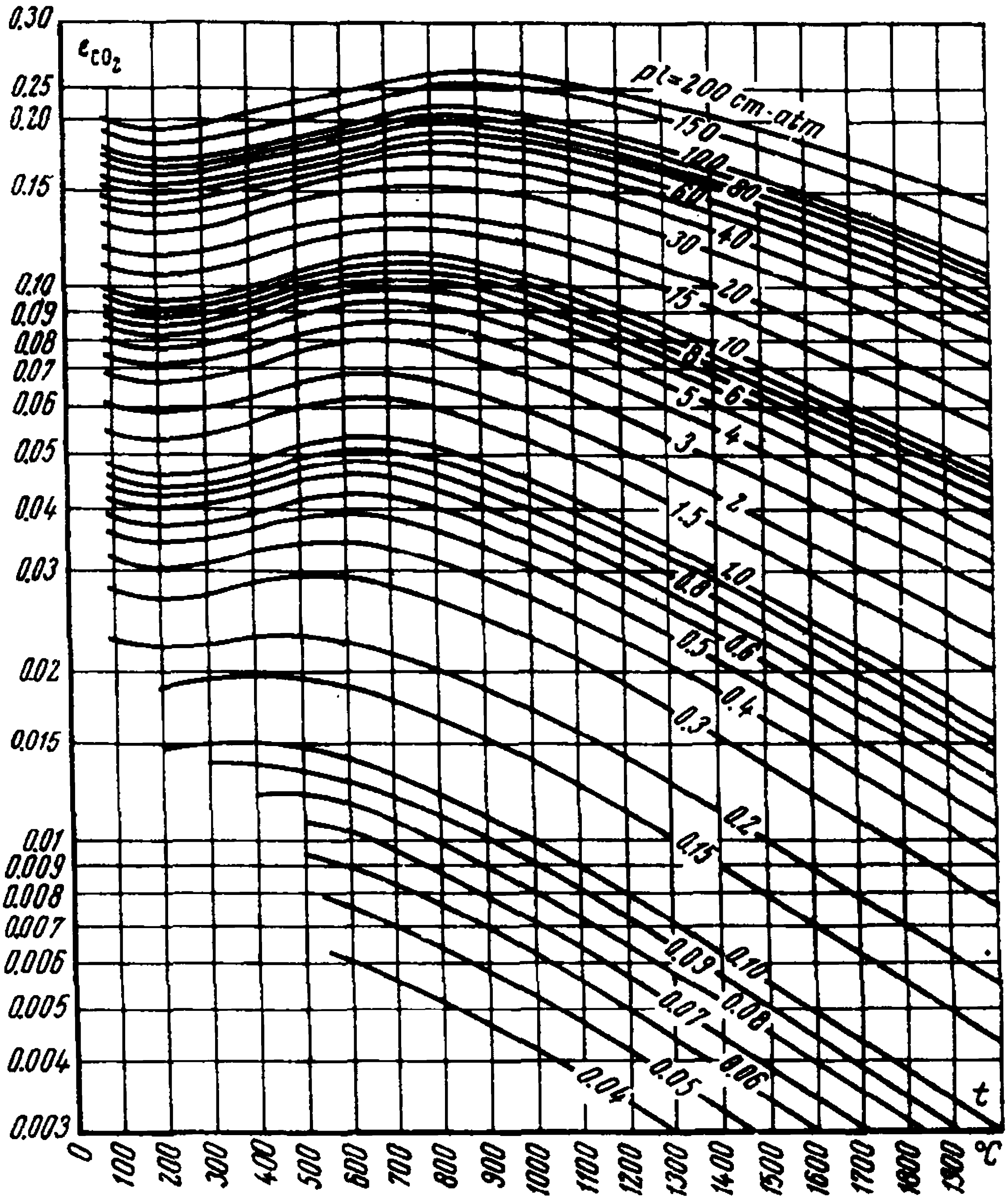

Fig. 11-1. Emissivity of carbon dioxide $\varepsilon_{CO_2} = f_1(t, pl)$ [29]

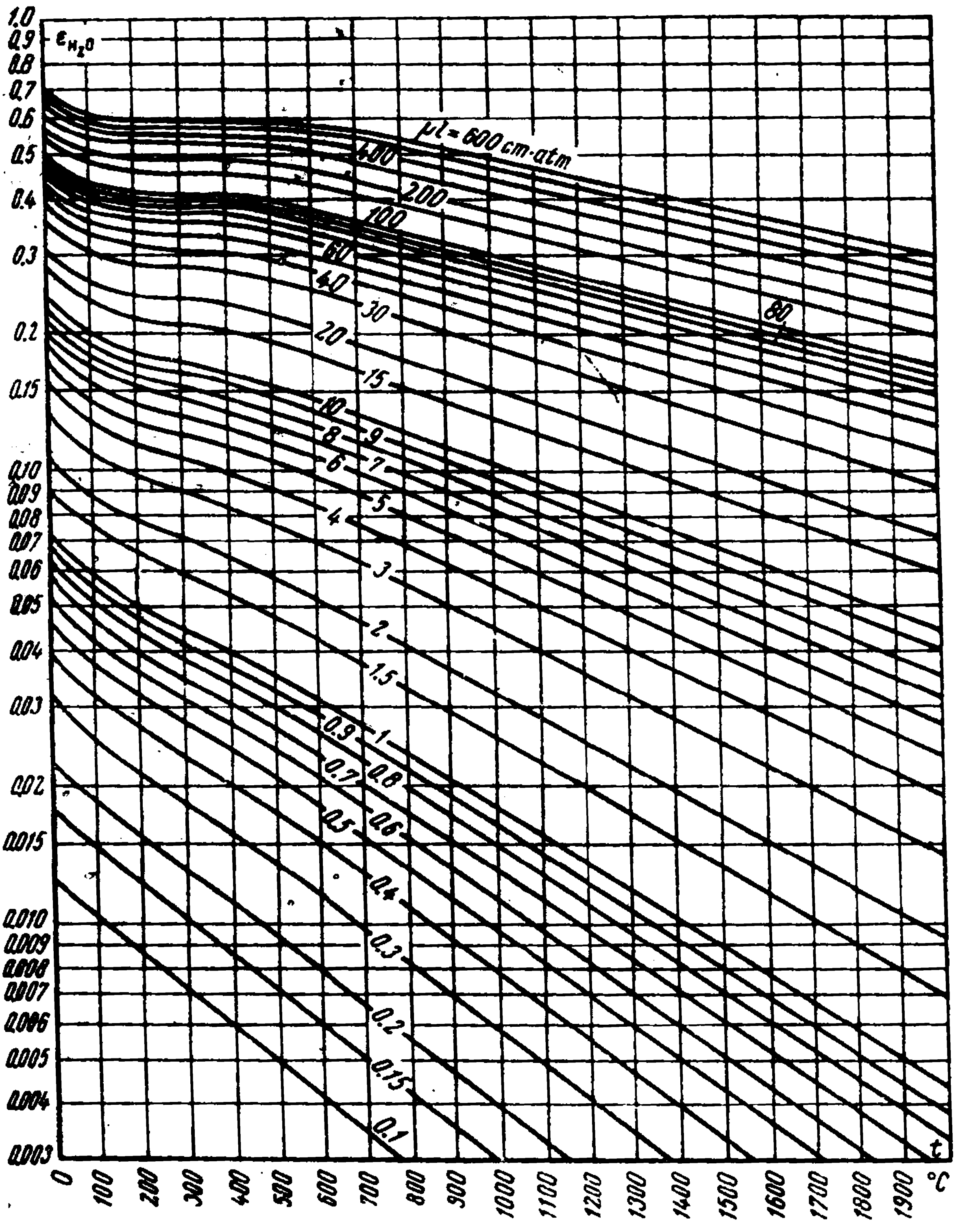

Fig. 11-2. Emissivity of water vapour (steam) $\varepsilon_{HO_2} = f_2\ (t, pl)$ [29]

and water vapour by the path of a ray are

$$p_{CO_2}l = 1.2 \times 10^4 \times 1.54 = 1.85 \times 10^4 \text{m} \times \text{Pa} = 18.9\,\text{cm}\cdot\text{atm};$$

$$p_{H_2O}l = 0.8 \times 10^4 \times 1.54 = 1.23 \times 10^4 \text{m} \times \text{Pa} = 12.5\,\text{cm}\cdot\text{atm}.$$

The emissivity of CO_2 and H_2O at a gas temperature $t_2 = 1200°C$ are found from the graphs in Figs. 11-1 and 11-2 [29]:

$$\varepsilon_{CO_2} = 0.11;$$

$$\varepsilon_{H_2O} = 0.10.$$

The emissivity of the mixture of gases

$$\varepsilon_g = \varepsilon_{CO_2} + \beta\varepsilon_{H_2O}.$$

From the graphs in Fig. 11-3 we find the correction $\beta = 1.05$, then $\varepsilon_g = 0.11 + 1.05 \times 0.10 = 0.215$.

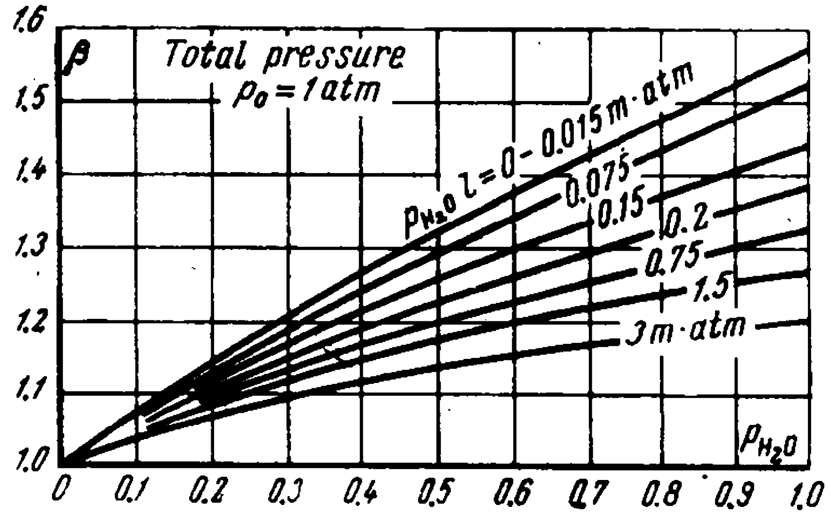

Fig. 11-3. Correction factor β for partial pressure for water vapour (steam) [29]

The radiation emitted by the products of combustion

$$E_{em.\,g} = \varepsilon_g C_0 \left(\frac{T_g}{100}\right)^4 = 0.215 \times 5.67 \left(\frac{1473}{100}\right)^4 = 57\,400\ \text{W/m}^2.$$

11-5. Calculate the emissivity of and the radiation emitted from the mixture of gases, if the mean temperature of the gases dropped to 1000°C, and all other conditions remain as in problem 11-4.

Answer

$$\varepsilon_g = 0.256;\quad E_{em.\,g} = 38\,200\ \text{W/m}^2.$$

11-6. Calculate the emissivity of and the radiation emitted from the mixture of gases, if the partial pressure of carbon dioxide $p_{CO_2} = 1 \times 10^4$ Pa and of water vapour $p_{H_2O} = 1 \times 10^4$ Pa, and all other conditions remain as in problem 11-4.

Answer

$$\varepsilon_g = 0.220; \quad E_{em.\,g} = 58\,700 \text{ W/m}^2.$$

11-7. Determine the coefficient of heat transfer by radiation from a stream of gas to the surface of tubes of the superheater of a steam boiler, if the gas temperature at the superheater inlet $t_{g1} = 1100°C$ and the superheater outlet gas temperature $t_{g2} = 800°C$. Assume the mean temperature of the heat-transfer surface to be constant and equal to $t_w = 500°C$, and the emissivity of the surface $\varepsilon_w = 0.8$. The tubes of the superheater are in a staggered arrangement (Fig. 11-4) with left-to-right pitches $s_1 = 2d$ and front-to-back pitches $s_2 = 2d$; the outside diameter of the tubes $d = 38$ mm. The gas contains 10% CO_2 and 4% H_2O. The total gas pressure $p = 98.1$ kPa.

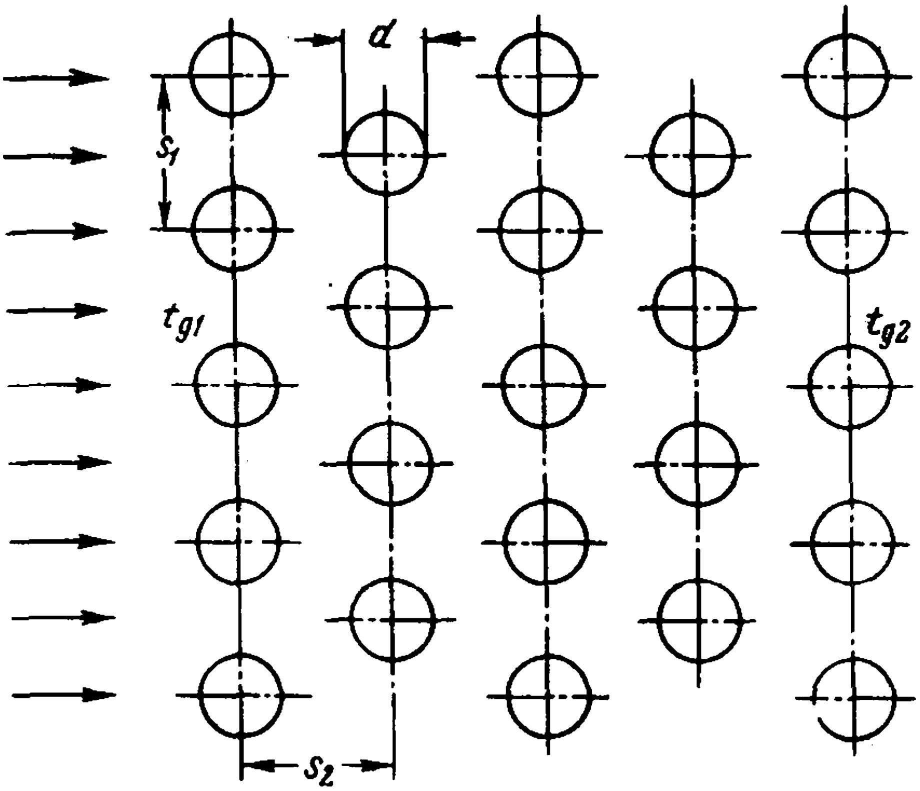

Fig. 11-4. Solution to problem 11-7

Answer

$$\alpha_{rad} = 11.5\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

Solution

The mean path of a ray in the intertubular space is determined from formula [28]:

$$l = 1.08d\left(\frac{s_1 s_2}{d^2} - 0.785\right) = 1.08 \times 0.038\,(2 \times 2 - 0.785) = $$

$$= 0.132\ \mathrm{m}.$$

The products of the partial pressure of carbon dioxide and water vapour by the mean path of a ray are:

$$p_{CO_2} l = 0.1 \times 9.81 \times 10^4 \times 0.132 = 0.129 \times 10^4\,\mathrm{m \cdot Pa} =$$

$$= 1.32\ \mathrm{cm \cdot atm};$$

$$p_{H_2O} = 0.04 \times 9.81 \times 10^4 \times 0.132 = 0.0518 \times 10^4\,\mathrm{m \cdot Pa}$$

$$= 0.528\ \mathrm{cm \cdot atm}.$$

The mean temperature of the gases

$$t_g = \frac{1}{2}(t_{g1} + t_{g2}) = \frac{1}{2}(1100 + 800) = 950\ {}^\circ\mathrm{C}.$$

Using this mean gas temperature, we find from the graphs in Figs. 11-1 and 11-2 the emissivities of CO_2 and H_2O;

$$\varepsilon_{CO_2} = 0.05;$$

$$\varepsilon_{H_2O} = 0.0105.$$

From the graph in Fig. 11-3 we find the correction β and calculate the emissivity of the gases at the mean temperature

$$\varepsilon_g = \varepsilon_{CO_2} + \beta\varepsilon_{H_2O} = 0.05 + 1.05 \times 0.0105 = 0.061.$$

At the temperature of the tube surfaces the absorptivity of the gases

$$A_g = \varepsilon_{CO_2}\left(\frac{T_g}{T_w}\right)^{0.65} + \beta\varepsilon_{H_2O} =$$

$$= 0.056\left(\frac{950 + 273}{500 + 273}\right)^{0.65} + 1.05 \times 0.021 = 0.0975,$$

where ε_{CO_2} and ε_{H_2O} are taken from the same graphs (Figs. 11-1 and 11-2) at the wall temperature.

The rate of heat transfer by radiation to the surface of the tubes

$$q_{rad}=\frac{1}{2}(e_w+1)C_0\left[e_g\left(\frac{T_g}{100}\right)^4-A_g\left(\frac{T_w}{100}\right)^4\right]. \qquad (11\text{-}3)$$

In the case considered

$$q_{rad}=\frac{1}{2}(0.8+1)\,5.67\left[0.061\left(\frac{950+273}{100}\right)^4-\right.$$
$$\left.-0.0975\left(\frac{500+273}{100}\right)^4\right]=5180\ \text{W/m}^2.$$

The coefficient of heat transfer by radiation

$$\alpha_{rad}=\frac{q_{rad}}{t_g-t_w}=\frac{5180}{950-500}=11.5\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

11-8. Solve problem 11-7 on condition that the longitudinal pitch between the tubes and the cross pitch s_2 are increased two times, i.e. $s_1 = s_2 = 4d$, and all other initial data remained unchanged.

Answer

$$\alpha_{rad} = 25.4\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

11-9. Solve problem 11-7 on condition that, due to a change in furnace operating conditions, the partial pressure of the water vapour increased 3 times, and all other initial data remained unchanged.

Answer

$$\alpha_{rad} = 15.6\ \text{W/m}^2\cdot{}^\circ\text{C}.$$

11-10. A tube bank, built up of a large number of long tubes having an outside diameter $d = 60$ mm, is exposed to cross flow of combustion products containing 12% carbon dioxide and 7% water vapour. The temperature of the combustion products is 1200°C and their total pressure is equal to 98.1 kPa.

The tubes of the bank are in a staggered arrangement with equal longitudinal and cross pitches, equal to $s_1 = s_2 = 2d$.

Determine the radiation emitted from the products of combustion per 1 m² of the tubes.

Answer

$$q_{em.g} = 2.03 \times 10^4\ \text{W/m}^2.$$

11-11. Calculate the fraction of heat transferred by radiation in the common process of heat transfer from flue gases to the tubes of the boiler bank of tubes considered in problem 6-13, on condition that the longitudinal and cross pitches between the tubes $s_1 = s_2 = 3d$ ($d_e = 80$ mm), the mean temperature of gases in the bank $t_g = 1000°C$, the temperature of tube surfaces $t_w = 400°C$, and their emissivity $\varepsilon_w = 0.8$. The flue gases contain 11% H_2O and 13% CO_2. The gas pressure $p = 98.1$ kPa. The local coefficient of heat transfer by convection α_c is taken from the answer to problem 6-13 (the change in α_c due to the other value of s_1/s_2 is to be ignored).

Answer

$$\varkappa_{rad} = 0.385\alpha_0.$$

11-12. Calculate the density of radiation from flue gases to the surface of a cylindrical gas flue having a diameter $d = 500$ mm. The gases contain 10% carbon dioxide and 5% water vapour. The total pressure of the gases is 98.1 kPa. The temperature of the gases at the flue inlet $t_{g1} = 800°C$ and at the outlet $t_{g2} = 600°C$; the mean temperature of the flue surface $t_w = 400°C$, and the emissivity of the surface $\varepsilon_w = 0.85$.

Answer

$$q_{rad} = 4630 \text{ W/m}^2.$$

Solution

The mean path of a ray

$$l = 0.9\frac{4V}{F} = 0.9\frac{4\frac{\pi d^2}{4}L}{\pi dL} = 0.9d = 0.45 \text{ m}.$$

In distinction to problem 11-7, in practical calculations, the emissivity of gases is determined on the basis of the geometrical mean temperature. This method is applied in the event of great temperature variation in the flue.

The geometrical mean temperature in the gas flue

$$\bar{T}_g = \sqrt[4]{(T_{g1})^2(T_{g2})^2} = \sqrt[4]{(800+273)^2(600+273)^2} = 967 \text{ K};$$

$$\bar{t}_g = \bar{T}_g - 273 = 694°C.$$

The products of the partial pressure of carbon dioxide CO_2 and water vapour H_2O by the mean path of a ray are equal to:

$$p_{CO_2}l = 0.1 \times 9.81 \times 10^4 \times 0.45 =$$
$$= 0.442 \times 10^4 \text{ m·Pa} = 4.5 \text{ cm·atm};$$
$$p_{H_2O}l = 0.05 \times 9.81 \times 10^4 \times 0.45 =$$
$$= 0.221 \times 10^4 \text{ m·Pa} = 2.25 \text{ cm·atm}.$$

At the mean gas temperature $\bar{t}_g \doteq 694°C$, from Figs. 11-1 through 11-3 we find that

$$\varepsilon_{CO_2} = 0.09; \quad \varepsilon_{H_2O} = 0.05 \quad \text{and} \quad \beta = 1.06.$$

At this mean temperature of the gases their emissivity

$$\varepsilon_g = \varepsilon_{CO_2} + \beta\varepsilon_{H_2O} = 0.09 + 1.06 \times 0.05 = 0.143.$$

At the wall temperature $t_w = 400°C$ from the same graphs we find that

$$\varepsilon_{CO_2} = 0.08 \quad \text{and} \quad \varepsilon_{H_2O} = 0.07.$$

At that wall temperature the absorptivity of the gases

$$A_g = \varepsilon_{CO_2}\left(\frac{\bar{T}_g}{T_w}\right)^{0.65} + \beta\varepsilon_{H_2O} = 0.08\left(\frac{967}{673}\right)^{0.65} +$$
$$+ 1.06 \times 0.07 = 0.175.$$

The rate of radiant heat flow to the wall of the flue from the gases

$$q_{rad} \doteq \frac{1}{2}(\varepsilon_w + 1)C_0\left[\varepsilon_g\left(\frac{\bar{T}_g}{100}\right)^4 - A_g\left(\frac{T_w}{100}\right)^4\right] =$$
$$= \frac{1}{2}(0.85 + 1)5.67(0.143 \times 9.7^4 - 0.175 \times 6.73^4) =$$
$$= 4630 \text{ W/m}^2.$$

11-13. Solve problem 11-12 on condition that the flue has a rectangular cross section with sides: $a = 0.5$ m and $b = 1$ m. All other initial data remain unchanged.

Answer

$$q_{rad} = 5420 \text{ W/m}^2.$$

11-14. Solve problem 11-12 on condition that the partial pressure of water vapour increased 2 times, all other initial data remaining unchanged.

Answer

$$q_{rad} = 5930 \text{ W/m}^2.$$

Chapter Twelve

CALCULATION OF HEAT EXCHANGERS

12-1. Grade MC oil flows into an oil cooler at a temperature $t'_{f1} = 70°C$ and cools there to a temperature $t''_{f1} = 30°C$. The inlet temperature of the cooling water $t'_{f2} = 20°C$.

Determine the temperature of the cooling water at the outlet of the oil cooler, if the rates of oil and water flow are respectively $G_1 = 1 \times 10^4$ kg/h and $G_2 = 2.04 \times 10^4$ kg/h. The loss of heat to surroundings is to be ignored.

Answer

$$t''_{f2} = 30°C.$$

12-2. To what temperature shall heat water in the oil cooler, if the rates of oil and water flow are the same, $G_1 = G_2$, and the temperature t'_{f1}, t''_{f1} and t'_{f2} are the same as in problem 12-1?

Answer

$$t''_{f2} = 40.4°C.$$

12-3. Determine the logarithmic mean temperature differences between the primary (hot) and secondary (cold) fluids under conditions 12-1 and 12-2, if the two fluids are in counterflow (Fig. 12-1 and Fig. 12-2).

Answer

$$\Delta t_{log} = 21.7°C \quad \text{and} \quad t_{log} = 18.1°C.$$

12-4. In an air heater air is heated from a temperature $t'_{f2} = 20°C$ to $t''_{f2} = 210°C$, and the hot gas cools from a temperature $t'_{f1} = 410°C$ to $t''_{f1} = 250°C$.

Determine the logarithmic mean temperature difference between the air and gas with the two fluids in parallel flow and in counterflow (Fig. 12-1).

Answer

$$\Delta t_{par} = 154°C; \quad \Delta t_{count} = 215°C.$$

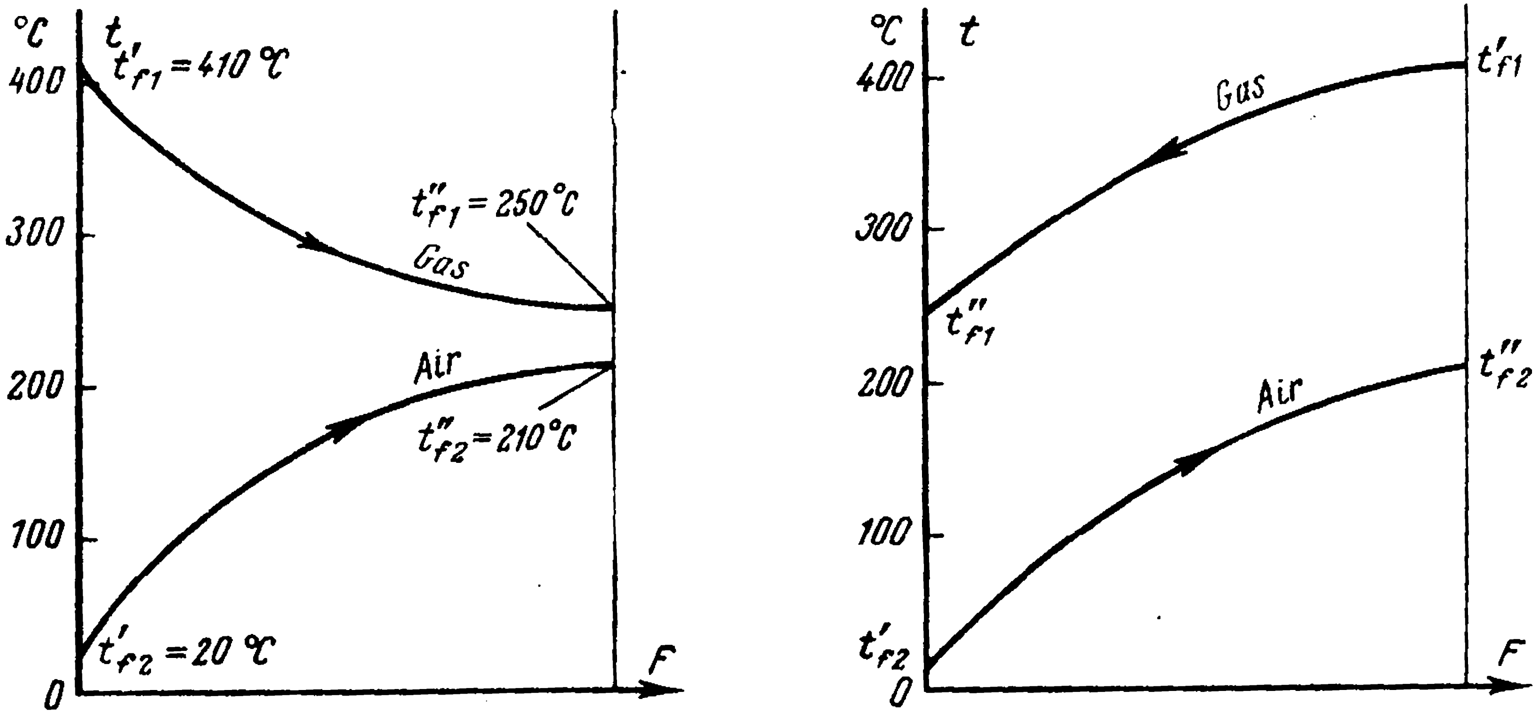

Fig. 12-1. Solution to problem 12-4

12-5. Under the conditions of problem 12-4 determine the logarithmic mean temperature difference, if the air and gas are in cross flow and each stream is properly mixed. Compare the result with the answer to problem 12-4.

Answer

$$\Delta t_{c.f} = 184°\text{C}; \quad \Delta t_{count} > \Delta t_{c.f} > t_{par}.$$

Hint. With the primary (hot) and secondary (cold) fluids in cross flow the logarithmic mean temperature difference

$$\Delta t_{c.f} = \Delta t_{count}\varepsilon_{c.f} \qquad (12\text{-}1)$$

where Δt_{count} = logarithmic mean temperature difference with the primary and secondary fluids in counterflow;

$\varepsilon_{c.f}$ = correction factor determined from the chart in Fig. A-3, depending on P and R:

$$P = \frac{\delta t_{f2}}{\Delta t'} \frac{t''_{f2} - t'_{f2}}{t'_{f1} - t'_{f2}};$$

$$R = \frac{\delta t_{f1}}{\delta t_{f2}} = \frac{t'_{f1} - t''_{f1}}{t''_{f2} - t'_{f2}}.$$

12-6. In a tubular steam-to-water heat exchanger dry saturated water vapour, steam, condenses at a pressure

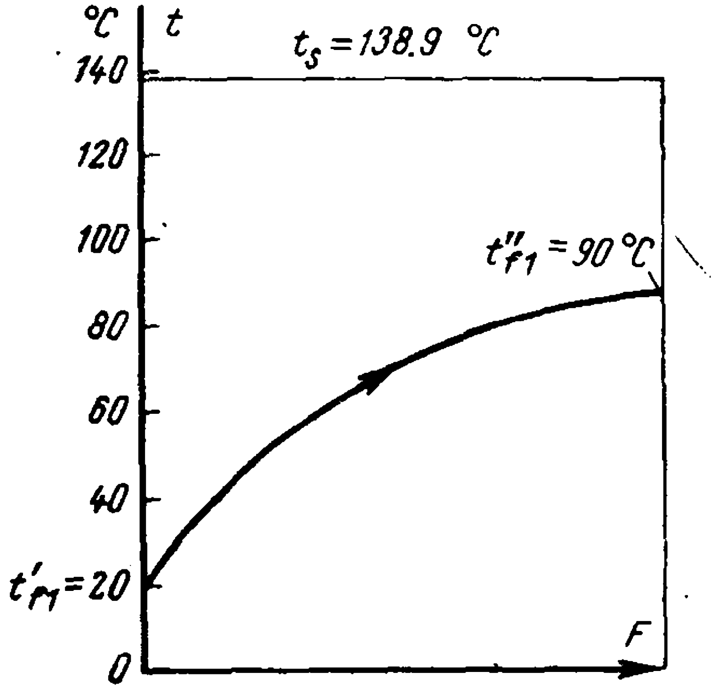

Fig. 12-2. Solution to problem 12-6

$p = 3.5 \times 10^5$ Pa on the external surface of tubes. The water moves in the tubes and heats from $t'_{f1} = 20°C$ to $t''_{f1} = 90°C$. Determine the logarithmic mean temperature difference for this heat exchanger (Fig. 12-2).

Answer

$$\Delta t_{log} = 78.9°C.$$

12-7. Determine the rate of steam flow through the steam-to-water heat exchanger, considered in problem 12-6, if the rate of water flow $G_1 = 8$ t/h. Assume that there is no supercooling of condensate.

Answer

$$G_2 = 1090 \text{ kg/h}.$$

12-8. Under conditions of problems 12-6 and 12-7 how shall change the logarithmic mean temperature difference, if the steam pressure is raised to $p = 7 \times 10^5$ Pa?

Answer

$$p = 3.5 \times 10^5 \text{ Pa}; \quad \Delta t_{log} = 78.9°C; \quad G_2 = 1090 \text{ kg/h};$$

$$p = 7 \times 10^5 \text{ Pa}; \quad \Delta t_{log} = 106°C; \quad G_2 = 1130 \text{ kg/h}.$$

12-9. Determine the heating surface of an economizer in which the primary (hot) fluid (gas) and the secondary fluid (water) are in counterflow, if the following quantities are known: the inlet gas temperature $t'_{f1} = 420°C$; the rate of gas flow through the economizer $G_1 = 220$ t/h; the heat capacity of the gases $c_{p1} = 1.045$ kJ/kg·°C; the water inlet temperature $t'_{f2} = 105°C$; the rate of water flow $G_2 = 120$ t/h; the amount of heat transferred from the hot gases to the water $Q = 13.5$ MW; the coefficient of heat transfer from the gases to the water $k = 79$ W/m²·°C.

Answer

$$F = 1100 \text{ m}^2.$$

12-10. Determine the heating surface of the economizer considered in problem 12-9, if the primary and secondary fluids are in parallel flow in it. Compare the result obtained with the answer to problem 12-9.

Answer

$$F = 1930 \text{ m}^2; \quad F_{count}/F_{par} = 0.57.$$

12-11. In a water-to-water heat exchanger of a counterflow arrangement having a heating surface $F = 2$ m² the inlet temperature of the heating water $t'_{f1} = 85°C$ and the rate of water flow $G_1 = 2000$ kg/h. The rate of heated water flow $G_2 = 1500$ kg/h and its inlet temperature $t'_{f2} = 25°C$.

Determine the amount of heat transferred from the hot to the cold water and their final temperatures, if it is known that the coefficient of heat transfer from the hot to the cold water $k = 1400$ W/m²·°C.

Answer

$$Q = 69.8 \text{ kW}; \quad t''_{f1} = 55°C; \quad t'_{f2} = 65°C.$$

12-12. Determine the heating surface and the number of sections of a water-to-water heat exchanger of the "tube-in-tube" type (Fig. 12-3). The heating (hot) water flows in the internal steel tube [($\lambda_w = 45$ W/m·°C)] with diameters $d_2/d_1 = 35/32$ mm and its inlet temperature $t'_{f1} = 95°C$. The rate of hot water flow $G_1 = 2130$ kg/h.

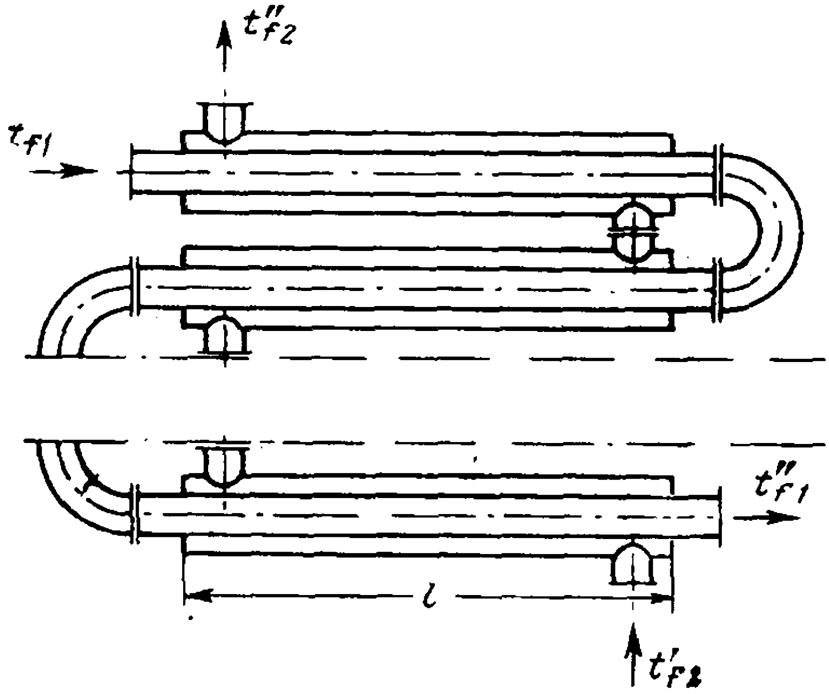

Fig. 12-3. Solution to problem 12-12

The cold water is in counterflow in respect to the hot water and it moves in the annulus formed between the two tubes; the cold water heats in the heat exchanger from $t'_{f2} = 15°C$ to $t''_{f2} = 45°C$. The inside diameter of the external tube $D = 48$ mm. The rate of cold water flow $G_2 = 3200$ kg/h. The length of a section of the heat exchanger $l = 1.9$ m.

The loss of heat from the external surface of the heat exchanger can be ignored.

Answer

$$F = 1.33 \text{ m}^2; \quad n = 7.$$

Solution

The heat capacity of water $c_p \approx 4.19$ kJ/kg·°C.

The amount of heat transferred in the heat exchanger

$$Q = G_2 c_{p2} (t''_{f2} - t'_{f2}) = \frac{3200}{3600} 4.19 (45 - 15) = 111 \text{ kW}.$$

The outlet temperature of the heating water

$$t''_{f1} = t'_{f1} - \frac{Q}{G_1 c_{p1}} = 95 - \frac{111 \times 3600}{2130 \times 4.19} = 50\,^\circ\text{C}.$$

Find the arithmetic mean temperatures of the hot and cold fluids and the physical properties of water at these temperatures:

$$t_{f1} = 0.5 (t'_{f1} + t''_{f1}) = 0.5 (95 + 50) = 72.5^\circ\text{C};$$

at that temperature

$$\rho_{f1} = 976 \text{ kg/m}^3; \quad \nu_{f1} = 0.403 \times 10^{-6} \text{ m}^2/\text{s};$$

$$\lambda_{f1} = 0.670 \text{ W/m}\cdot{}^\circ\text{C}; \quad \text{Pr}_{f1} = 2.47;$$

$$t_{f2} = 0.5 (t'_{f2} + t''_{f2}) = 0.5 (15 + 45) = 30^\circ\text{C};$$

at that temperature

$$\rho_{f2} = 996 \text{ kg/m}^3; \quad \nu_{f2} = 0.805 \times 10^{-6} \text{ m}^2/\text{s};$$

$$\lambda_{f2} = 0.618 \text{ W/m}\cdot{}^\circ\text{C}; \quad \text{Pr}_{f2} = 5.42.$$

The velocities of the hot and cold fluids

$$w_1 = \frac{4G_1}{\rho_{f1} \pi d_1^2 3600} = \frac{4 \times 2130}{976 \times 3.14 (3.2 \times 10^{-2})^2 3600} = 0.755 \text{ m/s};$$

$$w_2 = \frac{4G_2}{\rho_{f2} \pi (D^2 - d_2^2) 3600} = \frac{4 \times 3200}{996 \times 3.14 (4.8^2 - 3.5^2) 10^{-4} \times 3600} =$$

$$= 1.06 \text{ m/s}.$$

The Reynolds number for the hot fluid

$$\text{Re}_{f1} = \frac{w_1 d_1}{\nu_{f1}} = \frac{0.755 \times 3.2 \times 10^{-2}}{0.403 \times 10^{-6}} = 6 \times 10^4.$$

The hot water is in turbulent flow so that the Nusselt number and the heat-transfer coefficient are calculated by formula (5-7).

The Nusselt number

$$\mathrm{Nu}_{f1}=0.021\mathrm{Re}_{f1}^{0.8}\mathrm{Pr}_{f1}^{0.43}\left(\frac{\mathrm{Pr}_{f1}}{\mathrm{Pr}_{w1}}\right)^{0.25}.$$

Since the wall temperature is not known, then for a first approximation we assume a wall temperature

$$t_{w1}\approx 0.5\,(t_{f1}+t_{f2})=0.5\,(72.5+30)=51.2^\circ\mathrm{C}.$$

At that temperature the Prandtl number $\mathrm{Pr}_{w1}=3.5$; then

$$\mathrm{Nu}_{f1}=0.021\,(6\times 10^4)^{0.8}(2.47)^{0.43}\left(\frac{2.47}{3.5}\right)^{0.25}=188.$$

The coefficient of heat transfer from the hot water to the wall of a tube

$$\alpha_1=\mathrm{Nu}_{f1}\frac{\lambda_{f1}}{d_1}=188\,\frac{0.670}{3.2\times 10^{-2}}=3940\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The Reynolds number for the cold (heated) water

$$\mathrm{Re}_{f2}=\frac{w_2 d_e}{\nu_{f2}}=\frac{1.06\times 1.3\times 10^{-2}}{0.805\times 10^{-6}}=1.71\times 10^4,$$

where the equivalent diameter for the annular channel

$$d_e=D-d_2=48-35=13\ \mathrm{mm}.$$

The cold (heated) water is in turbulent flow so that the Nusselt number and the heat-transfer coefficient are calculated by formula (5-12) derived for heat transfer in fluids in turbulent flow in annular channels:

$$\mathrm{Nu}_{f2}=0.017\mathrm{Re}_{f2}^{0.8}\mathrm{Pr}_{f2}^{0.4}\left(\frac{\mathrm{Pr}_{f2}}{\mathrm{Pr}_{w2}}\right)^{0.25}\left(\frac{D}{d_2}\right)^{0.18}.$$

Assuming as a first approximation that $t_{w2}\approx t_{w1}$ and, consequently, that $\mathrm{Pr}_{w2}\approx\mathrm{Pr}_{w1}\approx 3.5$, we obtain:

$$\mathrm{Nu}_{f2}=0.017\,(1.71\times 10^4)^{0.8}(5.42)^{0.4}\left(\frac{5.42}{3.5}\right)^{0.25}\left(\frac{48}{35}\right)^{0.18}=95.$$

The coefficient of heat transfer from the tube to the cold water

$$\alpha_2 = \mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d_e} = 95\frac{0.618}{1.3\times10^{-2}} = 4500\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The overall heat-transfer coefficient

$$k = \frac{1}{\frac{1}{\alpha_1}+\frac{\delta_w}{\lambda_w}+\frac{1}{\alpha_2}} =$$

$$= \frac{1}{\frac{1}{3940}+\frac{1.5\times10^{-3}}{45}+\frac{1}{4500}} = 1970\ \mathrm{W/m^2\cdot{}^\circ C}$$

Since in the case considered $\frac{t'_{f1}-t''_{f2}}{t''_{f1}-t'_{f2}} = \frac{50}{35} < 1.5$, then calculations can be based with sufficient accuracy on the arithmetic mean temperature difference

$$\Delta t_a = t_{f1} - t_{f2} = 72.5 - 30 = 42.5^\circ\mathrm{C}.$$

The rate of heat flow from the hot to the cold water

$$q = k\Delta t_a = 1970 \times 42.5 = 8.37 \times 10^4\ \mathrm{W/m^2}.$$

The heating surface

$$F = \frac{Q}{q} = \frac{111}{83.7} = 1.33\ \mathrm{m^2}.$$

The number of sections

$$n = \frac{F}{\pi d_1 l} = \frac{1.33}{\pi\times3.2\times10^{-2}\times1.9} \approx 7.$$

The surface temperatures of a tube

$$t_{w1} = t_{f1} - \frac{q}{\alpha_1} = 72.5 - \frac{83\,700}{3940} = 51.3\ ^\circ\mathrm{C};$$

$$t_{w2} = t_{f2} - \frac{q}{\alpha_2} = 30 + \frac{83\,700}{4500} = 48.6\ ^\circ\mathrm{C}.$$

At these temperatures $\mathrm{Pr}_{w1} = 3.47$ and $\mathrm{Pr}_{w2} = 3.65$ and the corrections, allowing for a change in the physical properties of the fluid over the cross section of flow are as follows:

$$\left(\frac{\mathrm{Pr}_{f1}}{\mathrm{Pr}_{w1}}\right)^{0.25}=\left(\frac{2.47}{3.47}\right)^{0.25}=0.915 \text{ (assumed equal to 0.92);}$$

$$\left(\frac{\mathrm{Pr}_{f2}}{\mathrm{Pr}_{w2}}\right)^{0.25}=\left(\frac{5.42}{3.65}\right)^{0.25}=1.10 \text{ (assumed equal to 1.12).}$$

The coincidence is sufficiently accurate; it may be assumed that $F = 1.33\ \mathrm{m}^2$ and the number of sections $n = 7$.

12-13. In a sectional heat exchanger of the "tube-in-tube" type transformer oil is cooled with water.

The transformer oil flows along an inner brass tube of diameters $d_2/d_1 = 14/12$ mm with a velocity $w_1 = 4$ m/s. The transformer oil inlet temperature $t'_{f1} = 100°\mathrm{C}$. The cooling water is in counterflow along the annulus with a velocity $w_2 = 2.5$ m/s, and its temperature at the heat-exchanger inlet $t'_{f2} = 20°\mathrm{C}$. The inside diameter of the outer tube, or shell, of the heat exchanger $d_3 = 22$ mm.

Determine the total length of the heat-transfer surface with which oil outlet temperature shall be $t''_{f1} = 60°\mathrm{C}$.

The loss of heat through the external surface of the heat exchanger can be ignored.

Answer

$$l = 11.6 \text{ m.}$$

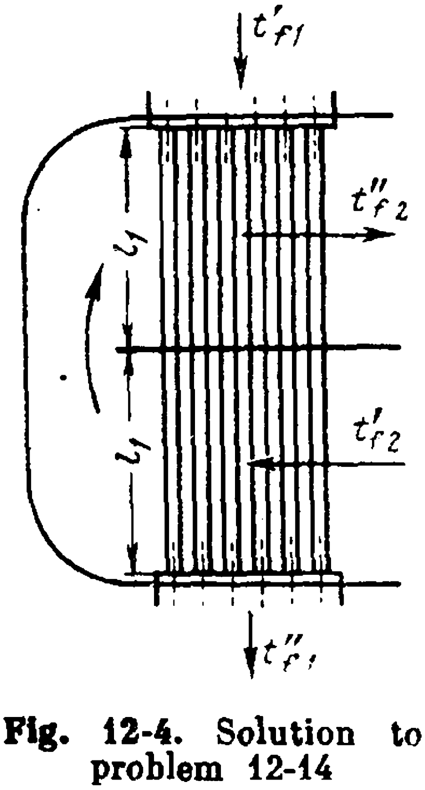

Fig. 12-4. Solution to problem 12-14

12-14. In a tubular two-pass air heater of a steam boiler (Fig. 12-4) air in the amount $G_2 = 21.5$ kg/s must be heated from an inlet temperature $t'_{f2} = 30°\mathrm{C}$ to an outlet temperature $t''_{f2} = 260°\mathrm{C}$.

Determine the required heating surface, the height of tubes in one pass, l_1, and the number of tubes arranged across and along the flow of air.

The flue gas (13% CO_2, 11% H_2O), in the amount of $G_1 = 19.6$ kg/s, moves in the steel tubes ($\lambda_w = 46.5\ \mathrm{W/m \cdot °C}$)

having diameters $d_2/d_1 = 53/50$ mm with a mean velocity $w_1 = 14$ m/s. At the air heater inlet flue gas temperature $t'_{f1} = 380°C$.

Air flows across the bank of tubes with a mean velocity in the narrow cross section of the bank $w_2 = 8$ m/s. The tubes are in a staggered arrangement in the bank with pitches $s_1 = s_2 = 1.3\,d_2$.

Answer

The heating surface $F = 1830$ m^2; the height of tubes in one pass $l_1 = 5.3$ m; the number of tubes arranged across the flow $n_1 = 38$, and along the flow—$n_2 = 29$.

Solution

The arithmetic mean temperature of air

$$t_{f2} = 0.5\,(t'_{f2} + t''_{f2}) = 0.5\,(30 + 260) = 145°\text{C}.$$

At that temperature the physical properties of air are as follows: $\rho_{f2} = 0.844$ kg/m^3, $c_{pf2} = 1.01$ kJ/kg·°C, $\lambda_{f2} = 3.52 \times 10^{-2}$ W/m·°C, $\nu_{f2} = 28.3 \times 10^{-6}$ m^2/s, $\text{Pr}_{f2} = 0.684$.

The amount of heat transferred from the flue gas to the air

$$Q = G_2 c_{pf2}\,(t''_{f2} - t'_{f2}) = 21.5 \times 1.01\,(260 - 30) = 5000\,\text{kW}.$$

Determine the temperature of the flue gas at the air heater outlet.

As a first approximation the mean temperature of the flue gas in the air heater is assumed to be $t_{f1} = 300°C$, and at that temperature $c_{pf1} \approx 1.12$ kJ/kg·°C and

$$t''_{f1} = t'_{f1} - \frac{Q}{G_1 c_{pf1}} = 380 - \frac{5000}{19.6 \times 1.12} = 152\,°\text{C},$$

then

$$t_{f1} = 0.5\,(t'_{f1} + t''_{f1}) = 0.5\,(380 + 152) = 266°\text{C}.$$

At that temperature $c_{pf1} = 1.11$ kJ/kg·°C and as a result of a second approximation we find that

$$t''_{f1} = 150°\text{C} \quad \text{and} \quad t_{f1} = 265°\text{C}.$$

At a temperature $t_{f1} = 265°C$ the physical properties of flue gas of the given composition are as follows:

$$\rho_{f1} = 0.622\ \text{kg/m}^3;\quad c_{pf1} = 1.11\ \text{kJ/kg·°C};$$

$$\lambda_{f1} = 0.0454\ \text{W/m·°C};\quad \nu_{f1} = 41.2 \times 10^{-6}\ \text{m}^2/\text{s};\quad \text{Pr}_{f1} = 0.66.$$

The Reynolds number for the flue gas

$$\mathrm{Re}_{f1}=\frac{w_1d_1}{\nu_{f1}}=\frac{14\times0.05}{41.2\times10^{-6}}=17\,000.$$

The Nusselt number and the coefficient of heat transfer from the flue gas to the walls of tubes are calculated by formula (5-7):

$$\mathrm{Nu}_{f1}=0.021\,\mathrm{Re}_{f1}^{0.8}\,\mathrm{Pr}_{f1}^{0.43}=0.021\,(1.7\times10^4)^{0.8}\,(0.66)^{0.43}=$$
$$=43.5;$$

$$\alpha_1=\mathrm{Nu}_{f1}\frac{\lambda_{f1}}{d_1}=43.5\,\frac{0.0454}{0.05}=39.5\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The Reynolds number for air

$$\mathrm{Re}_{f2}=\frac{w_2d_2}{\nu_{f2}}=\frac{8\times0.053}{28.3\times10^{-6}}=15\,000.$$

For cross flow the Nusselt number and the coefficient of heat transfer from the walls of tubes to the air are calculated by means of formula (6-4):

$$\mathrm{Nu}_{f2}=0.41\mathrm{Re}_{f2}^{0.6}\mathrm{Pr}_{f2}^{0.33}\varepsilon_s=$$
$$=0.41\,(1.5\times10^4)^{0.6}\,(0.684)^{0.33}=115,$$

where, for tubes in a staggered arrangement, and $s_1/s_2<$ <2 $\varepsilon_s=(s_1/s_2)^{1.6}$ and since $s_1=s_2$ then $\varepsilon_s=1$;

$$\alpha_2=\mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d_2}=115\,\frac{0.0352}{0.053}=76.2\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The overall heat-transfer coefficient

$$k=\frac{1}{\frac{1}{\alpha}+\frac{\delta_w}{\lambda_w}+\frac{1}{\alpha_2}}=\frac{1}{\frac{1}{39.5}+\frac{1.5\times10^{-3}}{46.5}+\frac{1}{76.2}}=$$
$$=26\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Since

$$\frac{t'_{f1}-t''_{f2}}{t''_{f1}-t'_{f2}}=\frac{380-150}{260-30}=1,$$

then the mean temperature difference (or temperature drop)

$$\Delta t_{\mathrm{log.\ count}}\approx\Delta t_a=t_{f1}-t_{f2}=265-145=120^\circ\mathrm{C}.$$

For the flue gas and air flows considered from the graph (Fig. A-4 in the appendix) we find:
at

$$P=\frac{t''_{f2}-t'_{f2}}{t'_{f1}-t'_{f2}}=\frac{260-30}{380-30}=0.658$$

and

$$R=\frac{t'_{f1}-t''_{f1}}{t''_{f2}-t'_{f2}}=\frac{380-150}{260-30}=1.0;$$

$$\varepsilon=0.88;$$

consequently,

$$\Delta t=\varepsilon\Delta t_{count}=0.88\times 120=105.5^{\circ}\mathrm{C}.$$

The heating surface of the air heater

$$F=\frac{Q}{k\Delta t}=\frac{5\times 10^{6}}{26\times 105.5}=1830\ \mathrm{m}^{2}.$$

The total number of tubes

$$n=\frac{4G_1}{\rho_{f1}\pi d_1^2 w_1}=\frac{4\times 19.6}{0.622\times 3.14\,(5\times 10^{-2})^{2}\,14}=1080.$$

The height of tubes in one pass

$$l_1=\frac{F}{2\pi d_1 n}=\frac{1830}{2\times 3.14\times 0.05\times 1080}=5.4\ \mathrm{m}.$$

The clear passage for air flow

$$f=\frac{G_2}{\rho_{f2}w_2}=\frac{21.5}{0.844\times 8}=3.2\ \mathrm{m}^{2}.$$

The number of tubes arranged across the flow

$$n_1=\frac{f}{l_1\,(s_1-d_2)}=\frac{3.2}{5.4\,(1.3\times 0.053-0.053)}\approx 38.$$

The number of tubes exposed to longitudinal flow

$$n_2=\frac{n}{n_1}=\frac{1080}{38}\approx 29.$$

12-15. Calculate and determine the main dimensions of a vertical four-pass tubular steam-to-water heat exchanger designed to heat water in the amount of $G_1 = 30$ t/h from an inlet temperature $t''_{f1} = 20\ ^{\circ}\mathrm{C}$ to an outlet temperature $t''_{f1} = 95\ ^{\circ}\mathrm{C}$.

The heated water flows in brass tubes ($\lambda = 104.5$ W/m·°C) having diameters $d_2/d_1 = 14/12$ mm with a velocity $w = 1.5$ m/s. The hot fluid is dry saturated steam at a pressure $p = 127.5$ kPa, condensing on the outer surface of the water-carrying tubes.

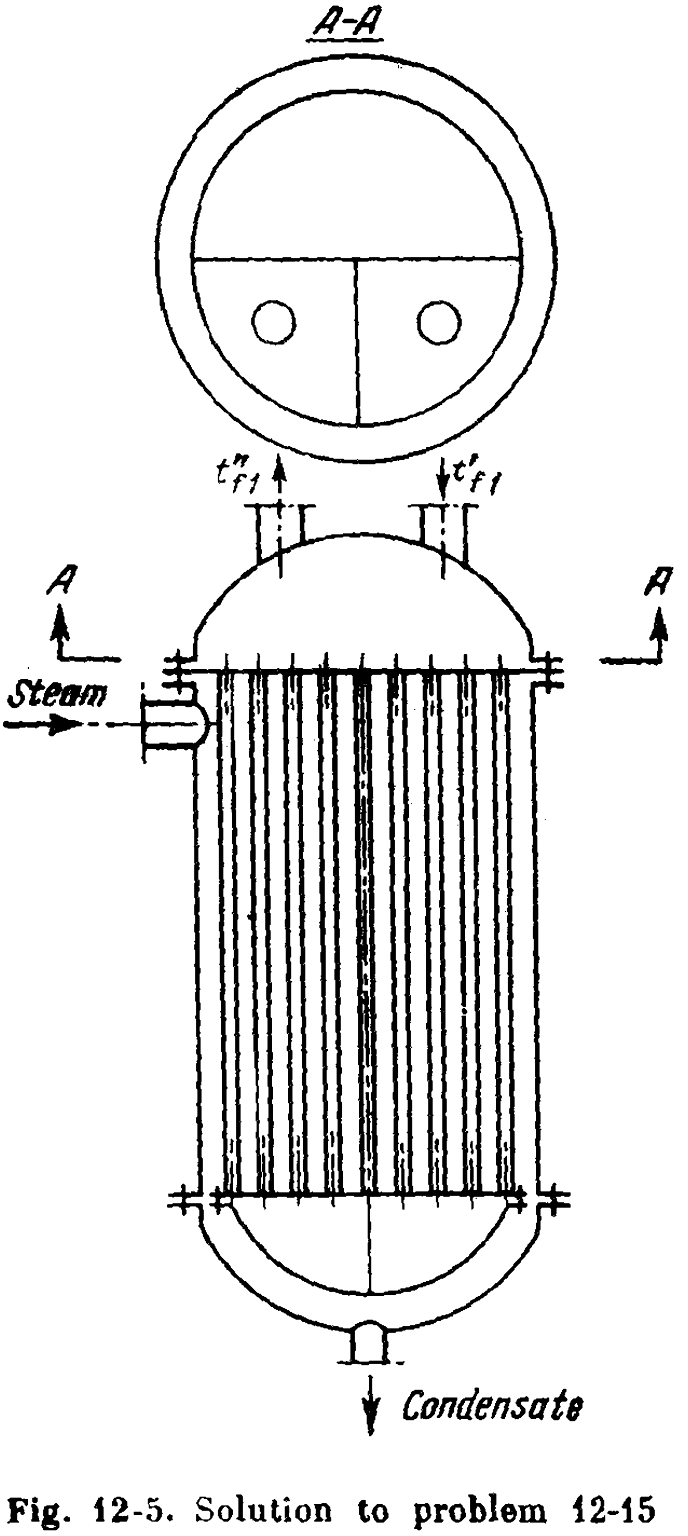

Fig. 12-5. Solution to problem 12-15

The loss of heat to surroundings is to be assumed equal to 2 per cent of the amount of heat added. The schematic layout of the heat exchanger is shown in Fig. 12-5.

Answer

The rate of steam flow $G_2 = 4310$ kg/h; the heating surface $F = 20$ m^2; the number of tubes $n = 200$; the height of tubes $H = 2.5$ m.

Solution

The amount of heat transferred from steam to water

$$Q = G_1 c_{p1} (t''_{f1} - t'_{f1}) = \frac{3\times 10^4}{3600} 4.187 (95-20) \approx 2620 \text{ kW}.$$

Find the rate of heat flow G_2. At the pressure $p = 127.5$ kPa $t_s = 106.6$ °C, $i'' = 2685$ kJ/kg, $i' = 447$ kJ/kg and

$$G_2 = \frac{Q}{0.98 (i''-i')} = \frac{2620 \times 10^3}{0.98 (2685-447)} = 1.2 \text{ kg/s}.$$

To calculate the coefficient of heat transfer to the outer surface of the tube due to steam condensation, we must know the temperature of the tube outer surface t_{w2} and the height of tube H. Since these quantities are not known, calculations should be carried out by the method of successive approximations. Determine the logarithmic mean temperature difference

$$\Delta t_{\log} = \frac{t''_{f1} - t'_{f1}}{\ln \frac{t_s - t'_{f1}}{t_s - t''_{f1}}} = \frac{95-20}{2.3 \log \frac{106.6-20}{106.6-95}} = 37.4 \text{ °C}.$$

for a first approximation assume

$$t_{w2} \approx t_s - \frac{\Delta t_{\log}}{2} = 106.6 - \frac{37.4}{2} \approx 88 \text{ °C}.$$

In addition, assume a height of tubes $H = 2$ m.

The reduced length of a tube

$$Z = \Delta t_2 H A.$$

At a saturation temperature $t_s = 106.6$ °C, from Table 8-1, we find $A = 57.6$ 1/m·°C and $B = 6.71 \times 10^{-3}$ m/W. Then.

$$Z = (t_s - t_{w2}) HA = (106.6 - 88) 2 \times 57.6 = 2140 < 2300.$$

The condensate is in laminar flow along the entire tube and calculation formula (8-5) is applicable:

$$Re = 3.8Z^{0.78} = 3.8\,(2140)^{0.78} = 1520;$$

$$\alpha_2 = \frac{Re}{\Delta t_2 HB} = \frac{1520}{18.6 \times 2 \times 6.71 \times 10^{-3}} = 6080\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

Determine the coefficient of heat transfer from the condensing steam to the heated water.

The arithmetic mean temperature of water

$$t_{f1} = 0.5\,(t'_{f1} + t''_{f1}) = 0.5\,(20 + 95) = 57.5\ {}^\circ\mathrm{C};$$

and at that temperature

$$\nu_{f1} = 0.498 \times 10^{-6}\ \mathrm{m^2/s};\quad \lambda_{f1} = 0.665\ \mathrm{W/m \cdot {}^\circ C};$$

$$\rho_{f1} = 984\ \mathrm{kg/m^3};\quad \text{and } Pr_{f1} = 3.12;$$

$$Re_{f1} = \frac{wd_1}{\nu_1} = \frac{1.5 \times 1.2 \times 10^{-2}}{0.498 \times 10^{-6}} = 3.62 \times 10^4.$$

The water is in turbulent flow; calculation formula (5-7) is used.

The drop of temperature across the wall of a tube is set approximately at 1 °C, then, $t_{w1} \approx t_{w2} - 1 = 87\ {}^\circ\mathrm{C}$ and $Pr_{w1} \approx 2.03$.

$$Nu_{f1} = 0.021 Re_{f1}^{0.8} Pr_{f1}^{0.43} (Pr_{f1}/Pr_{w1})^{0.25} =$$

$$= 0.021\,(3.62 \times 10^4)^{0.8}\,(3.12)^{0.43} \left(\frac{3.12}{2.03}\right)^{0.25} = 165;$$

$$\alpha_1 = Nu_{f1}\frac{\lambda_{f1}}{d_1} = 165\,\frac{0.655}{1.2 \times 10^{-2}} = 9000\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The overall heat-transfer coefficient

$$k = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{9000} + \frac{1 \times 10^{-3}}{104.5} + \frac{1}{6080}} =$$

$$= 3630\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

The mean rate of heat flow

$$q = k\Delta t_{log} = 3630 \times 37.4 \times 10^{-3} = 135.8\ \mathrm{kW/m^2}.$$

For a first approximation the heating surface

$$F = \frac{Q}{q} = \frac{2620}{135.8} = 19.3\ \mathrm{m^2}.$$

The number of tubes in one pass

$$m = \frac{4G_1}{\rho_{f1} w \pi d_1^2} = \frac{4 \times 8.34}{984 \times 1.5 \times 3.14\,(1.2 \times 10^{-2})^2} = 50.$$

There are four passes in the heat exchanger and the total number of tubes $n = 4 \times 50 = 200$.

For a first approximation the height of tubes

$$H = \frac{F}{\pi d_{av} n} = \frac{19.3}{3.14 \times 1.3 \times 10^{-2} \times 200} = 2.37 \text{ m}.$$

The surface temperature of tubes

$$t_{w2} = t_s - \frac{q}{\alpha_2} = 106.6 - \frac{135.8 \times 10^3}{6080} = 84.3\ ^\circ\text{C};$$

$$t_{w1} = t_{w2} - \frac{q}{\lambda}\delta = 84.3 - \frac{135.8 \times 10^3}{104.5}\, 1 \times 10^{-3} \approx 83\ ^\circ\text{C}.$$

Since the obtained numerical values of H, t_{w2} and t_{w1} do not coincide with these assumed for the first approximation, the calculation is repeated, assuming $H = 2.4$ m, $t_{w2} = 84.3$ °C and $t_{w1} = 83$ °C. The second calculation gives: $\alpha_1 = 8950$ W/m²·°C, $\alpha_2 = 6030$ W/m²·°C, $k = 3490$ W/m²·°C, $q = 130$ kW/m², $F = 20$ m²; for the second approximation the height of tubes $H = 2.45$ m.

For the second approximation the tube surface temperatures $t_{w2} = 85$°C and $t_{w1} = 83.8$°C. The values obtained coincide with sufficient accuracy with these previously assumed, so we assume finally that $F \doteq 20$ m² and $H = 2.5$ m.

12-16. Calculate the steam-to-water heat exchanger of the design considered in problem 12-15, assuming that heating steam pressure is raised to $p = 226$ kPa, all other conditions remaining unchanged.

Answer

$$G_2 = 1.22 \text{ kg/s};\quad F = 12.1 \text{ m}^2;\quad H = 1.5 \text{ m}.$$

12-17. Determine the heating surface and the length of individual sections of a continuous loop-type boiler economizer designed for heating feedwater in the amount of $G_2 = 230$ t/h from $t'_{f2} = 160$ °C to $t''_{f2} = 300$ °C (Fig. 12-6).

The water is in upward flow in steel tubes ($\lambda_w = 22$ W/m·°C), $d_1/d_2 = 44/51$ mm in diameter, with a mean velocity $w_2 = 0.6$ m/s.

The flue gas (13% CO_2, 11% H_2O) flows downward in the intertubular space with a mean velocity $w_1 = 13$ m/s, measured in the narrowest cross section of the bank of tubes. The rate of gas flow $G_1 = 500$ t/h. At the economizer inlet flue gas temperature $t'_{f1} = 800$°C. The tubes are in a staggered arrangement with a pitch $s_1 = 2.1d$ across the flow and $s_2 = 2d$ along the flow.

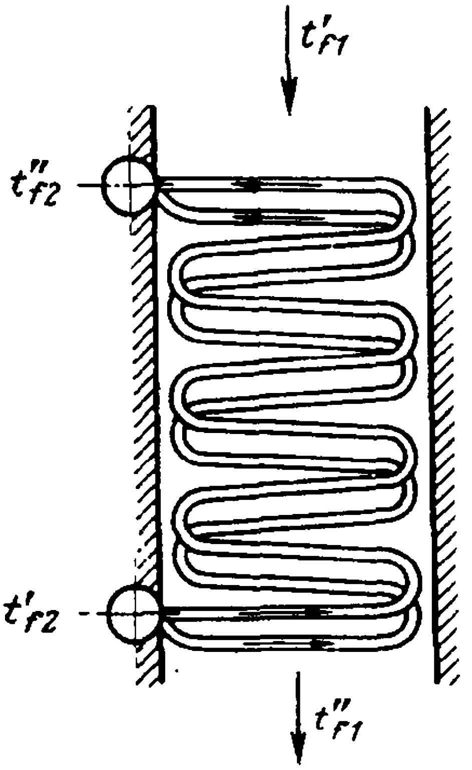

Fig. 12-6. Solution to problem 12-17

Answer

The heating surface $F = 1065$ m^2; the number of loops $n = 86$; the length of the loops $l = 77.5$ m.

Solution

The arithmetic mean water temperature

$$t_{f2} = 0.5\,(t'_{f2} + t''_{f2}) = 0.5\,(160 + 300) = 230\ ^\circ\mathrm{C}.$$

At that temperature the physical properties of water are:

$$\rho_{f2} = 827\ \mathrm{kg/m^3};\quad c_{pf2} = 4.68\ \mathrm{kJ/kg\cdot ^\circ C};$$
$$\lambda_{f2} = 0.637\ \mathrm{W/m\cdot ^\circ C};\quad \nu_{f2} = 0.145 \times 10^{-6}\ \mathrm{m^2/s};$$
$$\mathrm{Pr}_{f2} = 0.88.$$

The amount of heat transferred to feedwater in the economizer

$$Q = G_2 c_{pf2}\,(t''_{f2} - t'_{f2}) = \frac{230 \times 10^3}{3600}\,4.68\,(300 - 160) = 4.2 \times 10^4\ \mathrm{kW}.$$

The Reynolds number for water

$$\mathrm{Re}_{f2} = \frac{w_2 d_1}{\nu_{f2}} = \frac{0.6 \times 4.4 \times 10^{-2}}{0.145 \times 10^{-6}} = 1.82 \times 10^5.$$

The Nusselt number and the heat-transfer coefficient for water are determined by formula (5-7) in which $(\mathrm{Pr}_{f2}/\mathrm{Pr}_{w2})^{0.25} \approx 1$, taking into account the fact that the local heat-transfer coefficient on the water side exceeds considerably that on the flue gas side, consequently, the wall temperature is close to water temperature, assume that

$$\mathrm{Nu}_{f2} = 0.021\mathrm{Re}_{f2}^{0.8}\mathrm{Pr}_{f2}^{0.43} = 0.021\,(1.82 \times 10^{5})^{0.8} \times \times (0.88)^{0.43} = 314;$$

$$\alpha_2 = \mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d_1} = 314\,\frac{0.637}{4.4 \times 10^{-2}} = 4550\ \mathrm{W/m^2 \cdot {}^\circ C}.$$

To determine the temperature of the flue gas at the economizer outlet, assume for a first approximation that the heat capacity of the flue gas $c_{pf1} \approx 1.25$ kJ/kg·°C. Then,

$$t''_{f1} = t'_{f1} - \frac{Q}{G_1 c_{pf1}} = 800 - \frac{4.2 \times 10^{4}}{500 \times 10^{3} \times 1.25} = 558\ {}^\circ\mathrm{C}$$

and

$$t_{f1} = 0.5\,(t'_{f1} + t''_{f1}) = 0.5\,(800 + 558) = 679\ {}^\circ\mathrm{C}.$$

At that temperature $c_{pf1} = 1.234$ kJ/kg·°C, and as a result of a second approximation

$$t''_{f1} = 554\ {}^\circ\mathrm{C} \quad \text{and} \quad t_{f1} = 677\ {}^\circ\mathrm{C}.$$

At a temperature $t_{f1} = 677$°C the physical properties of flue gas of the given composition are as follows:

$$\rho_{f1} = 0.373\ \mathrm{kg/m^3};\quad \lambda_{f1} = 0.0808\ \mathrm{W/m \cdot {}^\circ C};$$
$$\nu_{f1} = 108 \times 10^{-6}\ \mathrm{m^2/s};\quad \mathrm{Pr}_{f1} = 0.61.$$

The Reynolds number for the flue gas

$$\mathrm{Re}_{f1} = \frac{w_1 d_2}{\nu_{f1}} = \frac{13 \times 5.1 \times 10^{-2}}{108 \times 10^{-6}} = 6130.$$

Find the Nusselt number and the coefficient of heat transfer by convection from the flue gas to the walls of tubes.

Since the number of rows of tubes arranged along the flow is not known, calculations are carried out for the third row of tubes. For clean tubes in a staggered arrangement, by formula (6-4),

$$\mathrm{Nu}_{f1} = 0.41\mathrm{Re}_{f1}^{0.6}\mathrm{Pr}_{f1}^{0.33}\varepsilon_s = 0.41\,(6130)^{0.6}\,(0.61)^{0.33} = 64.3,$$

where, since $s_1/s_2 = 1.05$, the correction $\varepsilon_s \approx 1$;

$$\alpha_1' = \mathrm{Nu}_{f1} \frac{\lambda_{f1}}{d_2} = 64.3 \frac{8.08 \times 10^{-2}}{5.1 \times 10^{-2}} = 102\ \mathrm{W/m^2 \cdot ^\circ C}.$$

In service, due to fouling of boiler heating surfaces, the intensity of heat transfer diminishes, which is accounted for by assumed [13]:

$$\alpha_1 = 0.8\alpha_1' = 0.8 \times 102 = 81.6\ \mathrm{W/m^2 \cdot ^\circ C}.$$

Determine now the coefficient of heat transfer by thermal radiation from the flue gas flow to the walls of tubes. The mean beam length

$$l = 1.08 d_2 \left(\frac{s_1 s_2}{d_2^2} - 0.785\right) =$$

$$= 1.08 \times 0.051\,(2 \times 2.1 - 0.785) = 0.188\ \mathrm{m}.$$

The product of the mean beam length by partial pressure of carbon dioxide and water vapour

$$p_{CO_2} l = 0.13 \times 0.188 = 0.0245\ \mathrm{m \cdot atm};$$

$$p_{H_2O} l = 0.11 \times 0.188 = 0.0207\ \mathrm{m \cdot atm}.$$

At the mean gas temperature $t_{f1} = 677\ ^\circ\mathrm{C}$ the emissivity of the flue gas is found from the graphs shown in Figs. 11-1 through 11-3:

$$\varepsilon_g = \varepsilon_{CO_2} + \beta\varepsilon_{H_2O} = 0.072 + 1.08 \times 0.042 = 0.117.$$

Taking into account that $\alpha_1 \ll \alpha_2$, assume $t_{w1} \approx t_{f2} + 20 \approx 250\ ^\circ\mathrm{C}$. At that temperature find from the same graphs the absorptivity of the flue gas at the surface temperature of the tubes:

$$A_g = \varepsilon_{CO_2} \left(\frac{T_{f1}}{T_{w1}}\right)^{0.65} + \beta\varepsilon_{H_2O} =$$

$$= 0.064 \left(\frac{677 + 273}{250 + 273}\right)^{0.65} + 1.08 \times 0.07 = 0.17.$$

The effective emissivity of the enclosure

$$\varepsilon_{w1}' - 0.5\,(\varepsilon_{w1} + 1) = 0.5\,(0.8 + 1) = 0.9.$$

The rate of radiation heat transfer

$$q_{rad} = \varepsilon'_{w1} C_0 \left[\varepsilon_g \left(\frac{T_{f1}}{100}\right)^4 - A_g \left(\frac{T_{w1}}{100}\right)^4\right] =$$

$$= 0.9 \times 5.7 \left[0.117 \left(\frac{677+273}{100}\right)^4 - \right.$$

$$\left. - 0.17 \left(\frac{250+273}{100}\right)^4\right] = 4230 \text{ W/m}^2.$$

The coefficient of heat transfer by radiation

$$\alpha_{rad} = \frac{q_{rad}}{t_{f1} - t_{w1}} = \frac{4230}{677-250} = 9.9 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The total coefficient of heat transfer from the flue gas to the walls of the tubes

$$\alpha_0 = \alpha_1 + \alpha_{rad} = 81.6 + 9.9 = 91.5 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

The overall heat-transfer coefficient

$$k = \frac{1}{\frac{1}{\alpha_0} + \frac{\delta_w}{\lambda_w} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{91.5} + \frac{3.5 \times 10^{-3}}{22} + \frac{1}{4550}} =$$

$$= 88.3 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Find the mean temperature difference, assuming the flue gas and water to be in counterflow:

$$\frac{t'_{f1} - t''_{f2}}{t''_{f1} - t'_{f2}} = \frac{800-300}{554-160} = 1.27 < 1.5.$$

The logarithmic temperature difference

$$\Delta t_{\log} \approx \Delta t_a = t_{f1} - t_{f2} = 677 - 230 = 447 \text{ }^\circ\text{C}.$$

The heating surface of the water economizer

$$F = \frac{Q}{k \Delta t_{\log}} = \frac{4.2 \times 10^4}{88.3 \times 447} 10^3 = 1070 \text{ m}^2.$$

The number of loops connected in parallel

$$n = \frac{4G_2}{\rho_{f2} \pi d_1^2 w_1 3600} = \frac{4 \times 230 \times 10^3}{827 \times 3.14 (4.4 \times 10^{-2})^2 0.6 \times 3600} \approx 86.$$

The length of an individual section (loop)

$$l_1 = \frac{F}{\pi d_2 n} = \frac{1070}{3.14 \times 5.1 \times 10^{-2} \times 86} = 77.5 \text{ m}.$$

12-18. Determine the number and the length of loops of the superheater of a steam boiler rated at 230 t/h, generating steam at a pressure $p = 9.8$ MPa and temperature $t''_{f2} = 510$ °C (Fig. 12-7).

Dry superheated steam flows into the superheater which is built-up of steel tubes having diameters $d_2/d_1 = 32/38$ mm ($\lambda_w = 22$ W/m·°C). Steam m[illegible] in the superheater tubes with a mean velocity $w_2 =$

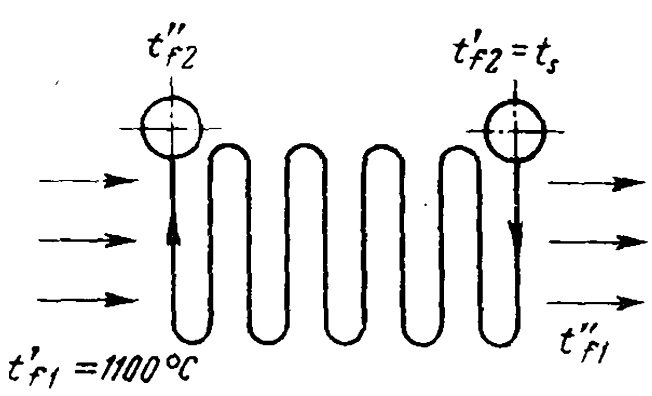

Fig. 12-7. Solution to problem 12-18

The flue gas (13% CO_2, 11% H_2O), in the amount $G_1 = 500$ t/h, is in crossflow through the bank of superheater tubes. The gas inlet temperature $t'_{f1} = 1100$ °C. The mean velocity of the flue gas in the narrow cross section of the bank of tubes $w_1 = 14$ m/s. The superheater tubes are in an in-line arrangement with a transverse pitch $s_1 = 2.3d_2$ and a longitudinal pitch $s_2 = 3d_2$.

The variation on steam pressure along the superheater can be ignored.

Answer

The heating surface of the superheater $F = 764$ m^2; the number of loops $n = 168$; the length of each loop $l_1 = 45.2$ m.

Solution

At the pressure $p = 9.8$ MPa the saturation temperature $t_s = t'_{f2} = 309.5$ °C. At that temperature the enthalpy of the steam [2] $i'_2 = 2728$ kJ/kg. At the superheater outlet steam temperature $t''_{f2} = 510$ °C and its enthalpy $i''_2 = 3401$ kJ/kg, hence, the amount of heat added to the saturated steam in the superheater

$$Q = G_2 (i''_2 - i'_2) = \frac{230 \times 10^3}{3600} (3401 - 2728) = 4.3 \times 10^4 \text{ kW}.$$

The arithmetic mean temperature of steam

$$t_{f2} = 0.5\,(t'_{f2} + t''_{f2}) = 0.5\,(309.5 + 510) \approx 410\ ^\circ\text{C}.$$

At that temperature the physical properties of steam are as follows [25]: $\rho_{f2} = 36.5$ kg/m³, $\lambda_{f2} = 0.0708$ W/m·°C; $\nu_{f2} = 0.704 \times 10^{-6}$ m²/s, $\text{Pr}_{f2} = 1.09$.

The Reynolds number for steam flow

$$\text{Re}_{f2} = \frac{w_2 d_1}{\nu_{f2}} = \frac{17 \times 2.8 \times 10^{-2}}{0.704 \times 10^{-6}} = 6.77 \times 10^5.$$

The Nusselt number and the coefficient of heat transfer from the wall to the steam:

$$\text{Nu}_{f2} = 0.021 \text{Re}_{f2}^{0.8} \text{Pr}_{f2}^{0.43} = 0.021\,(6.77 \times 10^5)^{0.8} \times$$

$$\times (1.09)^{0.43} = 1000;$$

$$\alpha_2 = \text{Nu}_{f2} \frac{\lambda_{f2}}{d_1} = 1000 \frac{0.0708}{0.028} = 2530 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Assuming for a first approximation the heat capacity of the flue gas $c_{pf1} = 1.31$ kJ/kg·°C, find the temperature of the gas at the superheater outlet:

$$t''_{f1} = t'_{f1} - \frac{Q}{G_1 c_{pf1}} = 1100 - \frac{4.3 \times 10^4 \times 3600}{500 \times 10^3 \times 1.31} = 863\ ^\circ\text{C};$$

then, $t_{f1} = 0.5\,(t'_{f1} + t''_{f1}) = 0.5\,(1100 + 863) \approx 982\ ^\circ\text{C}$.

At that temperature $c_{pf1} = 1.303$ kJ/kg·°C and no second approximation is required.

At the temperature $t_{f1} = 982\ ^\circ\text{C}$ the physical properties of the flue gas are as follows:

$$\rho_{f1} = 0.28 \text{ kg/m}^3;\quad \lambda_{f1} = 0.1075 \text{ W/m}\cdot{}^\circ\text{C};$$

$$\nu_{f1} = 170 \times 10^{-6} \text{ m}^2\text{/s};\quad \text{Pr}_{f1} = 0.58.$$

The Reynolds number for the flue gas

$$\text{Re}_{f1} = \frac{w_1 d_2}{\nu_{f1}} = \frac{14 \times 3.2 \times 10^{-2}}{170 \times 10^{-6}} = 2640.$$

Find the Nusselt number and the coefficient of heat transfer by convection from the flue gas to the walls of tubes.

Since the number of rows of tubes along the flow is not known, calculations are carried out for the third row of tubes. For clean tubes in an in-line arrangement by formula (6-4)

$$\mathrm{Nu}_{f1} = 0.26\mathrm{Re}_{f1}^{0.65}\mathrm{Pr}_{f1}^{0.33}\varepsilon_s,$$

where

$$\varepsilon_s = \left(\frac{s_2}{d_2}\right)^{-0.15} = (3)^{-0.15} = 0.85$$

and

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$$\mathrm{Nu}_{f1} = 0.26\,(2640)^{0.65}\,(0.58)^{0.33}\,0.85 = 31;$$

$$\alpha_1' = \mathrm{Nu}_{f1}\frac{\lambda_{f1}}{d_2} = 31\,\frac{0.1075}{0.032} = 104\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Fouling of the heating surface is accounted for by reducing somewhat the heat-transfer coefficient [13]:

$$\alpha_1 = 0.8\alpha_1' = 0.8 \times 104 = 83\ \mathrm{W/m^2\cdot{}^\circ C}.$$

Determine the coefficient of heat transfer by thermal radiation from the flue gas to the walls of tubes.

The mean beam length

$$l = 1.08d_2\left(\frac{s_1 s_2}{d_2^2} - 0.785\right) =$$
$$= 1.08 \times 0.032\,(2.3 \times 3 - 0.785) = 0.212\ \mathrm{m}.$$

The products of the mean beam length by the partial pressure of carbon dioxide and water vapour are as follows:

$$p_{\mathrm{CO_2}}l = 0.13 \times 0.212 = 0.0276\ \mathrm{m\cdot atm};$$
$$p_{\mathrm{H_2O}}l = 0.11 \times 0.212 = 0.0233\ \mathrm{m\cdot atm}.$$

At the mean temperature $t_{f1} = 982\ ^\circ\mathrm{C}$ the emissivity of the flue gas is found from the charts in Figs. 11-1 through 11-3:

$$\varepsilon_g = \varepsilon_{\mathrm{CO_2}} + \beta\varepsilon_{\mathrm{H_2O}} = 0.066 + 1.08 \times 0.037 = 0.106.$$

To calculate the absorptivity of the flue gas, assume a tube surface temperature $t_w \approx t_{f2} + 40 = 450\ ^\circ\mathrm{C}$. At that temperature find from the same graphs that

$$A_g = \varepsilon_{\mathrm{CO_2}}\left(\frac{T_{f1}}{T_w}\right)^{0.65} + \beta\varepsilon_{\mathrm{H_2O}} = 0.068\left(\frac{1255}{733}\right)^{0.65} +$$
$$+ 1.08 \times 0.067 = 0.169.$$

The effective emissivity of the enclosure

$$\varepsilon'_w = 0.5\,(\varepsilon_w + 1) = 0.5\,(0.8 + 1) = 0.9.$$

The rate of heat flow due to thermal radiation

$$q_{rad} = \varepsilon'_w C_0 \left[\varepsilon_g \left(\frac{T_{f1}}{100}\right)^4 - A_g \left(\frac{T_w}{100}\right)^4 \right] =$$

$$= 0.9 \times 5.7 \left[0.106 \left(\frac{1255}{100}\right)^4 - 0.169 \left(\frac{733}{100}\right)^4 \right] =$$

$$= 1.1 \times 10^4 \text{ W/m}^2.$$

The coefficient of heat transfer α_{rad} due to radiation

$$\alpha_{rad} = \frac{g_{rad}}{t_{f1} - t_w} = \frac{1.1 \times 10^4}{982 - 450} = 20.7 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

The summary coefficient of heat transfer from the flue gas to the walls of superheater tubes

$$\alpha_0 = \alpha_{rad} + \alpha_1 = 20.7 + 83 = 103.7 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

The overall heat-transfer coefficient

$$k = \frac{1}{\dfrac{1}{\alpha_0} + \dfrac{\delta_w}{\lambda_w} + \dfrac{1}{\alpha_2}} = \frac{1}{\dfrac{1}{103.7} + \dfrac{2 \times 10^{-3}}{22} + \dfrac{1}{2\,530}} = 98.5 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

The mean temperature difference is determined, assuming the flue gas and steam to be in counterflow. Allowing for

$$\frac{t'_{f1} - t''_{f2}}{t''_{f1} - t'_{f2}} = \frac{1100 - 510}{863 - 309.5} \approx 1.07 < 1.5,$$

it may be assumed that

$$\Delta t_{\log} \approx \Delta t_a = t_{f1} - t_{f2} = 982 - 410 = 572\ {}^\circ\text{C}.$$

The outside surface temperature of tubes

$$t_{w1} = t_{f1} + \frac{k \Delta t}{\alpha_0} = 982 - \frac{98.5 \times 572}{103.7} = 440\ {}^\circ\text{C}.$$

In calculating radiation heat transfer, it was assumed that $t_w = 450\ {}^\circ\text{C}$, and the coincidence between the assumed and calculated temperatures is sufficiently accurate so that no repeated calculation is necessary.

The heating surface of the superheater

$$F = \frac{Q}{k\Delta t_{\log}} = \frac{4.3 \times 10^7}{98.5 \times 572} = 764 \text{ m}^2.$$

The number of loops

$$n = \frac{4G_2}{\rho_{f2}\pi d_1^2 w_2 \times 3600} = \frac{4 \times 230 \times 10^3}{36.5 \times 3.14\, (2.8 \times 10^{-2})^2\, 17 \times 3600} = 168.$$

The length of each loop

$$l_1 = \frac{F}{\pi d_2 n} = \frac{764}{3.14 \times 3.2 \times 10^{-2} \times 168} = 45.2 \text{ m}.$$

12-19. Carbon dioxide, in the amount $G = 0.02$ kg/s and at a pressure $p = 10$ MPa, flows into a round tube having a diameter $d = 4$ mm, passes the section of hydrodynamic stabilization and at a temperature $t_{f1} = 30$ °C enters the heated section of the tube where the carbon dioxide is heated at a constant rate of heat flow $q_w = 8 \times 10^5$ W/m^2. Find the distribution of temperature along the tube for relative distances from the tube inlet $x/d = 2$, 4, 10, 20, 30, 38, 50, 60 and 75.

Answer

Calculation results are tabulated below.

x/d	2	6	10	20	30
t_{fx}, °C	31.2	32.4	35.6	39.9	43.0
α_x, W/m^2·°C	4510	4000	3500	3150	3050
t_{wx}, °C	208	232	264	294	305

x/d	38	50	60	75
t_{fx}, °C	45.1	48.2	51.6	59.0
α_x, W/m^2·°C	3200	3340	3360	3190
t_{wx}, °C	295	288	290	310

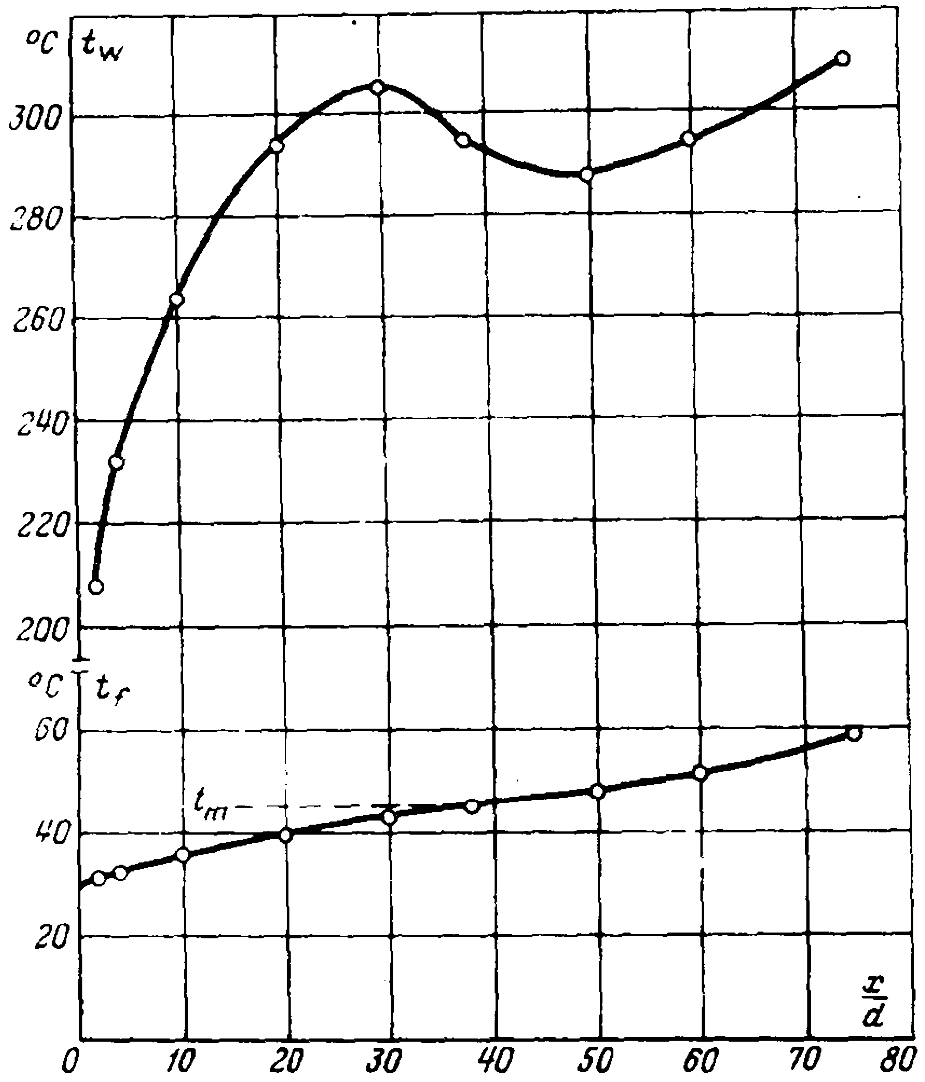

Fig. 12-8. Solution to problem 12-19

The variation of the wall temperature and of the bulk temperature of the carbon dioxide along the tube, and also the change in some characteristic quantities are presented graphically in Figs. 12-8 and 12-9.

Solution

The critical pressure of carbon dioxide $p_{cr} = 7.39$ MPa, hence, the process of heat transfer considered in the problem proceeds in the super critical region of the parameters of state. Since in that region the heat capacity of fluid varies materially with temperature, the variation of the bulk temperature of the carbon dioxide along the tube is determined by the variation in its enthalpy. With q_w = const the

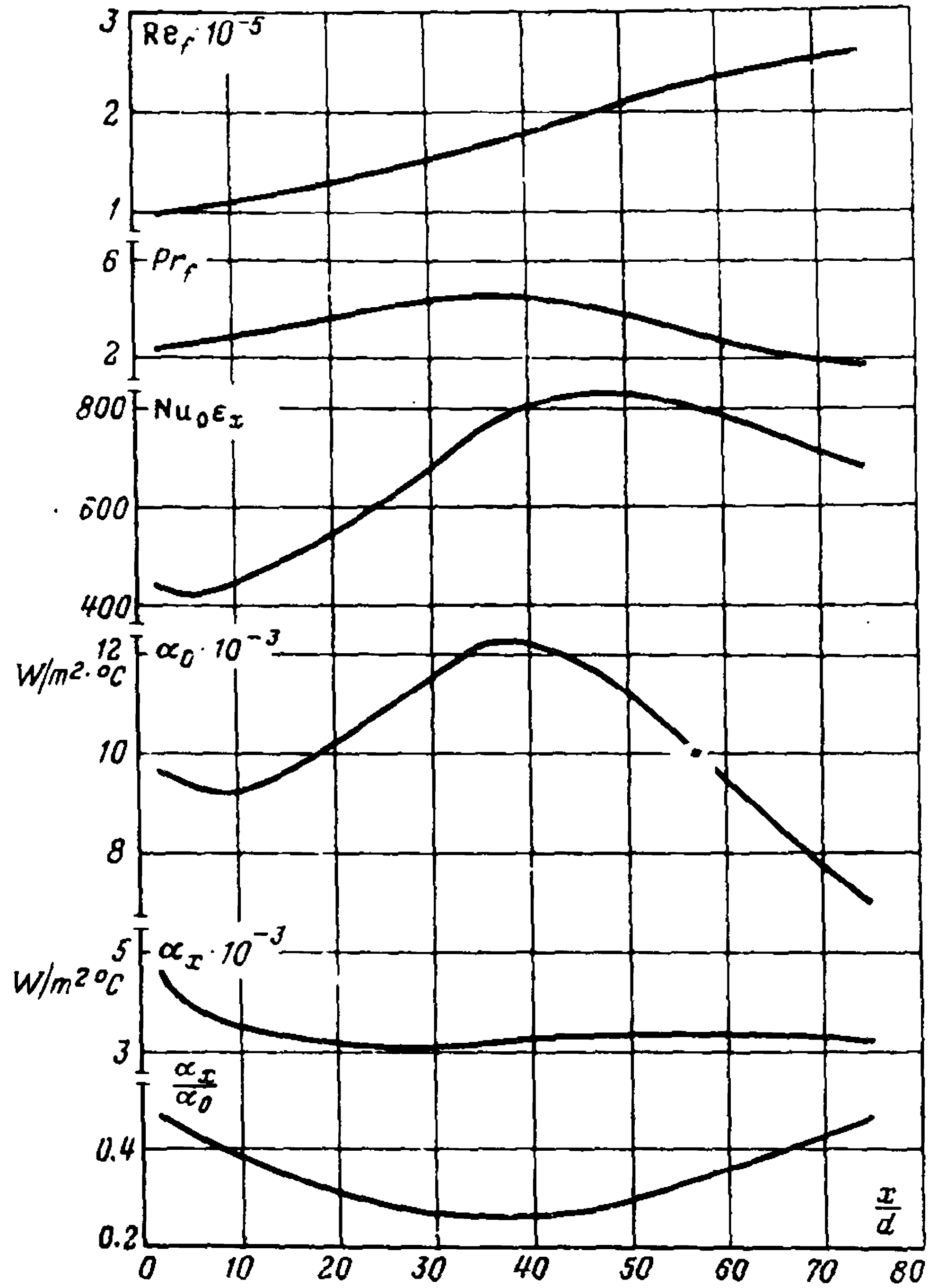

Fig. 12-9. Solution to problem 12-19

change in liquid enthalpy along the tube is linear and

$$i_{fx}=i_{f1}+\frac{q_w\pi d}{G}x.$$

With t_{fx} = 30°C and, accordingly, T_{f1} = 303 K i_{f1} = = 537.4 kJ/kg;

$$i_{fx}=573.4+\frac{8\times10^5\pi4\times10^{-3}}{2\times10^{-2}}x=573.4+502x \text{ kJ/kg}.$$

With $x/d = 2$ $x = 2 \times 4 \times 10^{-3} = 8 \times 10^{-3}$ m and $i_{fx} = 573.4 + 502 \times 8 \times 10^{-3} = 577.4$ kJ/kg.

From tables we find the temperature of carbon dioxide corresponding to this magnitude of enthalpy: $T_{fx} = 304.2$ K, or $t_{fx} = 31.2°C$. By analogy, for other values of x/d, we find:

x/d	2	4	10	20	30	38	50	60	75
T_{fx}, K	304.2	305.4	308.6	312.9	316.0	318.1	321.2	324.6	332.0
t_{fx}, °C	31.2	32.4	35.6	39.9	43.0	45.1	48.2	51.6	59.0

At $p = 10$ MPa the pseudocritical temperature of carbon dioxide $t_m \approx 45°C$. Therefore, near the cross section $x/d = 38$, where $t_{fx} = 45.1°C$, the heat capacity, c_{pf}, and the Prandtl number, Pr_f, are maximum and the other physical properties change drastically.

Tha local Reynolds number

$$\mathrm{Re}_x = \frac{4G}{\pi d \mu_{fx}} = \frac{4 \times 2 \times 10^{-2}}{\pi \times 4 \times 10^{-3} \mu_{fx}} = \frac{6.37}{\mu_{fx}}.$$

Calculate the unknown wall temperature for the cross section $x/d = 2$.

With $x/d = 2$ $T_{fx} = 304.2$ K and the physical properties of carbon dioxide are as follows: $\mu_{fx} = 6.55 \times 10^{-5}$ Pa·s, $\rho_{fx} = 761.1$ kg/m^3, $i_{fx} = 577.4$ kJ/kg, $c_{pfx} = 3.37$ kJ/kg·°C, $\lambda_{fx} = 8.74 \times 10^{-2}$ W/m·°C, $\mathrm{Pr}_{fx} = 2.52$;

$$\mathrm{Re}_x = \frac{6.37}{6.55 \times 10^{-5}} = 9.72 \times 10^4;$$

the pattern of flow is turbulent and calculation formula (5.17) is used:

$$\mathrm{Nu}_f = \mathrm{Nu}_0 \left(\frac{\bar{c}_p}{c_{pf}}\right)^n \left(\frac{\rho_w}{\rho_f}\right)^m \varepsilon_x,$$

where Nu_0, $\bar{c}_p$, n, m and ε_x are determined by the dependences given in Chapter 5 for formulae (5-15) and (5.17).

For stabilized heat transfer and constant physical properties, at $p \ll p_{cr}$, the Nusselt number

$$\mathrm{Nu}_0 = \frac{\frac{\xi}{8}\mathrm{Re}_f \mathrm{Pr}_f}{12.7\sqrt{\frac{\xi}{8}}(\mathrm{Pr}_f^{2/3}-1)+1.07};$$

$$\xi = (1.82 \log \mathrm{Re}_f - 1.64)^{-2} =$$

$$= [1.82 \log (9.72 \times 10^4) - 1.64^{-2}] = 1.81 \times 10^{-2};$$

$$\xi/8 = 2.26 \times 10^{-3}; \quad \sqrt{\xi/8} = 4.76 \times 10^{-2};$$

$$\mathrm{Nu}_0 = \frac{2.26 \times 10^{-3} \times 9.72 \times 10^4 \times 2.52}{12.7 \times 4.76 \times 10^{-2}(2.52^{2/3}-1)+1.07} = 348.$$

Correction for the initial section

$$\varepsilon_x = 0.86 + 0.54\left(\frac{d}{x}\right)^{0.4} = 0.86 + 0.54\left(\frac{1}{2}\right)^{0.4} = 1.27;$$

$$\mathrm{Nu}_0\varepsilon_x = 348 \times 1.27 = 442;$$

$$\alpha_0 = \mathrm{Nu}_0\varepsilon_x \frac{\lambda_{fx}}{d} = 442\,\frac{8.74 \times 10^{-2}}{4 \times 10^{-3}} = 9650 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

To determine the wall temperature, the local heat-transfer coefficient must be known, which, according to formula (5-17), depends on the unknown wall temperature. The calculation is. therefore, carried out applying the method of successive approximations, solving together equation (5-17) and the expression

$$t_{wx} = t_{fx} + \frac{q_{wx}}{\alpha_x}. \tag{a}$$

Considering that heat transfer is affected by the variation of the physical properties of carbon dioxide over the cross section of flow, which in formula (5-17) is accounted for by the factor

$$\varphi = \left(\frac{\bar{c}_p}{c_{pf}}\right)^n \left(\frac{\rho_w}{\rho_f}\right)^m,$$

resulting in a considerable reduction in the intensity of heat transfer, assume for a first approximation that $\varphi = 0.5$.

Then

$$\mathrm{Nu}_x = \mathrm{Nu}_0 \varepsilon_x \varphi = 442 \times 0.5 = 221;$$

$$\alpha_x = \mathrm{Nu}_x \frac{\lambda_{fx}}{d} = 221 \frac{8.47 \times 10^{-2}}{4 \times 10^{-3}} \approx 4820 \text{ W/m}^2\cdot{}^\circ\text{C}$$

and for the first approximation, by formula (a), the wall temperature

$$t_{wx} = 31.2 + \frac{8 \times 10^5}{4820} = 31.2 + 166 \approx 197\,^\circ\text{C}.$$

With $t_{wx} = 197^\circ\text{C}$, $T_{wx} = 470$ K, $\rho_w = 124.2$ kg/m^3, $i_w = 930.3$ kJ/kg.

The value of φ for a second approximation

$$\bar{c}_p = \frac{i_w - i_{fx}}{t_{wx} - t_{fx}} = \frac{930.3 - 577.4}{197 - 31.2} = 2.13 \text{ kJ/kg}\cdot{}^\circ\text{C}.$$

$$\frac{\bar{c}_p}{c_{pf}} = \frac{2.13}{3.37} = 0.632.$$

The pseudocritical temperature $T_m = 318$ K and in the cross section considered

$$\frac{T_f}{T_m} < 1, \text{ and } \frac{T_w}{T_m} = \frac{470}{318} = 1.48$$

and the exponent n is determined by formula

$$n = 0.22 + 0.18 \frac{T_w}{T_m} = 0.22 + 0.18 \times 1.48 = 0.486;$$

$$\frac{\rho_w}{\rho_f} = \frac{124.2}{761.1} = 0.163;$$

$$m = 0.35 - 0.05 \frac{p}{p_{cr}} = 0.35 - 0.05 \frac{10}{7.39} = 0.282;$$

$$\varphi = (0.632)^{0.486} (0.163)^{0.282} = 0.48.$$

For the second approximation the Nusselt number Nu_x, α_x and the wall temperature are as follows:

$$\mathrm{Nu}_x = 442 \times 0.48 = 212;$$

$$\alpha_x = \alpha_0 \varphi = 9650 \times 0.48 = 4640 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$t_{wx} = 31.2 + \frac{8 \times 10^5}{4640} = 31.2 + 172.4 \approx 204^\circ\text{C}.$$

If a wall temperature $t_{wx} = 204$ °C is assumed for a third approximation, then α_x shall reduce somewhat and t_{wx} increase. Therefore, a wall temperature, somewhat exceeding 204 °C, is assumed, namely $t_{wx} = 208$ °C.

At a wall temperature $t_{wx} = 208°\text{C}$ ($T_{wx} = 481\text{K}$) $\rho_w = 120.2\ \text{kg/m}^3$, $i_w = 943.3\ \text{kJ/kg}$ and as a result of the third approximation $\bar{c}_p = 2.07\ \text{kJ/kg}\cdot°\text{C}$, $\bar{c}_p/c_{pf} = 0.613$, $T_w/T_m = 1.51$, $n = 0.493$, $\rho_w/\rho_{fx} = 0.158$, $m = 0.282$ and

$$\varphi = (0.613)^{0.493}\,(0.158)^{0.282} = 0.468;$$

$$\text{Nu}_x = 442 \times 0.468 = 207;$$

$$\alpha_x = 9650 \times 0.468 = 4520\ \text{W/m}^2\cdot°\text{C};$$

$$t_{wx} = 31.2\frac{8\times10^5}{4520} = 208.2\ °\text{C}.$$

There is sufficiently good agreement between the value of wall temperature assumed and that obtained as a result of the third approximation, and no further calculations are required. Thus, with a relative distance $x/d = 2$ the wall temperature $t_{wx} = 208\ °\text{C}$.

The second calculation is made for the cross section $x/d = 50$.

With $x/d = 50$ $t_{fx} = 48.2\ °\text{C}$, $T_{fx} = 321.2\ \text{K}$, and the physical properties of carbon dioxide are as follows:

$$\mu_{fx} = 3.07\times10^{-5}\ \text{Pa}\cdot\text{s};\ \rho_{fx} = 422.1\ \text{kg/m}^3;\ i_{fx} = 673.8\ \text{kJ/kg};$$

$$c_{pfx} = 6.98\ \text{kJ/kg}\cdot°\text{C};\quad \lambda_{fx} = 5.42\times10^{-2}\ \text{W/m}\cdot°\text{C};$$

$$\text{Pr}_{fx} = 3.96.$$

The local Reynolds number

$$\text{Re}_x = \frac{6.37}{\mu_{fx}} = \frac{6.37}{3.07\times10^{-5}} = 2.08\times10^5.$$

The friction factor ξ, the Nusselt number Nu_0 and α_0:

$$\xi = [1.82\log(2.08\times10^5) - 1.64]^{-2} = 1.55\times10^{-2};$$

$$\xi/8 = 1.94\times10^{-3};\quad \sqrt{\xi/8} = 4.4\times10^{-2};$$

$$\text{Nu}_0 = \frac{1.94\times10^{-3}\times2.08\times10^5\times3.96}{12.7\times4.4\times10^{-2}\,(3.96^{2/3}-1)+1.07} = 835.$$

Since $x/d > 20$, then $\varepsilon_x = 1$ and

$$\alpha_0 = \text{Nu}_0\frac{\lambda_{fx}}{d} = 835\frac{5.42\times10^{-2}}{4\times10^{-3}} = 11\,300\ \text{W/m}^2\cdot°\text{C}.$$

Taking into consideration that in a cross section more removed from the inlet t_{wx} will be greater than at the relative distance $x/d = 2$ and, consequently, the effect of the variable properties of carbon dioxide on heat transfer results in a greater reduction of the rate of heat transfer, assume as a first approximation that $\varphi = 0.3$, then

$$\mathrm{Nu}_x = \mathrm{Nu}_0\varphi = 835 \times 0.3 = 250.5;$$

$$\alpha_x = \alpha_0\varphi = 11\,300 \times 0.3 = 3390 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$t_{wx} = t_{fx} + \frac{q_w}{\alpha_x} = 48.2 + \frac{8 \times 10^5}{3390} \approx 284\ {}^\circ\text{C}.$$

With $t_{wx} = 284$ °C ($T_{wx} = 557$ K) $\rho_w = 98.5$ kg/m^3; $i_w = 1032$ kJ/kg.

For a second approximation the magnitude of φ

$$\bar{c}_p = \frac{i_w - i_{fx}}{t_{wx} - t_{fx}} = \frac{1032 - 673.8}{284 - 48.2} = 1.52 \text{ kJ/kg}\cdot{}^\circ\text{C};$$

$$\frac{\bar{c}_p}{c_{pf}} = \frac{1.52}{6.98} = 0.218;$$

$$\frac{T_f}{T_m} = \frac{321.2}{318} \approx 1.01; \quad \frac{T_w}{T_m} = \frac{557}{318} = 1.75.$$

Since $1 < T_f / T_m < 1.2$ and $1 < T_w/T_m < 2.6$, then by formula (5-17)

$n = n_1 + (5n_1 - 2)(1 - T_f/T_m)$, where

$$n_1 = 0.22 + 0.18\left(\frac{T_w}{T_m}\right) = 0.22 + 0.18 \times 1.75 = 0.536$$

and

$$n = 0.536 + (5 \times 0.536 - 2)(1 - 1.01) = 0.53.$$

$\frac{\rho_w}{\rho_f} = \frac{98.5}{422.1} = 0.233.$ Just as before $p = 1 \times 10^{\cdot}$ Pa

$m = 0.282$ and

$$\varphi = \left(\frac{\bar{c}_p}{c_{pf}}\right)^n \left(\frac{\rho_w}{\rho_f}\right)^m = (0.218)^{0.53}(0.233)^{0.282} = 0.297.$$

For the second approximation Nu_x, α_x and t_{wx} are:

$$\mathrm{Nu}_x = \mathrm{Nu}_0\varphi = 835 \times 0.297 = 248;$$

$$\alpha_x = \alpha_0\varphi = 11\,300 \times 0.297 = 3350 \ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$t_{wx} = 48.2 + \frac{8 \times 10^5}{3350} = 48.2 + 239 = 287.2\ {}^\circ\mathrm{C}.$$

Since the temperature obtained, t_{wx}, differs from that assumed, calculations should be continued. For a third approximation assume a temperature $t_{wx} = 287°\mathrm{C}$ and repeat the calculation. At $t_{wx} = 287°\mathrm{C}$ $T_{wx} = 560$ K, $\rho_w = 97.9$ kg/m^3, $i_w = 1035.6$ kJ/kg.

Mean heat capacity $\bar{c}_p = 1.515$ kJ/kg·°C; $\bar{c}_p/c_{pf} = 0.217$;

$T_w/T_m = 1.76$; $n = 0.53$; $\rho_w/\rho_f = 0.232$ and

$$\varphi = (0.127)^{0.53}\,(0.232)^{0.282} = 0.295.$$

$$\mathrm{Nu}_x = 835 \times 0.295 = 247;$$

$$\alpha_x = 11\,300 \times 0.295 = 3340 \ \mathrm{W/m^2 \cdot {}^\circ C};$$

$$t_{wx} = 48.2 + \frac{8 \times 10^5}{3340} = 48.2 + 239.5 = 287.7\ {}^\circ\mathrm{C}.$$

The obtained value of t_{wx} agrees sufficiently well with the temperature assumed, so that it may be taken that at $x/d = 50$ $t_{wx} = 288$ °C.

The wall temperature at other cross-sections along the tube is calculated in a similar manner, and calculation results are tabulated on page 337 and presented graphically in Figs. 12-8 and 12-9.

12-20. A round tube having a diameter $d = 4$ mm carries carbon dioxide at a pressure $p = 10$ MPa and a rate of flow $G = 0.03$ kg/s. The carbon dioxide enters the heated section of the tube at a temperature $t_{f1} = 10°\mathrm{C}$ and is heated at a constant rate of heat flow through the wall, $q_w = 1.3 \times$

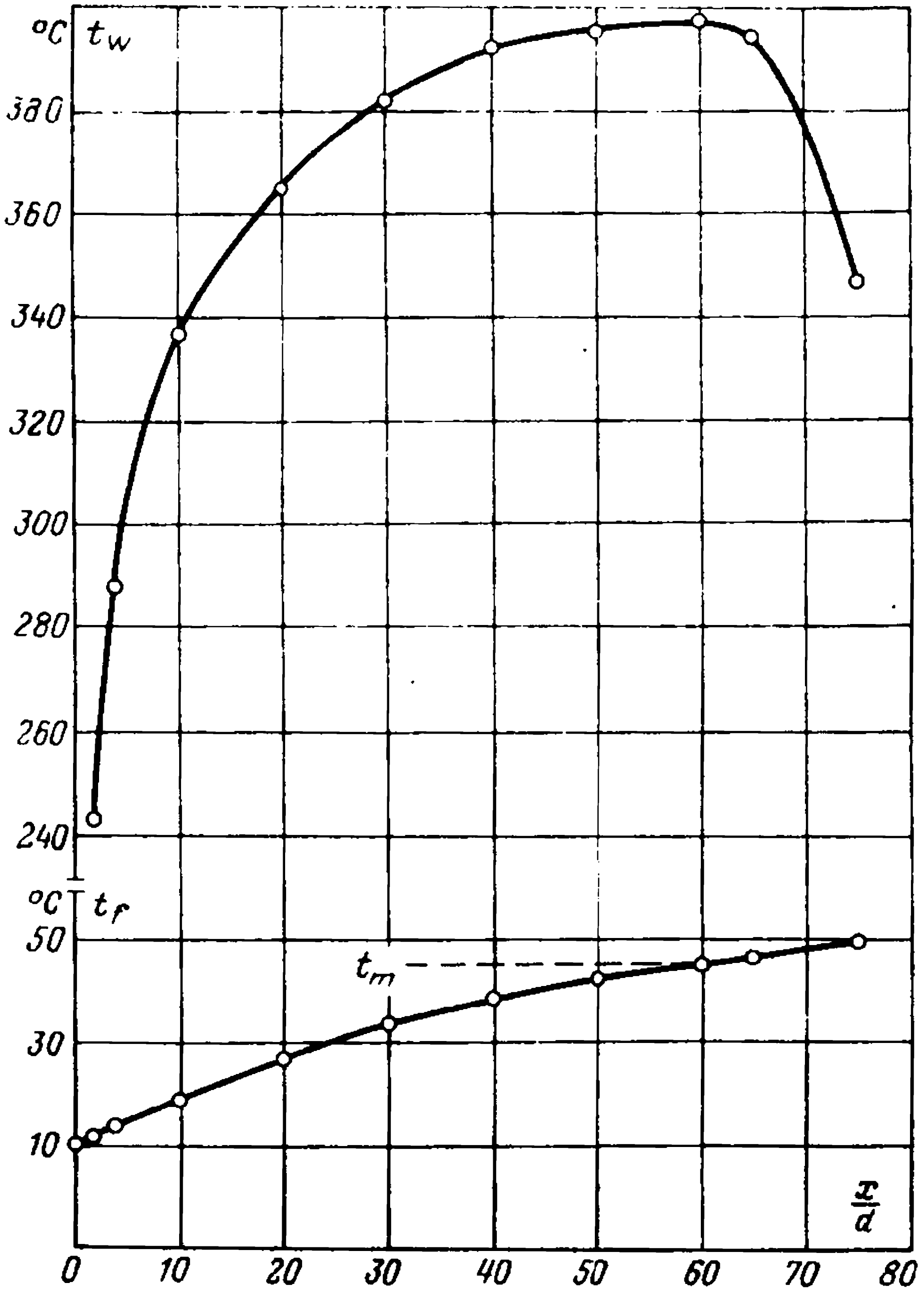

Fig. 12-10. Solution to problem 12-20

$\times\ 10^6$ W/m^2. The heated section of the tube is preceded by a non-heated section of hydrodynamic stabilization.

Find the distribution of temperature along the tube, assuming the following relative distances from the tube inlet $x/d = 2, 4, 10, 20, 30, 40, 50, 60, 65$ and 75.

Answer

Calculation results are presented graphically in Fig. 12-10 and tabulated below.

	x/d									
	2	4	10	20	30	40	50	60	65	75
t_f, °C	11.8	13.6	18.9	26.9	33.5	38.7	42.4	45.2	46.6	49.7
α_x/α_0	0.504	0.462	0.419	0.363	0.332	0.276	0.230	0.212	0.233	0.29
α_x, W/m²·°C	5630	4730	4080	3840	3730	3670	3680	3680	3730	4360
t_w, °C	243	288	337	365	382	393	396	398	395	347

12-21. Find the distribution of temperature over the cross section of a cylindrical uranium ($\lambda = 31$ W/m·°C) fuel element having an inside diameter $d_1 = 14$ mm and outside diameter $d_2 = 28$ mm. The inside and outside surfaces of the cylindrical fuel element are covered with tightly fitting claddings made from stainless steel ($\lambda_{cl} = 21$ W/m·°C) of thickness $\delta = 0.5$ mm. The rate of heat release in the uranium fuel element is assumed to be uniform over the cross section and equal to $q_v = 2 \times 10^8$ W/m³.

The coolant used in the fuel element is water moving along an inner circular channel and an outer annular channel with an outside diameter $d_3 = 34$ mm. The bulk temperature of water and its rate of flow in the inner channel are $t_{f1} = 180$ °C and $G_1 = 0.18$ kg/s, and in the annular channel—$t_{f2} = 200$ °C and $G_2 = 0.30$ kg/s.

It is to be assumed that there is no heat transfer through the outside surface of the circular channel, i.e. $q_3 = 0$.

Answer

The maximum temperature $t_0 = 307$ °C; uranium surface temperatures $t_1 = 268$ °C, $t_2 = 265$ °C. The drops in tem-

	x/d								
	2	4	10	20	30	38	50	60	75
i_f, kJ/kg	577.4	581.4	593.5	613.6	633.6	649.8	673.8	693.9	724.0
T_f, K	304.2	305.4	308.6	312.9	316.0	318.1	321.2	324.6	332.0
t_f, °C	31.2	32.4	35.6	39.9	43.0	45.1	48.2	51.6	59.0
$\mu_f \times 10^5$, Pa·s	6.55	6.34	5.77	4.88	4.13	3.63	3.07	2.74	2.44
ρ_f, kg/m³	761.1	747.8	707.3	634.9	561	501.2	422.1	364.2	296.9
c_{pf}, kJ/kg·°C	3.37	3.53	4.08	5.51	7.28	8.04	6.98	5.19	3.21
$\lambda_f \times 10^2$, W/m·°C	8.74	8.59	8.17	7.48	6.81	6.23	5.42	4.78	4.05
Pr_f	2.52	2.60	2.88	3.59	4.42	4.68	3.96	2.98	1.94
$\mathrm{Re}_f \times 10^{-5}$	0.972	1.01	1.105	1.30	1.54	1.76	2.08	2.32	2.61
Nu_{0e_x}	442	428	448	546	677	787	835	795	685
ψ	0.468	0.435	0.382	0.308	0.265	0.260	0.295	0.353	0.460
Nu_x	207	186.5	171	168.5	179.5	205	247	281	315
α_x, W/m²·°C	4510	4000	3500	3150	3050	3200	3340	3360	3190
t_{wx}, °C	208	232	264	294	305	295	288	290	310

perature across the claddings $t_1 - t_{w1} = 20\ ^\circ\mathrm{C}$, $t_2 - t_{w2} = 15\ ^\circ\mathrm{C}$.

Solution

The coefficient of heat transfer from the cladding to the cooling water in the inner channel.

The physical properties of water at $t_{f1} = 180\ ^\circ\mathrm{C}$: $\rho_{f1} = 886.9\ \mathrm{kg/m^3}$, $\lambda_{f1} = 0.674\ \mathrm{W/m\cdot ^\circ C}$, $\nu_{f1} = 0.173 \times 10^{-6}\ \mathrm{m^2/s}$, $\mathrm{Pr}_{f1} = 1.0$.

The cross-sectional area of the inner channel

$$f_1 = \pi (r_1 - \delta)^2 = \pi (6.5 \times 10^{-3})^2 = 132.5 \times 10^{-6}\ \mathrm{m}^2,$$

where

$$r_1 - \delta = \frac{d_1}{2} - \delta = \frac{14 \times 10^{-3}}{2} - 0.5 \times 10^{-3} = 6.5 \times 10^{-3}\ \mathrm{m}.$$

The velocity of water flow in the channel

$$w_1 = \frac{G_1}{f_1 \rho_{f1}} = \frac{0.18}{132.5 \times 10^{-6} \times 886.9} = 1.535\ \mathrm{m/s}.$$

The Reynolds number

$$\mathrm{Re}_{f1} = \frac{w_1 (d_1 - 2\delta)}{\nu_{f1}} = \frac{1.535\,(14 - 2 \times 0.5)\,10^{-3}}{0.173 \times 10^{-6}} = 1.15 \times 10^5.$$

The Nusselt number is determined by formula (5-7):

$$\mathrm{Nu}_{f1} = 0.021 \mathrm{Re}_{f1}^{0.8} \mathrm{Pr}_{f1}^{0.43} \left(\frac{\mathrm{Pr}_{f1}}{\mathrm{Pr}_{w1}}\right)^{0.25} = 0.021\,(1.15 \times 10^5)^{0.8} \times \times 1.0 = 236,$$

where it is assumed for a first approximation that the correction for the variation in physical properties over the cross section of flow $(\mathrm{Pr}_{f1}/\mathrm{Pr}_{w1})^{0.25} \approx 1$.

$$\alpha_1 = \mathrm{Nu}_{f1} \frac{\lambda_{f1}}{(d_1 - 2\delta)} = 236\, \frac{0.674}{13 \times 10^{-3}} = 1.22 \times 10^4\ \mathrm{W/m^2 \cdot ^\circ C}.$$

The coefficient of heat transfer from the surface of the cladding to the cooling water in the outer annular channel.

With $t_{f2} = 200$ °C $\rho_{f2} = 863$ kg/m^3; $\lambda_{f2} = 0.663$ W/m·°C;

$$\nu_{f2} = 0.158 \times 10^{-6}\ \mathrm{m^2/s};\quad \mathrm{Pr}_{f2} = 0.93;$$

$$f_2 = \pi\,[r_3^2 - (r_2 + \delta)^2] =$$

$$= \pi\,(17^2 - 14.5^2)\,10^{-6} = 248 \times 10^{-6}\ \mathrm{m^2};$$

$$w_2 = \frac{G_2}{f_2\rho_{f2}} = \frac{0.3}{248 \times 10^{-6} \times 863} = 1.4\ \mathrm{m/s};$$

$$\mathrm{Re}_{f2} = \frac{w_2 d_e}{\nu_{f2}} = \frac{1.4 \times 5 \times 10^{-3}}{0.158 \times 10^{-6}} = 4.42 \times 10^4,$$

where

$$d_e = \frac{4f_2}{\pi\,[d_3 + (d_2 + 2\delta)]} = d_3 - (d_2 + 2\delta) = 34 - 29 = 5 \times 10^{-3}\ \mathrm{m}.$$

Since heat is transferred only from the inside surface of the annular channel, the Nusselt number is determined by formula (5-24) suitable for the case of one-side heating

$$\mathrm{Nu}_{1,1} = \mathrm{Nu}_t\,(1 - \varphi)\left(\frac{d_1}{d_2}\right)^n \xi,$$

where, replacing subscripts in accordance with the problem considered

$$\mathrm{Nu}_{1,1} = \mathrm{Nu}_{f2};\quad \mathrm{Nu}_t = \mathrm{Nu}_{2t};$$

$$\frac{d_1}{d_2} = \frac{d_2 + 2\delta}{d_3} = \frac{28 + 1}{34} = 0.853;$$

$$\varphi = \frac{0.255}{1 + 1.1\mathrm{Pr}_{f2}^{0.9}} = \frac{0.255}{1 + 1.1 \times 1.0} = 0.121;$$

$$n = -0.16\mathrm{Pr}_{f2}^{-0.15} = -0.16$$

and since $(d_2 + 2\delta)/d_3 > 0.2$, then $\xi = 1$.

Calculated by the formula (5-7) derived for a round tube, the Nusselt number in the annular channel

$$\mathrm{Nu}_{2t} = 0.021\mathrm{Re}_{f2}^{0.8}\mathrm{Pr}_{f2}^{0.43}\left(\frac{\mathrm{Pr}_{f2}}{\mathrm{Pr}_{w2}}\right)^{0.25} =$$

$$= 0.021\,(4.42 \times 10^4)^{0.08}\,(0.93)^{0.43} = 102,$$

where it was assumed for the first approximation that $(\mathrm{Pr}_{f2}/\mathrm{Pr}_{w2})^{0.25} = 1$.

$$\mathrm{Nu}_{f2} = 102\,(1 - 0.121)\,(0.853)^{-0.16} = 92;$$

$$\alpha_2 = \mathrm{Nu}_{f2}\frac{\lambda_{f2}}{d_e} = 92\frac{0.663}{5\times10^{-3}} = 1.22\times10^4\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The effective heat-transfer coefficients, accounting for the thermal resistance of the claddings

$$\frac{1}{\alpha_{cf1}} = \frac{d_1}{\alpha_1(d_1-2\delta)} + \frac{d_1}{2\lambda_{cl}}\ln\frac{d_1}{d_1-2\delta} =$$

$$= \frac{14\times10^{-3}}{1.22\times10^4\times13\times10^{-3}} + \frac{14\times10^{-3}}{2\cdot21}\ln\frac{14}{13} = 11.24\times10^{-5};$$

$$\alpha_{ef1} = 8880\ \mathrm{W/m^2\cdot{}^\circ C}.$$

$$\frac{1}{\alpha_{ef2}} = \frac{d_2}{\alpha_2(d_2+2\delta)} + \frac{d_2}{2\lambda_{cl}}\ln\frac{d_2+2\delta}{d_2} =$$

$$= \frac{28\times10^{-3}}{12.2\times10^3\times29\times10^{-3}} + \frac{28\times10^{-3}}{2.21}\ln\frac{29}{28} = 10.35\times10^{-5};$$

$$\alpha_{ef2} = 9680\ \mathrm{W/m^2\cdot{}^\circ C}.$$

The neutral section radius

$$r_0^2 = \frac{(t_{f2}-t_{f1}) + \frac{q_v}{2}\left[\frac{r_1}{\alpha_{ef1}} + \frac{r_2}{\alpha_{ef2}} + \frac{1}{2\lambda}(r_2^2 - r_1^2)\right]}{\frac{q_v}{2}\left(\frac{1}{\alpha_{ef1}r_1} + \frac{1}{\alpha_{ef2}r_2} + \frac{1}{\lambda}\ln\frac{r_2}{r_1}\right)} =$$

$$= \frac{(200-180) + \frac{2\times10^3}{2}\left[\frac{7\times10^{-3}}{8880} + \frac{14\times10^{-3}}{9680} + \frac{1}{2\times31}(14^2-7^2)10^{-6}\right]}{\frac{2\times10^8}{2}\left(\frac{1}{8880\times7\times10^{-3}} + \frac{1}{9680\times14\times10^{-3}} + \frac{1}{31}\ln\frac{14}{17}\right)}$$

$$= 105\times10^{-6}\ \mathrm{m^2};$$

$$r_0 = 10.25\ \mathrm{mm}.$$

The rate of heat flow through the inside surface of the urnaium fuel element and through the inside surface of the cladding

$$q_1=\frac{q_v r_1}{2}\left(\frac{r_0^2}{r_1^2}-1\right)=\frac{2\times10^8\times7\times10^{-3}}{2}\left(\frac{105}{7^2}-1\right)=$$

$$=7.98\times10^5\ \mathrm{W/m^2};$$

$$q_{w1}=q_1\frac{d_1}{d_1-2\delta}=7.98\times10^5\,\frac{14}{13}=8.60\times10^5\ \mathrm{W/m^2}.$$

The inside surface temperature of the cladding

$$t_{w1}=t_{f1}+\frac{q_{w1}}{\alpha_1}=180+\frac{8.6\times10^5}{1.22\times10^4}=180+70.5\approx251\ ^\circ\mathrm{C}.$$

The rates of heat flow q_2 and q_{w2} and the outside surface temperature of the uranium fuel element are as follows:

$$q_2=\frac{q_v r_2}{2}\left(1-\frac{r_0^2}{r_2^2}\right)=\frac{2\times10^8\times14\times10^{-3}}{2}\left(1-\frac{105}{14^2}\right)=$$

$$=6.5\times10^5\ \mathrm{W/m^2}.$$

$$q_{w2}=q_2\frac{d_2}{d_2+2\delta}=6.5\times10^5\,\frac{28}{29}=6.28\times10^5\ \mathrm{W/m^2};$$

$$t_{w2}=t_{f2}+\frac{q_{w2}}{\alpha_2}=200+\frac{6.28\times10^5}{1.22\times10^4}=200+51.4\approx251\ ^\circ\mathrm{C}.$$

The found values of t_{w1} and t_{w2} can be used to refine the corrections for the variation in the physical properties over the cross section of flow in the calculation formulae for the heat-transfer coefficients. With $t_{w1}=251\ ^\circ\mathrm{C}$ $\mathrm{Pr}_{w1}=0.86$, and with $t_{w2}=254\ ^\circ\mathrm{C}$ $\mathrm{Pr}_{w2}=0.865$, then, the corrections

$$\left(\frac{\mathrm{Pr}_{f1}}{\mathrm{Pr}_{w1}}\right)^{0.25}=\left(\frac{1}{0.86}\right)^{0.25}=1.04.$$

$$\left(\frac{\mathrm{Pr}_{f2}}{\mathrm{Pr}_{w2}}\right)^{0.25}=\left(\frac{0.93}{0.86}\right)^{0.25}=1.02.$$

Although the corrections are small, nevertheless they tell on temperature distribution. Recalculating, with account taken of the corrections, we obtain:

$$\alpha_1 = 1.27 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad \alpha_{ef1} = 9170 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$\text{Nu}_{2t} = 104; \quad \alpha_2 = 1.245 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C};$$

$$\alpha_{ef2} = 9850 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad r_0 = 10.26 \text{ mm};$$

$$q_1 = 8.04 \times 10^5 \text{ W/m}^2; \quad q_{w1} = 8.65 \times 10^5 \text{ W/m}^2;$$

$$t_{w1} = 248\ {}^\circ\text{C};$$

$$q_2 = 6.48 \times 10^5 \text{ W/m}^2; \quad q_{w2} = 6.25 \times 10^5 \text{ W/m}^2;$$

$$t_{w2} = 250\ {}^\circ\text{C}.$$

Since the temperatures changed but slightly, no further recalculations are required.

The inside and outside surface temperatures of uranium

$$ql_1 = q_1\pi d_1 = 8.04 \times 10^5 \times \pi \times 14 \times 10^{-3} =$$

$$= 3.53 \times 10^4 \text{ W/m};$$

$$t_1 = t_{w1} + \frac{q_{l1}}{2\pi\lambda_{cl}} \ln \frac{d_1}{d_1 - 2\delta} =$$

$$= 248 + \frac{3.53 \times 10^4}{2\pi \times 21} \ln \frac{14}{13} = 248 + 20 = 268\ {}^\circ\text{C};$$

$$t_2 = t_{w2} + \frac{q_2 d_2}{2\lambda_{cl}} \ln \frac{d_2 + 2\delta}{d_2} =$$

$$= 250 + \frac{6.48 \times 10^5 \times 28 \times 10^{-3}}{2 \times 21} \ln \frac{29}{28} = 250 + 14.9 \approx 265\ {}^\circ\text{C}.$$

The maximum temperature in the fuel element

$$t_0 = t_1 \frac{q_v}{4\lambda} \left[2r_0^2 \ln \frac{r_0}{r_1} - (r_0^2 - r_1^2)\right] =$$

$$= 268 + \frac{2 \times 10^8}{4 \times 31} \left[2\,(10.26)^2 \ln \frac{10.26}{7} - (10.26^2 - 7^2)\right] 10^{-6} =$$

$$= 268 + 39.2 \approx 307\ {}^\circ\text{C}.$$

12-22. Determine the distribution of temperature over the cross section of the fuel element considered in problem 12-21, if the rate of water flow in the inner channel is halved, i.e. $G_1 = 0.09$ kg/s, and all other conditions remain unchanged.

Answer

The maximum temperature $t_0 = 327$ °C; uranium surface temperatures $t_1 = 298$ °C, $t_2 = 274$ °C. The temperature drops across the claddings $t_1 - t_{w1} = 16$ °C, $t_2 - t_{w2} = 17$°C. Thus, the general temperature level increased at the expense of a smaller heat-transfer coefficient on the inside wall, with temperature t_1 increasing by 30 °C and temperature t_2—by 9 °C.

12-23. Determine the distribution of temperature along the outer and inner channels of a fuel element with two coolant passes (of the Field's tube type). Water flows downward into the outer annular channel, turns at the end of it and flows upward in the inner annulus, then leaves the fuel element at the top.

The main dimensions of the fuel element (Fig. 12-11) are: length of each pass $l = 2.7$ m; diameters of the inner annular channel $d_1 = 14$ mm, $d_2 = 20$ mm; diameters of the outer annular channel $d_3 = 22$ mm, $d_4 = 28$ mm; the outside diameter of the fuel element $d_5 = 34$ mm.

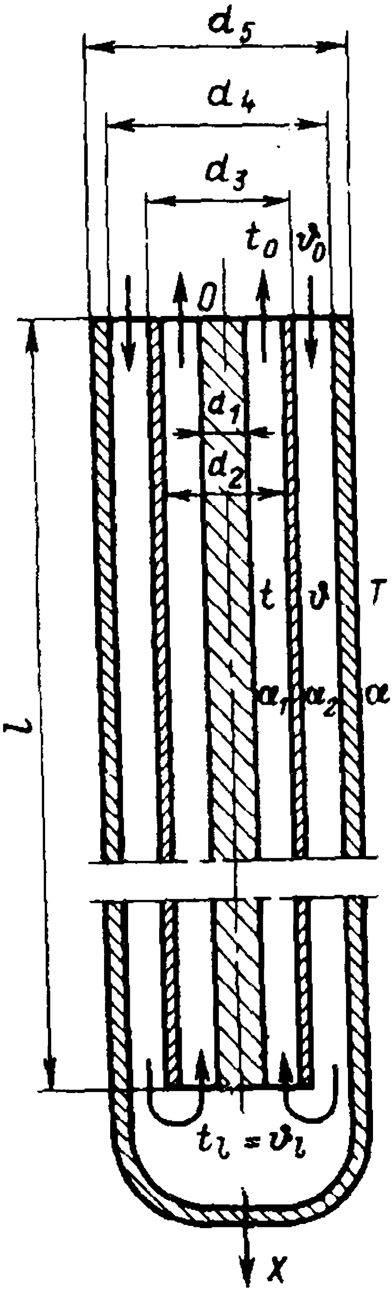

Fig. 12-11. Solution to problem 12-23

Calculate, assuming the following conditions: the rate of heat flow on the surface of the central fuel rod $q_w = 8 \times 10^5$ W/m^2; the water velocity in the inner annulus $w_1 = 2$ m/s; the water temperature at the inlet into the outer annular channel $\vartheta_0 = 90$ °C; the temperature of water to which the outside surface of the inner channel is exposed, T, is constant along the length of the channel and equal to 86 °C; the thermal conductivity of the walls $\lambda = 21$ W/m·°C.

Assume the local and overall heat-transfer coefficients to be constant along the fuel element, and when finding their numerical values use physical properties based on the lengthwise mean water temperature in a given coolant channel.

The local coefficients of heat transfer on both sides of the inside wall of the fuel element should be assumed equal, $\alpha_3 = \alpha_2$. Assume a constant heat capacity of water and $c_p = 4.25 \times 10^3$ J/kg·°C.

Determine the temperature of water at the end of the first pass, ϑ_l, and at the outlet from the second pass, t_0, and also the coordinate x_m and the magnitude of ϑ_m—the maximum temperature of the coolant in the first pass.

Answer

$\vartheta_l = 103$ °C; $t_0 = 156$ °C; $x_m = 2$m; $\vartheta_m = 104$ °C.

Solution

To determine the coefficient of heat transfer in the inner annular channel α_1, assume for a first approximation a mean water temperature $\bar{t}$ in this channel. This can be done on the basis of the following approximate calculation.

The cross-sectional area and the equivalent diameter of the inner annular channel are as follows:

$$f_1 = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(20^2 - 14^2)\,10^{-6} = 1.6 \times 10^{-4} \text{ m}^2;$$

$$d_{e1} = d_2 - d_1 = 20 - 14 = 6 \text{ mm}.$$

The rate of water flow in the channel

$$G = \rho_1 f_1 w_1 \approx 965 \times 1.6 \times 10^{-4} \times 2 \approx 0.31 \text{ kg/s},$$

where $\rho_1 \approx 965$ kg/m³ at an inlet temperature $\vartheta_0 = 90$ °C.

Then, in the absence of heat losses, the water outlet temperature

$$t_0 \approx \vartheta_0 + \frac{q_w \pi d_1 l}{G c_p} = 90 + \frac{8 \times 10^5 \pi 14 \times 10^{-3} \times 2.7}{0.31 \times 4.25 \times 10^3} = 160 \text{ °C}.$$

Considering the rejection of heat from the outer channel, it may be approximately assumed that $t_0 \approx 150$ °C and $\bar{t} \approx 0.5\,(90 + 150) = 120$ °C. Thus, for a first approximation we assume that $\bar{t} = 120$ °C. At that temperature the physical properties of water: $\rho_1 = 943$ kg/m³, $\nu_1 = 0.252 \times \times 10^{-6}$ m²/s, $\lambda_1 = 0.686$ W/m·°C, $\text{Pr}_1 = 1.47$.

The Reynolds number

$$\text{Re}_1 = \frac{w_1 d_{e1}}{\nu_1} = \frac{2 \times 6 \times 10^{-3}}{0.252 \times 10^{-6}} = 4.76 \times 10^4.$$

Taking into consideration that a number of simplifying assumptions have been made, the Nusselt number and the heat-transfer coefficient are determined with the aid of formula (5-7) derived for round tubes, introducing the equivalent diameter. In addition, with a small error, we assume that the correction $(\text{Pr}_f/\text{Pr}_w)^{0.25} = 1$. Then

$$\text{Nu}_1 = 0.021 \text{Re}_1^{0.8} \text{Pr}_1^{0.43} =$$

$$= 0.021 (4.76 \times 10^4)^{0.8} (1.47)^{0.43} = 136;$$

$$\alpha_1 = \text{Nu}_1 \frac{\lambda_1}{d_{e1}} = 136 \frac{0.686}{6 \times 10^{-3}} = 1.55 \times 10^4 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

The rate of water flow in the channel

$$G = \rho_1 f_1 w_1 = 943 \times 1.6 \times 10^{-4} \times 2 = 0.302 \text{ kg/s}.$$

For the outer channel we assume as a first approximation that $\bar{\vartheta} = \vartheta_0 = 90$ °C. At that temperature $\rho_2 = 965$ kg/m³, $\nu_2 = 0.326 \times 10^{-6}$ m²/s, $\lambda_2 = 0.680$ W/m·°C, $\text{Pr}_2 = 1.95$.

The cross-sectional area and equivalent diameter of the outer annular channel are as follows:

$$f_2 = \frac{\pi}{4} (d_4^2 - d_3^2) = \frac{\pi}{4} (28^2 - 22^2) = 2.35 \times 10^{-4} \text{ m}^2;$$

$$d_{e2} = d_4 - d_3 = 28 - 22 = 6 \text{ mm}.$$

The water velocity and the Reynolds number in the outer channel

$$w_2 = \frac{G}{\rho_2 f_2} = \frac{0.302}{965 \times 2.35 \times 10^{-4}} = 1.33 \text{ m/s};$$

$$\text{Re}_2 = \frac{w_2 d_{e2}}{\nu_2} = \frac{1.33 \times 6 \times 10^{-3}}{0.326 \times 10^{-6}} = 2.44 \times 10^4.$$

The Nusselt number and the heat-transfer coefficient

$$\text{Nu}_2 = 0.021 (2.44 \times 10^4)^{0.8} (1.95)^{0.43} = 90;$$

$$\alpha_2 = \text{Nu}_2 \frac{\lambda_2}{d_{e2}} = 90 \frac{0.68}{6 \times 10^{-3}} = 1.02 \times 10^4 \text{ W/m}^2 \cdot {}^\circ\text{C}.$$

The coefficient of heat transfer from the outside wall of the fuel element $\alpha_3 = \alpha_2 = 1.02 \times 10^4$ W/m²·°C, as given.

The overall coefficient of heat transfer through the walls, separating the annular channels k_1 and through the outside wall k_2, is:

$$k_1 = \frac{\pi}{\frac{1}{\alpha_1 d_2} + \frac{1}{2\lambda} \ln \frac{d_3}{d_2} + \frac{1}{\alpha_2 d_3}} =$$

$$= \frac{3.14}{\frac{1}{1.55 \times 10^4 \times 2 \times 10^{-2}} + \frac{2.3}{2 \times 21} \log \frac{22}{20} + \frac{1}{1.02 \times 10^4 \times 2.2 \times 10^{-2}}} =$$

$$= 317 \text{ W/m}^2\cdot{}^\circ\text{C}$$

$$k_2 = \frac{\pi}{\frac{1}{\alpha_2 d_4} + \frac{1}{2\lambda} \ln \frac{d_5}{d_4} + \frac{1}{\alpha_3 d_5}} =$$

$$= \frac{3.14}{\frac{1}{1.02 \times 10^4 \times 2.8 \times 10^{-2}} + \frac{2.3}{2 \times 21} \log \frac{34}{28} + \frac{1}{1.02 \times 10^4 \times 3.4 \times 10^{-2}}} =$$

$$= 285 \text{ W/m}^2\cdot{}^\circ\text{C}.$$

Assuming, in accordance with given conditions, that $k_1 = \text{const}$ and $k_2 = \text{const}$, the temperature distribution along the channels is calculated with the aid of formulae (5-31) through (5-33).

The rate of heat flow per length of 1 m of the central fuel rod

$$q_l = q_w \pi d_1 = 8 \times 10^5 \times 3.14 \times 14 \times 10^{-3} =$$

$$= 3.51 \times 10^4 \text{ W/m}.$$

We determine the values of the quantities in formula (5-31):

$$W = Gc_p = 0.302 \times 4.25 \times 10^3 = 1280 \text{ W/}^\circ\text{C};$$

$$\frac{q_l}{k_2} = \frac{3.51 \times 10^4}{285} = 123\,^\circ\text{C}; \quad \frac{q_l}{W} = \frac{3.51 \times 10^4}{1280} = 27.4\,^\circ\text{C/m};$$

$$C_1 = \vartheta_0 - T - \frac{q_l}{k_2} = 90 - 86 - 123 = -119\,^\circ\text{C};$$

$$\varepsilon_1=\frac{1}{2W}\left(-k_2+\sqrt{k_2^2+4k_1k_2}\right)=$$

$$=\frac{1}{2\times1280}\left(-285+10^2\sqrt{2.85^2+4\times3.17\times2.85}\right)=$$

$$=\frac{1}{2560}(-285+665)=0.148;$$

$$\varepsilon_2=\frac{1}{2W}\left(-k_2-\sqrt{k_2^2+4k_1k_2}\right)=$$

$$=\frac{1}{2560}(-285-665)=-0.371;$$

$$e^{\varepsilon_2 l}=e^{-0.371\times2.7}=0.368;$$

$$e^{\varepsilon_1 l}=e^{0.148\times2.7}=1.49;$$

$$A_1=\frac{C_1\varepsilon_1 e^{\varepsilon_2 l}-\frac{ql}{W}}{\varepsilon_1 e^{\varepsilon_2 l}-\varepsilon_2 e^{\varepsilon_1 l}}=\frac{-119\times0.148\times0.368-27.4}{0.148\times0.368+0.371\times1.49}=-55.8\,^{\circ}\mathrm{C};$$

$$A_2=\frac{C_1\varepsilon_2 e^{\varepsilon_1 l}-\frac{ql}{W}}{\varepsilon_1 e^{\varepsilon_2 l}-\varepsilon_2 e^{\varepsilon_1 l}}=\frac{-119\times0.371\times1.49-27.4}{0.6075}=-63\,^{\circ}\mathrm{C}.$$

The temperature of water at the end of the first pass is calculated by formula (5-31)

$$\vartheta_l=t_l=A_1e^{\varepsilon_1 l}+A_2e^{\varepsilon_2 l}+T+\frac{ql}{k_2}=$$

$$=-55.8\times1.49-63\times0.368+86+123=+102.5\,^{\circ}\mathrm{C}.$$

The coordinate of maximum temperature in the first pass is calculated by formula (5-33)

$$x_m=\frac{1}{\varepsilon_1-\varepsilon_2}\ln\left(-\frac{A_2\varepsilon_2}{A_1\varepsilon_1}\right)=\frac{1}{0.148+0.371}\ln\left(\frac{63\times0.371}{55.8\times0.148}\right)=$$

$$=2.01\ \mathrm{m}.$$

With $x=x_m$ the maximum temperature in the first pass is calculated with the aid of formula (5-31)

$$\vartheta_m=A_1e^{\varepsilon_1 x_m}+A_2e^{\varepsilon_2 x_m}+T+\frac{ql}{k_2}=$$

$$=-55.8e^{0.148\times2.01}-63e^{-0.371\times2.01}+86+123=103.7\ \mathrm{C}$$

The distribution of temperature along the inner channel is described by equation (5-32)

$$t = B_1 e^{\varepsilon_1 x} + B_2 e^{\varepsilon_2 x} + T + \frac{q_l}{k_1} + \frac{q_l}{k_2}.$$

$$\frac{q_l}{k_1} = \frac{3.51\times 10^4}{317} = 111\ ^\circ\mathrm{C};$$

$$C_2 = t_l - T - \frac{q_l}{k_1} - \frac{q_l}{k_2} = 102.5 - 86 - 111 - 123 = -217.5\ ^\circ\mathrm{C};$$

$$B_1 = -\frac{C_2\varepsilon_2 + \frac{q_l}{W}}{(\varepsilon_1-\varepsilon_2)e^{\varepsilon_1 l}} = -\frac{217.5\times 0.371 + 27.4}{(0.148+0.371)1.49} = -140\ ^\circ\mathrm{C},$$

$$B_2 = \frac{C_2\varepsilon_1 + \frac{q_l}{W}}{(\varepsilon_1-\varepsilon_2)e^{\varepsilon_2 l}} = \frac{-217.5\times 0.148 + 27.4}{(0.148+0.371)0.368} = -25.1\ ^\circ\mathrm{C}.$$

At $x=0$ the water outlet temperature is calculated by formula (5-32)

$$t_0 = B_1 + B_2 + T + \frac{q_l}{k_1} + \frac{q_l}{k_2} =$$
$$= -140 - 25.1 + 86 + 111 + 123 = 154.9\ ^\circ\mathrm{C}.$$

Check the values of the physical properties of water assumed in calculating the local heat-transfer coefficients α_1 and α_2. Taking into account the distribution of temperature along the outer channel, its mean value can be estimated in the following way:

$$\bar{\vartheta} \approx 0.5\,[0.5\,(\vartheta_0 + \vartheta_m) + 0.5\,(\vartheta_m + \vartheta_l)] \approx$$
$$\approx 0.5\,[0.5\,(90 + 103.7) + 0.5\,(103.7 + 102.5)] \approx 100\ ^\circ\mathrm{C}.$$

It was assumed above that $\bar{\vartheta} = 90\ ^\circ\mathrm{C}$. Taking into account non-linear temperature variation, the lengthwise mean water temperature in the inner channel can be estimated as $\bar{t} \approx 135\ ^\circ\mathrm{C}$; it was assumed that $\bar{t} = 120\ ^\circ\mathrm{C}$.

With $\bar{t} = 135^\circ\mathrm{C}$ the physical properties of water are as follows: $\rho_1 = 930$ kg/m³, $\lambda_1 = 0.685$ W/m·°C, $\nu_1 = 0.225 \times 10^{-6}$ m²/s, $\mathrm{Pr}_1 = 1.31$.

With $\bar{\vartheta} = 100\ ^\circ\mathrm{C}$ $\rho_2 = 958$ kg/m³, $\lambda_2 = 0.683$ W/m·°C, $\nu_2 = 0.295 \times 10^{-6}$ m²/s, $\mathrm{Pr}_2 = 1.75$.

Repeated calculation, using the specified values of the physical properties of the coolant, yields the following results:

$$G = 0.298 \text{ kg/s}; \quad w_2 = 1.32 \text{ m/s};$$
$$\alpha_1 = 1.61 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C}; \quad \alpha_2 = 1.06 \times 10^4 \text{ W/m}^2\cdot{}^\circ\text{C};$$
$$k_1 = 326 \text{ W/m}\cdot{}^\circ\text{C}; \quad k_2 = 291 \text{ W/m}\cdot{}^\circ\text{C};$$
$$\vartheta_l = 103.2\ {}^\circ\text{C}; \quad x_m = 2\text{m}; \quad \vartheta_m = 104.3\ {}^\circ\text{C};$$
$$t_0 = 156.3\ {}^\circ\text{C}.$$

The obtained results agree sufficiently well with these of the first calculation, and no recalculations are needed.

12-24. Determine the distribution of water temperature along the length of annular channels in the fuel element with two coolant passes, considered in problem 12-23, if the velocity of water in the inner channel is doubled, increased from 2 to 4 m/s. Assume the heat capacity of water to be constant and $c_p = 4.23 \times 10^3$ J/kg·°C. All other conditions of problem 12-23 remain unchanged.

Answer

$$\vartheta_l = 95\ {}^\circ\text{C}; \quad x_m = 2.1 \text{ m}; \quad \vartheta_m = 96\ {}^\circ\text{C};$$
$$t_0 = 124\ {}^\circ\text{C}.$$

12-25. Determine the distribution of coolant and wall temperature along the channel of the core of a nuclear reactor. The cylindrical fuel element of the reactor (Fig. 12-12) has an outside diameter $d = 21$ mm and a length $l = 2.8$ m. It is made from uranium ($\lambda = 31$ W/m·°C). The surface of the fuel element is covered with a tightly fitting stainless steel ($\lambda_w = 21$ W/m·°C) cladding of thickness $\delta = 0.5$ mm. The volumetric rate of heat release in the uranium fuel element, q_v, is assumed to be constant over the cross section and vary along the length of the element, obeying the cosine law (the reactor is not fitted with

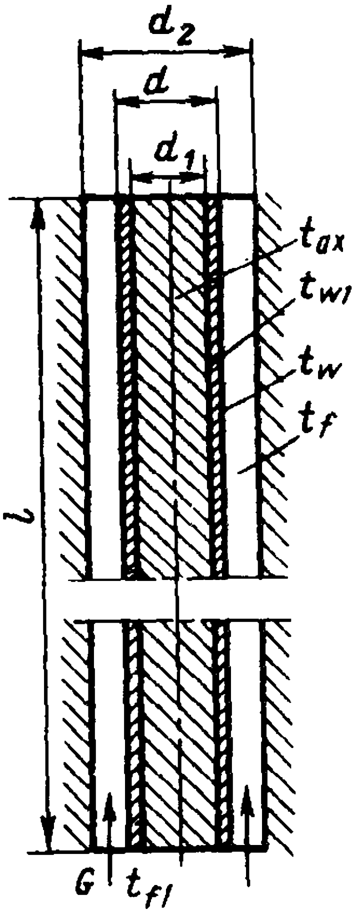

Fig. 12-12. Solution to problem 12-25

end closures or caps). If the origin of coordinates is placed midlength of the fuel element, then at $x = 0$ $q_{v0} = 4.9 \times \times 10^7$ W/m³.

The fuel element is cooled with carbon dioxide; the coolant flows along an annular channel having an outside diameter $d_2 = 30$ mm. The pressure and rate of flow of the carbon dioxide coolant $p = 2$ MPa and $G = 0.25$ kg/s and its inlet temperature $t_{f1} = 150$ °C. The transfer of heat through the outside wall of the annular cooling channel can be ignored.

In solving the problem assume a lengthwise constant coefficient of heat transfer from the surface of the cladding to the coolant and determine its approximate value with the aid of the formula derived for calculating heat transfer in round tubes and without the correction for the temperature factor.

Determine the temperature of the carbon dioxide coolant t_f, the inside and outside surface temperatures of the cladding, t_w and t_{w1}, the temperature at the centreline of the fuel element t_{ax} at the following distances from the inlet: 0.5, 1.0, 1.4, 1.8 and 2.3 m ($x = -0.9, -0.4, 0.0, 0.4, 0.9$ m), and also determine the coordinates and the maximum temperatures $t_{w,\ m}$, $t_{w1,m}$, and $t_{axis,m}$.

At $p = 2$ MPa and $t = 150$-300 °C the physical properties of carbon dioxide are tabulated below [25]:

t °C	ρ, kg/m³	c_p, kJ/kg·°C	$\mu \times 10^6$, Pa·s	$\lambda \times 10^4$, W/m·°C	Pr
150	26.3	1.00	20.0	260	0.768
200	23.2	1.02	22.5	278	0.825
250	20.8	1.04	24.6	307	0.832
300	17.9	1.07	26.5	341	0.832

Answer

The coordinates and maximum temperatures:

$$x_{w,m} = 0.309 \text{ m};\quad t_{w,\ m} = 384\ ^\circ\text{C};$$
$$x_{w1,m} = 0.298 \text{ m};\quad t_{w1,m} = 390\ ^\circ\text{C};$$
$$x_{ax\ m} = 0.240 \text{ m};\quad t_{ax,m} \doteq 431\ ^\circ\text{C}.$$

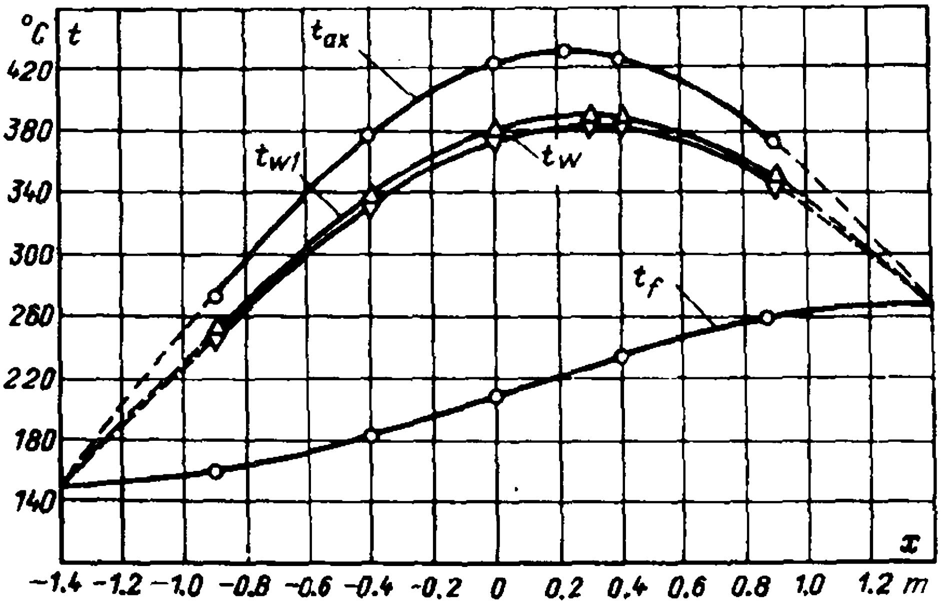

Fig. 12-13. Solution to problem 12-25

The data on temperature distribution along the cooling channel are presented graphically in Fig. 12-13 and tabulated below:

x	−1.4	−0.9	−0.4	0	0.4	0.9	1.4
t_f, °C	150	159	183.5	209	234.5	260	268
t_w, °C	—	246.5	331.5	373	382.5	347	—
t_{wl}, °C	—	249.5	336.5	379	387.5	350	—
t_{ax}, °C	—	272.5	375.5	422.5	426.5	373	—

Solution

The rate of heat transfer from length of 1 m at midlength of the cooling channel ($x = 0$)

$$q_0 = q_{vc}\frac{\pi d_1^2}{4} = 4.9\times10^7\,\frac{3.14\times21^2}{4}\,10^{-6} = 1.7\times10^4\ \text{W/m}.$$

At $x = l/2$ the outlet temperature of the carbon dioxide is calculated with the aid of formula (5-34)

$$t_{f2} - t_{f1} = \frac{2}{\pi}\,\frac{q_0 l}{Gc_p} = \frac{2}{\pi}\,\frac{1.7\times10^4\times2.8}{0.25\times1.02\times10^3} = 118\ ^\circ\text{C},$$

where in the 150 to 250 °C temperature range $c_p = 1.02 \times \times 10^3$ J/kg·°C:

$$t_{f2} = t_{f1} + 118 = 150 + 118 = 268 \text{ °C}.$$

At $x = 0$ the temperature of carbon dioxide

$$t_{f,0} = t_{f1} + \frac{q_0 l}{\pi G c_p} = 0.5\,(t_{f1} + t_{f2}) = 209 \text{ °C}.$$

At the pressure $p = 2$ MPa and $t_{f,0} = 209$ °C the physical properties of carbon dioxide are as follows: $c_{pf} = 1.02 \times \times 10^3$ J/kg·°C, $\rho_f = 22.7$ kg/m³, $\mu_f = 22.8 \times 10^{-6}$ Pa·s, $\lambda_f = 2.83 \times 10^{-2}$ W/m·°C, $\Pr_f = 0.826$.

The equivalent diameter of the annular channel

$$d_e = d_2 - d = 30 - 22 = 8 \text{ mm},$$

where $d = d_1 + 2\delta = 21 + 2 \times 0.5 = 22$ mm.

The Reynolds number

$$\mathrm{Re}_f = \frac{w d_e}{\nu_f} = \frac{G d_e}{\rho_f f \nu_f} = \frac{4G}{\pi (d_2 + d)\, \mu_f},$$

where $f = \pi\,(d_2^2 - d^2)/4$ is the cross-sectional area of the channel.

Substituting into the expression for Re_f the numerical values of the quantities found in it, we obtain:

$$\mathrm{Re}_f = \frac{4 \times 0.25}{\pi\,(30 + 22)\,10^{-3}\,22.8 \times 10^{-6}} = 2.69 \times 10^5.$$

The Nusselt number and the heat-transfer coefficient are determined approximately by formula (5-7)

$$\mathrm{Nu}_f = 0.021 \mathrm{Re}_f^{0.8} \Pr_f^{0.43} =$$
$$= 0.021\,(2.69 \times 10^5)^{0.8}\,(0.826)^{0.43} = 424;$$
$$\alpha = \mathrm{Nu}_f \frac{\lambda_f}{d_e} = 424\,\frac{2.83 \times 10^{-2}}{8 \times 10^{-3}} = 1500 \text{ W/m}^2\cdot\text{°C}.$$

The difference between the temperature at the centreline of the fuel element and that of the carbon dioxide coolant, at $x = 0$,

$$\Delta t_{ax,\,0} = t_{ax,\,0} - t_{f,\,0} = \frac{q_0}{\pi}\left(\frac{1}{4\lambda} + \frac{1}{2\lambda_w}\ln\frac{d}{d_1} + \frac{1}{\alpha d}\right) =$$

$$= \frac{1.7\times10^4}{\pi}\left(\frac{1}{4.31} + \frac{1}{2.21}\ln\frac{22}{21} + \frac{1}{1.5\times10^3\times22\times10^{-3}}\right) =$$

$$= 43.6 + 5.9 + 164 = 213.5\ ^\circ\mathrm{C}.$$

Thus,

$$\Delta t_{w,\,0} = t_{w,\,0} - t_{f,\,0} = 164\ ^\circ\mathrm{C};$$

$$\Delta t_{w1,\,0} = t_{w1,\,0} - t_{f,\,0} = 169.9\ ^\circ\mathrm{C};$$

$$\Delta t_{ax,\,0} = t_{ax,\,0} - t_{f,\,0} = 213.5\ ^\circ\mathrm{C}.$$

At $x = 0$ the outside and inside surface temperatures of the cladding and the axial temperature of the fuel element:

$$t_{w,0} = t_{f,0} + \Delta t_{w,0} = 209 + 164 = 373\ ^\circ\mathrm{C};$$

$$t_{w1,0} = t_{f,0} + \Delta t_{w1,0} = 209 + 169.9 = 378.9\ ^\circ\mathrm{C};$$

$$t_{ax,0} = t_{f,0} + \Delta t_{ax,0} = 209 + 213.5 = 422.5\ ^\circ\mathrm{C}.$$

The maximum temperature at the heat-transfer surface, $t_{w,m}$, is determined by formula (5-35):

$$\frac{t_{w,\,m} - t_{f1}}{t_{f2} - t_{f1}} = \frac{1}{2} + \sqrt{\frac{1}{4} + \left(\frac{\Delta t_{w,\,0}}{t_{f2} - t_{f1}}\right)^2}$$

or

$$t_{w,\,m} = t_{f,\,0} + \sqrt{\left(\frac{t_{f2} - t_{f1}}{2}\right)^2 + \Delta t_{w,\,0}^2} =$$

$$= 209 + \sqrt{\left(\frac{118}{2}\right)^2 + \Delta t_{w,\,0}^2} = 209 + \sqrt{3481 + \Delta t_{w,\,0}^2}; \qquad \text{(a)}$$

$$t_{w,\,m} = 209 + \sqrt{3481 + 164^2} = 383\ ^\circ\mathrm{C}.$$

The coordinate of the maximum temperature on the heat-transfer surface, $x_{w,m}$, is determined by formula (5-36)

$$x_{w,\,m} = \frac{l}{\pi}\arctan\left(\frac{t_{f2} - t_{f1}}{2\Delta t_{w,\,0}}\right) = \frac{2.8}{\pi}\arctan\left(\frac{59}{\Delta t_{w,\,0}}\right); \qquad \text{(b)}$$

$$x_{w,\,m} = 0.89\arctan\left(\frac{59}{164}\right) = 0.308\ \mathrm{m}.$$

The maximum inside surface temperature of the cladding and the maximum axial temperature of the fuel element and their coordinates are determined by expressions (a) and (b), substituting in them the quantity $\Delta t_{w,0}$ with $\Delta t_{w1,0}$ and $\Delta t_{ax,0}$ respectively.

$$t_{w1,\,m}=209+\sqrt{3481+\Delta t_{w1,\,0}^2}=$$

$$=209+\sqrt{3481+169.9^2}=389\,^\circ\mathrm{C};$$

$$x_{w1,\,m}=0.89\arctan\left(\frac{59}{\Delta t_{w1,\,0}}\right)=0.89\arctan\left(\frac{59}{169.9}\right)=0.298\,\mathrm{m};$$

$$t_{ax,\,m}=209+\sqrt{3481+213.5^2}=431\,^\circ\mathrm{C};$$

$$x_{ax,\,m}=0.89\arctan\frac{59}{213.5}=0.240\ \mathrm{m}.$$

The temperatures of the carbon dioxide coolant at different coordinates x are found with the aid of formula (5-34):

$$t_f=t_{f1}+\frac{q_0 l}{\pi G c_p}\left(\sin\frac{\pi x}{l}+1\right)=t_{f1}+\frac{t_{f2}-t_{f1}}{2}\left(\sin\frac{\pi x}{l}+1\right)=$$

$$=150+59\,[\sin(1.22x)+1].$$

Substitution of the value of x yields:

$x=-1.4$ $t_f=t_{f1}=150\,^\circ\mathrm{C}$; $x=-0.9$ $t_f=159\,^\circ\mathrm{C}$;

$x=-0.4$ $t_f=183.4\,^\circ\mathrm{C}$; $x=0$ $t_f=209\,^\circ\mathrm{C}$;

$x=0.9$ $t_f=259.7\,^\circ\mathrm{C}$; $x=1.4$ $t_f=268\,^\circ\mathrm{C}$.

The temperatures of the heat-transfer surface t_w at different coordinates x are found from formula

$$t_w=t_f+\Delta t_{w,\,0}\cos\frac{\pi x}{l}. \qquad (12\text{-}2)$$

With $x=-0.9$

$$t_w=159+164\cos(-1\ 122\times 0.9)=246.3^\circ\mathrm{C}.$$

By analogy, determine the wall temperature t_w at $x=-0.4$; 0; 0.4 and 0.9 m.

The temperatures of the inside surface of the cladding and at the axis of the fuel element are determined as a function of x by means of formula (12-2), substituting in the latter the quantity $\Delta t_{w,0}$ with $\Delta t_{w1,0}$ and $\Delta t_{ax,0}$ respectively.

At $x = -0.9$

$$t_{w1} = t_f + \Delta t_{w1,0} \cos\frac{\pi x}{l} = 159 + 169.9 \cos(-1.01) = 249.3\ ^\circ\mathrm{C}.$$

$$t_{ax} = t_f + \Delta t_{ax,0} \cos\frac{\pi x}{l} = 159 + 213.5 \cos(-1.01) = 272\ ^\circ\mathrm{C}.$$

The temperatures t_{w1} and t_{ax} at other coordinates x are determined in a similar way.

Calculation results are given in the answer to the problem. The variation of t_f, t_w, t_{w1}, and t_{ax} along the channel is shown in Fig. 12.13.

12-26. Determine the distribution of coolant and wall temperature along the fuel element considered in problem 12-25, if the coolant is sodium.

The maximum volumetric rate of heat release in uranium $q_{v0} = 1.2 \times 10^8$ W/m³. The rate of sodium flow $G = 0.66$ kg/s and its channel inlet temperature $t_{f1} = 300$ °C. All other given conditions remain unchanged.

Calculate the numerical values of quantities as in problem 12-25.

Answer

Coordinates and maximum temperatures:

$$x_{w,m} = 1.21\ \mathrm{m};\quad t_{w,m} = 389\ ^\circ\mathrm{C};$$
$$x_{w1,m} = 0.955\ \mathrm{m};\quad t_{w1,m} = 394.1\ ^\circ\mathrm{C};$$
$$x_{ax,m} = 0.289\ \mathrm{m};\quad t_{ax,m} = 482.5\ ^\circ\mathrm{C}.$$

The data on temperature distribution along the channel are tabulated below.

x, m	−1.4	−0.9	−0.4	0	0.4	0.9	1.4
t_f, °C	300	306.7	325	344	363	381.3	388
t_w, °C	—	311.8	333.6	353.6	371.6	386	—
t_{w1}, °C	—	319.5	346.7	368.1	384.7	394	—
t_{ax}, °C	—	376.2	436.8	475	474.8	450.8	

APPENDICES

Table 1

International System of Units (SI)

Quantity	Unit	Dimensions
	Fundamental units	
Length	meter	m
Mass	kilogram	kg
Time	second	s
Current	ampere	amp
Thermodynamic temperature	kelvin	K*
Luminous intensity	candela	cd
	Some derived units	
Area	square meter	m^2
Volume	cubic meter	m^3
Velocity	meter per second	m/s
Acceleration	meter per second per second	m/s^2
Density	kilogram per cubic meter	kg/m^3
Force	newton	N; $(kg \cdot m/s^2)$
Pressure	pascal	Pa; (N/m^2)
Dynamic viscosity	pascal-second	$Pa \cdot s$; $(N \cdot s/m^2)$
Kinematic viscosity	square meter per second	m^2/s
Work, energy and heat	joule	J; $(N \cdot m)$
Power, heat flux	watt	W; (J/s)
Specific heat	joule per kilogram-kelvin	$J/kg \cdot K$*

Table 1 (continued)

Quantity	Unit	Dimensions
Heat involved in change of phase, enthalpy	joule per kilogram	J/kg
Rate of heat flow	watt per square meter	W/m^2
Thermal conductivity	watt per meter-kelvin	$W/m \cdot K$*
Local and overall heat-transfer coefficients	watt per square meter-kelvin	$W/m^2 \cdot K$*
Radiation factor	watt per square meter-kelvin in the fourth power	$W/m^2 \cdot K^4$

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* In addition to the thermodynamic temperature scale, K, the international practical scale, °C, is also used.

Table 2

Conversion Factors Relating MKGCS to SI Units

Energy	1 kcal = 4.187 kJ
Force	1 kgf = 9.81 N
Specific gravity	$1\ kgf/m^3 = 9.81\ N/m^3$
Density	$1\ kgf \cdot s^2/m^4 = 9.81\ kg/m^3$
Pressure	$1\ kgf/cm^2 = 0.981 \times 10^5\ Pa$
Dynamic viscosity	$1\ kgf \cdot s/m^2 = 9.81\ Pa \cdot s$
Heat capacity	$1\ kcal/(kgf \cdot {}^\circ C) = 4.187\ kJ/kg \cdot {}^\circ C$
Enthalpy, change-of-phase heat	1 kcal/kgf = 4.187 kJ/kg
Heat flux	1 kcal/h = 1.163 W
Rate of heat flow	$1\ kcal/(m^2 \cdot h) = 1.163\ W/m^2$
Volumetric rate of heat flow	$1\ kcal/(m^3 \cdot h) = 1.163\ W/m^3$
Thermal conductivity	$1\ kcal/(m \cdot h \cdot {}^\circ C) = 1.163\ W/m \cdot {}^\circ C$
Heat-transfer coefficient	$1\ kcal/(m^2 \cdot h \cdot {}^\circ C) = 1.163\ W/m^2 \cdot {}^\circ C$
Radiation factor	$1\ kcal/(m^2 \cdot h \cdot K^4) = 1.163\ W/m^2 \cdot K^4$

Table 3

Density ρ, Thermal Conductivity λ and Heat Capacity c of Building, Insulating and Some Other Materials [24]

Material	ρ, kg/m^3	t*, °C	λ, W/m·°C	c, kJ/kg·°C
Aluminium foil at air layer thickness of 10 mm	—	—	$0.0302+0.85\times10^{-4}t$	—
Asbestos fibre:				
3rd grade	340	—	$0.087+0.24\times10^{-3}t$	0.816
6th grade	650	—	$0.11+0.19\times10^{-3}t$	0.816
Asbestos board	900	—	$0.16\text{-}0.17\times10^{-3}t$	0.816
Asbestos cord	800	—	$0.13\text{-}0.15\times10^{-3}t$	0.816
Asbestos slate:				
with high asbestos content	1800	20	0.17-0.35	—
with 10-50% dry asbestos content	1800	20	0.64-0.52	—
Asphalt (bitumen)	2120	0-30	0.60-0.74	1.67
Concrete, crushed-stone	2000	0	1.28	0.84
Ditto, dry	1600	0	0.84	—
Reinforced concrete, compacted	2200	0	1.55	0.84
Slag concrete	1500	0	0.70	0.80
Paper, ordinary	—	20	0.14	1.51
Cotton wool	80	30	0.042	—
Gypsum (formed, dry)	1250	20	0.43	0.8-0.92
Clay	2000-1600	20	0.9-0.7	0.84
Refractory clay	1845	450	1.04	1.09

Gravel	1840	20	0.36	—
Wood laminate	—	35-70	0.21	—
Wood:				
oak across grain	825	0-15	0.20-0.21	2.39
oak along grain	819	12-50	0.35-0.43	2.39
pine across grain	546	0-50	0.14-0.16	2.72
pine along grain	—	20-25	0.35-0.72	2.72
Coal:				
gas	1420	20-100	3.6-4.0	—
ordinary, solid	1200-1350	20	0.24-0.27	—
Pulverized coal	730	30-150	0.12-0.13	—
Cardboard	—	20	0.14-0.35	1.51
Cambric (varnished)	—	38	0.157	—
Brick:				
common, machine moulded	1800	0	0.77	0.88
common, hand moulded	1700	0	0.70	0.88
lime brick	1900	0	0.81	0.84
Common brickwork:				
with cold mortar	1700	0	0.81	0.88
with warm mortar	1600	0	0.67	0.84
Lime brick masonry:				
with cold mortar	1900	0	0.87	0.84
with warm mortar	1700	0	0.76	0.80
Rubble masonry of medium-density stone	2000	0	1.28	0.88
Carbolite, black	1150	50	0.231	—

Table 8 (continued)

Material	ρ, kg/m³	t*, °C	λ, W/m·°C	c, kJ/kg·°C
Leather	—	20	0.14-0.16	—
Coke, powdery	449	100	0.191	1.21
Scale, boiler:				
gypsum-rich	2000-2700	100	0.7-2.3	—
lime-rich	1000-2500	100	0.15-2.3	—
silicate-rich	300-1200	100	0.08-0.23	—
Quartz, crystalline:				
across axis	—	0	0.72	—
along axis	—	0	1.94	—
Lamp black	165	40	0.07-0.12	—
Ice	917	0	2.2	2.26
Ice	928	—100	3.5	1.17
Linen fabric	—	—	0.088	—
Magnesium, segment-shaped for pipe insulation	266	50-200	0.073-0.084	—
Chalk	2000	50	0.9	0.88
Micanite	—	20	0.21-0.41	—
Marble	2800	0	3.5	0.92
Paraffin	920	20	0.27	—
Sand, river, fine (dry)	1520	0-160	0.30-0.38	0.80
Sand, river, fine (damp)	1650	20	1.13	2.09

Pressboard	—	20-50	0.26-0.22	—
Plexiglass	—	20	0.184	—
Cork, plate, dry	148-198	80	0.042-0.053	1.76
Cork fines, piece size 4-5 mm	85	0.60	0.044-0.058	1.76
Rubber:				
hard, ordinary	1200	0-100	0.157-0.160	1.38
soft	—	20	0.13-0.16	1.38
Sugar, granulated	1600	0	0.58	1.26
Sulphur, rhombic	—	21	0.28	0.762
Slate	—	94	1.49	—
Mical (across layers)	2600-3200	20	0.49-0.58	—
Snow:				
loose	200	—	0.10	2.09
compacted	400	—	0.46	2.09
Glass:				
mirror	2550	0-100	0.78-0.88	0.779
ordinary	2500	20	0.74	0.67
thermometric	2590	20	0.96	—
pyrex	—	0	1.04	—
ditto	—	400	1.55	—
quartz	—	400	1.76	—
ditto	—	800	2.40	—
ditto	—	1200	3.05	—
Glass wool	154-206	88	0.051-0.059	—

Table 3 (concluded)

Material	ρ, kg/m^3	t*, °C	λ, W/m·°C	c, kJ/kg·°C
Textolite	1300-1400	20	0.23-0.34	1.46-1.51
Porcelain	2400	95	1.04	1.09
Ditto	2400	1055	1.96	1.09
Fibre, red	1290	20-100	0.46-0.50	—
Fibrolite	360-440	80	0.073-0.128	—
Celluloid	1400	30	0.21	—
Silk	100	0-93	0.043-0.06	—
Ebonite	1200	20	0.157-0.17	—
Slag:				
boiler room	1000	0	0.29	0.75
blast-furnace, granulated	500	0	0.15	0.75
Plaster:				
lime	1600	0	0.70	0.84
cement-sand	1800	0	1.20	0.84
Plywood, bonded	600	0	0.15	2.51
Charcoal, lump	190	80	0.074	—

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* Temperature by which properties are taken.

Table 4

Density ρ, Thermal Conductivity λ, Heat Capacity c and Maximum Working Temperature t for Main Refractory Products [24]

Name of refractory	$\rho\times10^{-3}$, kg/m^3	λ, W/m·°C	c, kJ/kg·°C	t, °C
Refractory brick	1.8-1.9	$0.84+0.0006t$	$0.88+0.00230t$	1350-1450
Foamed fireclay	0.95	$0.28+0.00023t$	—	1350
Foamed fireclay	0.60	$0.10+0.000145t$	—	—
Brick:				
dinas	0.9-1.95	$0.9+0.0007t$	$0.8+0.00025t$	1700
magnesite	2.6-2.8	$4.65-0.0017t$	$1.05+0.0003t$	1650-1700
magnesite chrome	2.75-2.85	1.86-1.98 (0-600 °C)	—	1700
chromium	3.0-3.1	$1.3+0.00041t$	$0.8+0.0003t$	1650-1700
Products;				
sillimanite (mullite)	2.2-2.4	$1.69-0.00023t$	$0.8+0.00025t$	1650
corundum (alundum)	2.3-2.6	$2.09+0.0019t$	$0.80+0.0004t$	1600-1700
zirconium	3.3	$1.30+0.00064t$	$0.54-0.00012t$	1750-1800
carborundum (carbofracs)	2.3-2.6	$21-0.010t$	$0.96+0.000146t$	1400-1500
carbon	1.35-1.6	$23+0.035t$	0.8	2000
graphite	1.6	$16-30.041t$	0.8	2000

Table 5

Density ρ, Thermal Conductivity λ and Maximum Application Temperature t for Main Insulating Materials and Products [24]

Name of material or product	ρ, kg/m³		λ, W/m·°C	t, °C
	power	mastic		
Insulating materials				
Asbestos, flaked, 6th grade	800	—	$0.130+0.00019t$	700
Asbozonolite	350	500-550	$0.143+0.00019t$	700
Asbozurite	450	700	$0.1622+0.000169t$	200-300
Asbomica	400-500	580-650	$0.120+0.000148t$	600
Asbotermite	400-430	550-570	$0.109+0.000145t$	500-550
Diatomite, crushed	400-500	—	$0.091+0.00028t$	800
Zonolite (vermiculite)	150-250	—	$0.072+0.000262t$	900-1100
Mineral wool	180-250	—	$0.046 \div 0.058$ at 50 °C	500
Novoasbozurite	400-450	580-650	$0.144+0.00014t$	250
Newell	180-200	405-485	$0.87+0.000084t$	325-370
Sovelite	230-250	440-520	$0.0901+0.000087t$	400-450
Peat dust	200-350	—	$0.06 \div 0.08$	100
Ferrigypsum (ferron paste)	—	400-550	$0.07 \div 0.08$	650
Slag wool (0 grade)	170-200	—	$0.06+0.000145t$	750

Insulating products				
Vermiculate slabs	—	350-380	$0.081+0.00015t$	700-750
Vulcanite slabs	—	400	$0.080+0.00021t$	550-600
Felt, construction	—	300	0.05 at 0 °C	90
Diatomaceous earth brick	—	500-600	$0.113+0.00023t$	850
Diatomaceous earth shells and segments	—	500-600	$0.113+0.00023t$	850
Products "novoizol"	—	400-450	$0.073+0.00028t$	400
Pressboard made of reed	—	260-360	0.10 at 0 °C	100
Mineral felt	—	250-300	0.058-0.076 at 50 °C	—
Foamed concrete blocks	—	400-500	0.099-0.122 at 50 °C	300
Foamed-diatomaceous earth brick	—	230-430	0.07-0.09 at 70 °C	600-800
Foamed glass (gasglass)	—	290-450	0.124-0.160 at 70 °C	600-800
Slabs "orgizol"	—	280-350	$0.078-0.00012t$	600
Cork slabs	—	250	0.07 at 0 °C	120
Sovelite slabs	—	400-450	$0.079-0.00019t$	450-500
Pressed straw mats	—	260-360	0.10 at 0 °C	100
Peat slabs	—	170-250	$0.046+0.00014t$	100-120
Slag and mineral cork	—	270-350	0.064-0.081 at 50 °C	150

Table 6

Thermal Conductivity of Steel λ, W/m·°C, Depending on Temperature [24, 25]

Name and grade of steel	Temperature °C								
	100	200	300	400	500	600	700	800	900
Carbon 15	54.4	50.2	46.0	41.9	37.7	33.5	—	—	—
Carbon 30	50.2	46.0	41.9	37.7	33.5	29.3	—	—	—
Chromium-molybdenum X10C2M (ЭИ107)	18.4	—	21.7	—	—	24.6	25.5	—	—
Chromium-nickel-tungsten 4X14HB2M (ЭИ69)	15.5	16.9	19.2	20.2	21.2	22.0	—	—	—
Chromium-nickel 1X18H9T (ЭЯ1Т) *	16.0	17.6	19.2	20.8	22.3	23.8	25.5	27.6	—
Chromium-nickel X25H20C2 (ЭИ283)	14.6	—	—	—	21.6	23.5	25.1	27.1	28.8
Chromium stainless:									
1X13 (Ж$_1$)	24.0	23.6	23.3	23.3	23.7	24.4	—	—	—
2X13 (Ж$_2$)	24.3	25.8	26.3	26.4	26.6	26.4	26.2	26.7	27.6
3X13 (Ж$_3$)	25.1	25.6	25.6	25.6	25.6	25.6	24.6	—	—
4X13 (Ж$_4$)	28.0	29.1	29.3	29.2	28.8	28.4	28.0	—	—
X17 (Ж$_{17}$)	24.4	—	—	—	—	—	—	—	—
X28 (Ж$_{27}$)	20.9	21.7	22.7	23.4	24.3	25.0	—	—	—

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* For various specimens of grade 1X18H9T steel the values of λ vary within ±20 per cent. The table gives average values of λ.

Table 7

Thermal Conductivity λ, W/m·°C, of Metals and Alloys Depending on Temperature [24]

Name of metal or alloy	Temperature, °C							
	0	20	100	200	300	400	500	600
Aluminium	202	—	206	229	272	319	371	422
Aluminium alloys:								
92% Al, 8% Mg	102	106	123	148	—	—	—	—
80% Al, 20% Si	158	160	169	174	—	—	—	—
Duralumin:								
94-96% Al, 3-5% Cu, 0.5% Mg	159	165	181	194	—	—	—	—
Brass:								
90% Cu, 10% Zn	102	—	117	134	149	166	180	195
70% Cu, 30% Zn	106	—	109	110	114	116	120	121
67% Cu, 33% Zn	100	—	107	113	121	128	135	151
60% Cu, 40% Zn	106	—	120	137	152	169	186	200
Copper (99.9%)	393	—	385	378	371	365	359	354
Monel metal:								
29% Cu, 67% Ni, 2% Fe	—	22.1	24.4	27.6	30	34	—	—
German silver:								
62% Cu, 15% Ni, 22% Zn	—	25.0	31	40	45	49	—	—
Nichrome:								
90% Ni, 10% Cr	17.1	17.4	19.0	20.9	22.8	24.6	—	—
80% Ni, 20% Cr	12.2	13.6	13.8	15.6	17.2	19.0	—	22.6
Ferrous nichrome:								
61% Ni, 15% Cr, 20% Fe, 4% Mn	—	11.6	11.9	12.2	12.4	12.7	—	13.1
61% Ni, 16% Cr, 23% Fe	11.9	12.1	13.2	14.6	16.0	17.4	—	—
Soft steel	63	—	57	52	46	36	—	31

Table 8

Thermal Conductivity of Alloys [24]

Alloy	t, °C	λ, W/m·°C
Aluminium bronze: 95% Cu, 5% Al	20	82
Bronze:		
90% Cu, 10% Sn	20	42
75% Cu, 25% Sn	20	26
88% Cu, 10% Sn, 2% Zn	20	48
Phosphor bronze:		
92.8% Cu, 5% Sn, 0.15% P, 2% Zn	20	79
91.7% Cu, 8% Sn, 0.3% P	20	45
87.2% Cu, 12.4% Sn, 0.4% P	20	36
Invar: 35% Ni, 65% Fe	20	11.0
Constantan:		
60% Cu, 40% Ni	20	22.7
60% Cu, 40% Ni	100	25.6
Manganin:		
84% Cu, 4% Ni, 12% Mn	20	21.3
84% Cu, 4% Ni, 12% Mn	100	26.4
Magnesium alloys:		
92% Mg, 8% Al	20-200	62-79
88% Mg, 10% Al, 2% Si	20-200	58-76
92% Mg, 8% Cu	20-200	126-132
Copper alloys:		
70% Cu, 30% Mg	20	13
90% Cu, 10% Ni	20-100	58-76
80% Cu, 20% Ni	20-100	34-41
40% Cu, 60% Ni	20-100	22-26
Rose metal: 50% Bi, 25% Pb, 25% Sn	20	16
Wood metal: 48% Bi, 26% Pb, 13% Sn, 13% Cd	20	13
Nickel alloys:		
70% Ni, 28% Cu, 2% Fe	20	35
60% Ni, 12% Cu, 26% Fe	20	13.5
Nickel silver	0	29.3
Ditto	100	37
Iridioplatinum: 90% Pt, 10% Ir	0-100	30.9-31
Electron: 93% Mg, 4% Zn, 0.5% Cu	20	116
Rhodic platinum: 90% Pt, 10% Rh	0-100	30-30.6

Table 9

Physical Properties of Dry Air
($B = 760$ mm Hg $\approx 1.01 \times 10^5$ Pa) [13]

t, °C	ρ, kg/m³	c_p, kJ/kg·°C	$\lambda \times 10^2$, W/m·°C	$a \times 10^6$, m²/s	$\mu \times 10^6$, Pa·s	$\nu \times 10^6$, m²·s	Pr
—50	1.584	1.013	2.04	12.7	14.6	9.23	0.728
—40	1.515	1.013	2.12	13.8	15.2	10.04	0.728
—30	1.453	1.013	2.20	14.9	15.7	10.80	0.723
—20	1.395	1.009	2.28	16.2	16.2	12.79	0.716
—10	1.342	1.009	2.36	17.4	16.7	12.43	0.712
0	1.293	1.005	2.44	18.8	17.2	13.28	0.707
10	1.247	1.005	2.51	20.0	17.6	14.16	0.705
20	1.205	1.005	2.59	21.4	18.1	15.06	0.703
30	1.165	1.005	2.67	22.9	18.6	16.00	0.701
40	1.128	1.005	2.76	24.3	19.1	16.96	0.699
50	1.003	1.005	2.83	25.7	19.6	17.95	0.698
60	1.060	1.005	2.90	26.2	20.1	18.97	0.696
70	1.029	1.009	2.96	28.6	20.6	20.02	0.694
80	1.000	1.009	3.05	30.2	21.1	21.09	0.692
90	0.972	1.009	3.13	31.9	21.5	22.10	0.690
100	0.946	1.009	3.21	33.6	21.9	23.13	0.688
120	0.898	1.009	3.34	36.8	22.8	25.45	0.686
140	0.854	1.013	3.49	40.3	23.7	27.80	0.684
160	0.815	1.017	3.64	43.9	24.5	30.09	0.682
180	0.779	1.022	3.78	47.5	25.3	32.49	0.681
200	0.746	1.026	3.93	51.4	26.0	34.85	0.680
250	0.674	1.038	4.27	61.0	27.4	40.61	0.677
300	0.615	1.047	4.60	71.6	29.7	48.33	0.674
350	0.566	1.059	4.91	81.9	31.4	55.46	0.676
400	0.524	1.068	5.21	93.1	33.0	63.09	0.678
500	0.456	1.093	5.74	115.3	36.2	79.38	0.687
600	0.404	1.114	6.22	138.3	39.1	96.89	0.699
700	0.362	1.135	6.71	163.4	41.8	115.4	0.706
800	0.329	1.156	7.18	188.8	44.3	134.8	0.713
900	0.301	1.172	7.63	216.2	46.7	155.1	0.717
1000	0.277	1.185	8.07	245.9	49.0	177.1	0.719
1100	0.257	1.197	8.50	276.2	51.2	199.3	0.722
1200	0.239	1.210	9.15	316.5	53.5	233.7	0.724

Table 10

Water Boiling Temperature Depending on Pressure [2]

$p \times 10^{-5}$, Pa	t_s, °C	$p \times 10^{-5}$, Pa	t_s, °C	$p \times 10^{-5}$, Pa	t_s, °C	$p \times 10^{-5}$, Pa	t_s, °C	$p \times 10^{-5}$, Pa	t_s, °C
1	99.64	26	226.03	72	287.71	122	325.90	172	353.21
2	120.33	27	228.06	74	289.58	124	327.15	174	353.17
3	133.54	28	230.04	76	291.41	126	328.39	176	355.11
4	143.62	29	231.96	78	293.22	128	329.61	178	356.04
5	151.84	30	233.83	80	294.98	130	330.81	180	356.96
6	158.84	32	237.44	82	296.71	132	332.00	182	357.87
7	164.96	34	240.88	84	298.40	134	333.18	184	358.78
8	170.42	36	244.16	86	300.07	136	334.34	186	359.67
9	175.35	38	247.31	88	301.71	138	335.49	188	360.56
10	179.88	40	250.33	90	303.32	140	336.63	190	361.44
11	184.05	42	253.24	92	304.90	142	337.75	192	362.31
12	187.95	44	256.05	94	306.45	144	338.86	194	363.17
13	191.60	46	258.75	96	307.98	146	339.96	196	364.02
14	195.04	48	261.37	98	309.49	148	341.04	198	364.87
15	198.28	50	263.91	100	310.96	150	342.11	200	365.71
16	201.36	52	266.38	102	312.42	152	343.18	202	366.54
17	204.30	54	268.77	104	313.86	154	344.23	204	367.37
18	207.10	56	271.10	106	315.28	156	345.27	206	368.18
19	209.78	58	273.36	108	316.67	158	346.30	208	368.99
20	212.37	60	275.56	110	318.04	160	347.32	210	369.79
21	214.84	62	277.71	112	319.39	162	348.33	212	370.58
22	217.24	64	279.80	114	320.73	164	349.32	214	371.4
23	219.56	66	281.85	116	322.05	166	350.31	216	372.2
24	221.77	68	283.85	118	323.35	168	351.29	218	372.9
25	223.93	70	285.80	120	324.63	170	352.26	220	373.7
								Critical 221.29	state 374.15

Table 11

Physical Properties of Saturated Water [2 and 13]

t, °C	$p\times10^{-5}$, Pa	ρ, kg/m³	i', kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^2$, W/m·°C	$a\times10^8$, m²/s	$\mu\times10^6$, Pa·s	$\nu\times10^6$, m²/s	$\beta\times10^4$, K⁻¹	$\sigma\times10^4$, N/m	Pr
0	1.013	999.9	0	4.212	55.1	13.1	1788	1.789	−0.63	756.4	13.67
10	1.013	999.7	42.04	4.191	57.4	13.7	1306	1.306	+0.70	741.6	9.52
20	1.013	998.2	83.91	4.183	59.9	14.3	1004	1.006	1.82	726.9	7.02
30	1.013	995.7	125.7	4.174	61.8	14.9	801.5	0.805	3.21	712.2	5.42
40	1.013	992.2	167.5	4.174	63.5	15.3	653.3	0.659	3.87	696.5	4.31
50	1.013	988.1	209.3	4.174	64.8	15.7	549.4	0.556	4.49	676.9	3.54
60	1.013	983.2	251.1	4.179	65.9	16.0	469.9	0.478	5.11	662.2	3.98
70	1.013	977.8	293	4.187	66.8	16.3	406.1	0.415	5.70	643.5	2.55
80	1.013	971.8	335.0	4.195	67.4	16.6	355.1	0.365	6.32	625.9	2.21
90	1.013	965.3	377.0	4.208	68.0	16.8	314.9	0.326	6.95	607.2	1.95
100	1.013	958.4	419.1	4.220	68.3	16.9	282.5	0.295	7.52	588.6	1.75
110	1.43	951.0	461.4	4.233	68.5	17.0	259.0	0.272	8.08	569.0	1.60
120	1.98	943.1	503.7	4.250	68.6	17.1	237.4	0.252	8.64	548.4	1.47
130	2.70	934.8	546.4	4.266	68.6	17.2	217.8	0.233	9.19	528.8	1.36
140	3.61	926.1	589.1	4.287	68.5	17.2	201.1	0.217	9.72	507.2	1.26
150	4.76	917.0	632.2	4.313	68.4	17.3	186.4	0.203	10.3	486.6	1.17
160	6.18	907.4	675.4	4.346	68.3	17.3	173.6	0.191	10.7	466.0	1.10
170	7.92	897.3	719.3	4.380	67.9	17.3	162.8	0.181	11.3	443.4	1.05
180	10.03	886.9	763.3	4.417	67.4	17.2	153.0	0.173	11.9	422.8	1.00
190	12.55	876.0	807.8	4.459	67.0	17.1	144.2	0.165	12.6	400.2	0.96

Table 11 (continued)

t, °C	$p \times 10^{-5}$, Pa	ρ, kg/m³	i', kJ/kg	c_p, kJ/kg·°C	$\lambda \times 10^2$, W/m·°C	$a \times 10^8$, m²/s	$\mu \times 10^6$, Pa·s	$\nu \times 10^6$, m²/s	$\beta \times 10^4$, K⁻¹	$\sigma \times 10^4$, N/m	Pr
200	15.55	863.0	852.5	4.505	66.3	17.0	136.4	0.158	13.3	376.7	0.93
210	19.08	852.8	897.7	4.555	65.5	16.9	130.5	0.153	14.1	354.1	0.91
220	23.20	840.3	943.7	4.614	64.5	16.6	124.6	0.148	14.8	331.6	0.89
230	27.98	827.3	990.2	4.681	63.7	16.4	119.7	0.145	15.9	310.0	0.88
240	33.48	813.6	1037.5	4.756	62.8	16.2	114.8	0.141	16.8	285.5	0.87
250	39.78	799.0	1085.7	4.844	61.8	15.9	109.9	0.137	18.1	261.9	0.86
260	46.94	784.0	1135.1	4.949	60.5	15.6	105.9	0.135	19.7	237.4	0.87
270	55.05	767.9	1185.3	5.070	59.0	15.1	102.0	0.133	21.6	214.8	0.88
280	64.19	750.7	1236.8	5.230	57.4	14.6	98.1	0.131	23.7	191.3	0.90
290	74.45	732.3	1290.0	5.485	55.8	13.9	94.2	0.129	26.2	168.7	0.93
300	85.92	712.5	1344.9	5.736	54.0	13.2	91.2	0.128	29.2	144.2	0.97
310	98.70	691.1	1402.2	6.071	52.3	12.5	88.3	0.128	32.9	120.7	1.03
320	112.90	667.1	1462.1	6.574	50.6	11.5	85.3	0.128	38.2	98.10	1.11
330	128.65	640.2	1526.2	7.244	48.4	10.4	81.4	0.127	43.3	76.71	1.22
340	146.08	610.1	1594.8	8.165	45.7	9.17	77.5	0.127	53.4	56.70	1.39
350	165.37	574.4	1671.4	9.504	43.0	7.88	72.6	0.126	66.8	38.16	1.60
360	186.74	528.0	1761.5	13.984	39.5	5.36	66.7	0.126	109	20.21	2.35
370	210.53	450.5	1892.5	40.321	33.7	1.86	56.9	0.126	264	4.709	6.79

Table 12

Physical Properties of Saturated Steam [2 and 13]

t, °C	$p \times 10^{-5}$, Pa	ρ'', kg/m^3	i'', kJ/kg	r, kJ/kg	c_p, kJ/kg·°C	$\lambda \times 10^2$, W/m·°C	$a \times 10^6$, m^2/s	$\mu \times 10^6$, Pa·s	$\nu \times 10^6$, m^2/s	Pr
100	1.013	0.598	2675.9	2256.8	2.135	2.372	18.58	11.97	20.02	1.08
110	1.43	0.826	2691.4	2230.0	2.177	2.489	13.83	12.46	15.07	1.09
120	1.98	1.121	2706.5	2202.8	2.206	2.593	10.50	12.85	11.46	1.09
130	2.70	1.496	2720.7	2174.3	2.257	2.686	7.972	13.24	8.85	1.11
140	3.61	1.966	2734.1	2145.0	2.315	2.791	6.130	13.54	6.89	1.12
150	4.76	2.547	2746.7	2114.4	2.395	2.884	4.728	13.93	5.47	1.16
160	6.18	3.258	2758.0	2082.6	2.479	3.012	3.722	14.32	4.39	1.18
170	7.92	4.122	2768.9	2049.5	2.583	3.128	2.939	14.72	3.57	1.12
180	10.03	5.157	2778.5	2015.2	2.709	3.268	2.339	15.11	2.93	1.25
190	12.55	6.394	2786.4	1978.8	2.856	3.419	1.872	15.60	2.44	1.30
200	15.55	7.862	2793.1	1940.7	3.023	3.547	1.492	15.99	2.03	1.36
210	19.08	9.588	2798.2	1900.5	3.199	3.722	1.214	16.38	1.71	1.41
220	23.20	11.62	2801.5	1857.8	3.408	3.896	0.983	16.87	1.45	1.47
230	27.98	13.99	2803.2	1813.0	3.634	4.094	0.806	17.36	1.24	1.54
240	33.48	16.76	2803.2	1765.6	3.881	4.291	0.658	17.76	1.06	1.61

Table 12 (continued)

t, °C	$p\times10^{-5}$, Pa	ρ'', kJ/m^3	i'', kJ/kg	r, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^2$, W/m·°C	$a\times10^6$, m^2/s	$\mu\times10^6$, Pa·s	$\nu\times10^6$, m^2/s	Pr
250	39.78	19.98	2801.1	1715.8	4.158	4.512	0.544	18.25	0.913	1.68
260	46.94	23.72	2796.5	1661.4	4.468	4.803	0.453	18.84	0.794	1.75
270	55.05	28.09	2789.8	1604.4	4.815	5.106	0.378	19.32	0.688	1.82
280	64.19	33.19	2779.7	1542.9	5.234	5.489	0.317	19.91	0.600	1.90
290	74.45	39.15	2766.4	1476.3	5.694	5.827	0.261	20.60	0.526	2.01
300	85.92	46.21	2749.2	1404.3	6.280	6.268	0.216	21.29	0.461	2.13
310	97.70	54.58	2727.4	1325.2	7.118	6.838	0.176	21.97	0.403	2.29
320	112.90	64.72	2700.2	1238.1	8.206	7.513	0.141	22.86	0.353	2.50
330	128.65	77.10	2665.9	1139.7	9.881	8.257	0.108	23.94	0.310	2.86
340	146.08	92.76	2621.9	1027.1	12.35	9.304	0.0811	25.21	0 272	3.35
350	165.37	113.6	2564.5	893.1	16.24	10.70	0.0580	26.58	0.234	4.03
360	186.74	144.0	2481.2	719.7	23.03	12.79	0.0386	29.14	0.202	5.23
370	210.53	203.0	2330.9	438.4	56.52	17.10	0.0150	33.75	0.166	11.10

Table 13

Physical Properties of Water at a Pressure $p = 240 \times 10^5$ Pa * [23]

t, °C	$v \times 10^3$, m³/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda \times 10^3$, W/m·°C	$\mu \times 10^7$, Pa·s	Pr
350	1.611	1630	7.16	471	750	1.14
351	1.620	1637	7.25	468	746	1.16
352	1.629	1645	7.34	465	741	1.17
353	1.639	1652	7.44	462	736	1.18
354	1.648	1660	7.55	459	731	1.19
355	1.658	1167	7.67	456	726	1.21
356	1.669	1675	7.79	453	721	1.24
357	1.680	1683	7.93	450	716	1.26
358	1.691	1691	8.07	447	711	1.28
359	1.703	1699	8.23	443	705	1.31
360	1.716	1707	8.42	440	700	1.34
361	1.729	1715	8.60	436	694	1.37
362	1.742	1724	8.81	433	688	1.40
363	1.757	1733	9.04	429	682	1.44
364	1.772	1743	9.30	425	676	1.48
365	1.788	1752	9.58	421	670	1.52
366	1.805	1762	9.91	416	663	1.58
367	1.824	1772	10.28	412	656	1.64
368	1.844	1782	10.71	407	649	1.71
369	1.865	1793	11.20	402	642	1.79
370	1.889	1805	11.79	396	634	1.89
371	1.915	1817	12.49	390	625	2.00
372	1.944	1830	13.33	383	616	2.14
373	1.976	1844	14.37	376	606	2.32
374	2.014	1859	15.72	367	596	2.55
375	2.057	1875	17.44	358	576	2.80
376	2.109	1894	19.84	346	564	3.23
377	2.174	1915	23.40	332	549	3.87
378	2.259	1942	29.26	314	532	4.96
379	2.389	1976	40.71	293	509	7.07
380	2.596	2028	68.38	269	468	11.90
381	3.068	2125	119.36	242	426	21.01

Table 13 (continued)

t, °C	$v\times10^3$, m^3/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^3$, W/m·°C	$\mu\times10^7$, Pa·s	Pr
382	3.698	2234	86.59	219	383	15.14
383	4.134	2302	56.17	204	363	9.90
384	4.458	2349	41.57	194	351	7.52
385	4.718	2386	33.38	187	343	6.12
386	4.940	2416	28.17	181	338	5.26
387	5.136	2443	24.56	176	333	4.65
388	5.311	2466	21.89	172	329	4.19
389	5.471	2486	19.84	168	326	3.85
390	5.619	2505	18.21	165	324	3.58
391	5.758	2523	16.87	162	322	3.35
392	5.888	2539	15.76	160	320	3.15
393	6.011	2554	14.81	157	318	3.00
394	6.128	2569	14.00	155	317	2.86
395	6.240	2582	13.29	153	316	2.74
396	6.347	2595	12.66	151	314	2.63
397	6.450	2608	12.11	149	313	2.54
398	6.550	2620	11.62	148	312	2.45
399	6.646	2631	11.17	146	312	2.39
400	6.738	2642	10.76	144	311	2.31
401	6.828	2653	10.40	143	310	2.26
402	6.916	2663	10.06	142	310	2.20
403	7.001	2673	9.75	140	309	2.15
404	7.084	2682	9.46	139	308	2.10
405	7.165	2692	9.20	138	308	2.06
406	7.244	2701	8.95	136	308	2.02
407	7.322	2710	8.72	135	307	1.98
408	7.397	2718	8.50	134	307	1.94
409	7.471	2727	8.30	133	306	1.91
410	7.544	2735	8.12	132	306	1.88
411	7.615	2743	7.94	131	306	1.85
412	7.685	2751	7.77	130	306	1.83
413	7.754	2758	7.61	129	306	1.80
414	7.821	2766	7.46	128	305	1.78

Table 13 (concluded)

t, °C	$v\times10^3$, m³/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^3$, W/m·°C	$\mu\times10^7$, Pa·s	Pr
415	7.888	2773	7.32	127	305	1.73
416	7.953	2781	7.19	126	305	1.73
417	8.017	2787	7.06	126	305	1.72
418	8.081	2795	6.94	125	305	1.70
419	8.143	2802	6.82	124	305	1.68
420	8.205	2808	6.71	123	305	1.66
422	8.326	2821	6.50	122	304	1.63
424	8.443	2834	6.31	120	304	1.60
426	8.558	2847	6.14	119	304	1.58
428	8.670	2859	5.98	117	304	1.55
430	8.779	2871	5.83	116	304	1.53
432	8.886	2882	5.69	115	305	1.51
434	8.991	2893	5.56	113	305	1.49
436	9.094	2904	5.44	112	305	1.48
438	9.195	2915	5.32	111	305	1.47
440	9.294	2926	5.22	110	305	1.45
442	9.392	2936	5.12	109	306	1.43
444	9.488	2946	5.02	108	306	1.42
446	9.582	2956	4.93	108	306	1.40
448	9.675	2966	4.85	107	307	1.39
450	9.766	2976	4.77	106	307	1.37
452	9.857	2985	4.70	106	307	1.36
454	9.946	2994	4.62	106	308	1.35
456	10.03	3004	4.56	105	308	1.34
458	10.12	3013	4.49	105	308	1.32
460	10.21	3022	4.43	104	309	1.31
465	10.41	3043	4.23	103	310	1.29
470	10.62	3064	4.16	103	311	1.26
475	10.82	3085	4.05	102	312	1.24
480	11.01	3105	3.95	102	314	1.22
485	11.20	3125	3.85	101	315	1.20
490	11.38	3144	3.77	101	316	1.18
495	11.56	3162	3.69	101	318	1.16
500	11.74	3181	3.62	101	319	1.15

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* $p_{cr}=221.2\times10^5$ Pa; $t_{cr}=374.1$ °C. At $p=240\times10^5$ Pa $t_m=380.7$ °C.

Table 14

Physical Properties of Water at Pressure $p = 300 \times 10^5$ Pa * [23]

t, °C	$v \times 10^3$, m³/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda \times 10^3$, W/m·°C	$\mu \times 10^7$, Pa·s	Pr
350	1.554	1611	6.44	494	778	1.01
352	1.568	1624	6.54	489	770	1.03
354	1.582	1637	6.64	484	762	1.05
356	1.597	1651	6.76	478	754	1.06
358	1.612	1664	6.88	473	746	1.08
360	1.629	1679	7.02	467	737	1.11
362	1.646	1693	7.17	462	729	1.13
364	1.665	1707	7.34	456	720	1.16
366	1.684	1722	7.53	450	711	1.19
368	1.705	1737	7.75	444	701	1.22
370	1.727	1753	7.99	437	692	1.26
372	1.751	1769	8.26	430	682	1.31
374	1.777	1786	8.59	423	671	1.36
376	1.806	1804	8.95	415	654	1.41
378	1.837	1822	9.39	407	643	1.48
380	1.872	1841	9.90	398	632	1.57
381	1.891	1851	10.19	393	626	1.62
382	1.911	1862	10.51	388	620	1.68
383	1.932	1872	10.86	383	614	1.74
384	1.955	1884	11.25	378	608	1.81
385	1.980	1895	11.67	372	601	1.89
386	2.006	1907	12.14	366	594	1.97
387	2.034	1919	12.66	359	587	2.07
388	2.065	1932	13.23	352	580	2.18
389	2.098	1946	13.87	345	572	2.30
390	2.135	1960	14.59	338	563	2.43
391	2.175	1975	15.40	330	555	2.59
392	2.219	1991	16.30	322	546	2.76
393	2.268	2008	17.32	315	536	2.95
394	2.322	2026	18.46	307	526	3.17

Table 14 (continued)

t, °C	$v\times10^3$, m^3/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^3$, W/m·°C	$\mu\times10^7$, Pa·s	Pr
395	2.383	2045	19.68	299	516	3.40
396	2.450	2065	20.90	291	505	3.62
397	2.526	2086	22.24	284	494	3.88
398	2.610	2109	23.55	276	483	4.12
399	2.703	2133	24.79	269	471	4.34
400	2.804	2159	25.71	262	460	4.52
401	2.914	2185	26.48	255	448	4.66
402	3.031	2212	26.69	248	438	4.70
403	3.152	2238	26.43	242	428	4.67
404	3.276	2264	25.79	236	418	4.58
405	3.400	2290	24.89	230	410	4.43
406	3.523	2314	23.84	225	403	4.29
407	3.644	2337	22.74	220	396	4.10
408	3.762	2360	21.63	215	390	3.92
409	3.876	2381	20.56	211	385	3.76
410	3.986	2401	19.54	206	380	3.60
411	4.094	2420	18.58	202	376	3.45
412	4.198	2438	17.70	199	372	3.31
413	4.298	2455	16.88	196	368	3.18
414	4.395	2472	16.12	192	365	3.06
415	4.489	2487	15.43	190	363	2.95
416	4.580	2503	14.79	187	360	2.85
417	4.669	2517	14.20	184	358	2.76
418	4.755	2531	13.66	182	356	2.67
419	4.838	2544	13.10	179	354	2.60
420	4.920	2557	12.69	177	352	2.52
422	5.076	2582	11.86	173	349	2.39
424	5.225	2605	11.15	169	346	2.28
426	5.367	2626	10.52	166	344	2.18
428	5.503	2647	9.28	163	342	2.10

Table 14 (concluded)

t, °C	$v\times10^3$, m³/kg	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^3$, W/m·°C	$\mu\times10^7$, Pa·s	Pr
430	5.634	2666	9.49	160	340	2.02
432	5.759	2685	9.06	157	338	1.95
434	5.880	2703	8.67	154	337	1.89
436	5.998	2720	8.33	152	336	1.84
438	6.111	2736	8.01	150	335	1.79
440	6.222	2752	7.73	148	334	1.75
442	6.329	2767	7.47	146	333	1.71
444	6.433	2782	7.23	144	333	1.67
446	6.535	2796	7.01	142	332	1.64
448	6.634	2810	6.81	140	332	1.61
450	6.731	2823	6.62	139	331	1.58
452	6.826	2836	6.45	137	331	1.56
454	6.918	2849	6.29	136	331	1.53
456	7.009	2861	6.14	134	330	1.51
458	7.098	2874	6.00	133	330	1.49
460	7.185	2885	5.87	131	330	1.48
462	7.271	2897	5.74	130	330	1.46
464	7.355	2908	5.63	129	330	1.44
466	7.438	2920	5.52	127	330	1.43
468	7.520	2930	5.42	126	330	1.42
470	7.600	2941	5.32	125	330	1.40
475	7.795	2967	5.10	122	330	1.38
480	7.983	2992	4.90	121	331	1.34
485	8.165	3016	4.73	119	331	1.32
490	8.342	3040	4.58	118	332	1.29
495	8.514	3062	4.44	117	333	1.26
500	8.681	3084	4.32	116	334	1.24

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* $p_{cr} = 221.2\cdot10^5$ Pa; $t_{cr} = 374.1$ °C; At $p = 300\times10^5$ Pa $t_m = 401.6$ °C.

Table 15

Physical Properties of Carbon Dioxide at a Pressure $p = 100 \times 10^5$ Pa *

T, K	ρ, kg/m³	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^2$, W/m·°C	$a\times10^8$, m²/s	$\mu\times10^5$, Pa·s	$\beta\times10^4$, K⁻¹	Pr
275	964.6	502.0	2.20	11.9	5.59	11.1	52.9	2.06
280	938.1	513.1	2.28	11.3	5.31	10.3	58.4	2.07
285	909.6	524.8	2.38	10.8	5.01	9.52	65.6	2.09
290	878.2	537.0	2.51	10.3	4.67	8.76	75.4	2.14
295	843.0	550.0	2.70	9.77	4.30	8.00	89.4	2.21
300	802.1	564.1	2.99	9.23	3.85	7.23	111	2.34
301	793.0	567.1	3.06	9.11	3.75	7.07	117	2.38
302	783.6	570.2	3.15	9.00	3.65	6.91	123	2.42
303	773.7	573.4	3.24	8.88	3.55	6.75	131	2.46
304	763.3	576.7	3.35	8.76	3.43	6.58	139	2.51
305	752.4	580.1	3.47	8.64	3.31	6.41	149	2.57
306	740.9	583.7	3.61	8.52	3.19	6.24	160	2.64
307	728.7	587.3	3.76	8.39	3.06	6.06	173	2.72
308	715.7	591.2	3.95	8.26	2.92	5.88	188	2.81
309	701.8	595.3	4.18	8.12	2.77	5.69	206	2.92
310	686.7	599.5	4.23	7.97	2.62	5.50	228	3.05
311	670.4	604.1	4.74	7.81	2.46	5.29	254	3.21
312	652.5	609.0	5.11	7.65	2.29	5.08	286	3.39
313	632.9	614.4	5.55	7.47	2.12	4.86	326	3.61
314	611.2	620.2	6.08	7.27	1.96	4.63	373	3.87
315	587.2	626.6	6.68	7.05	1.80	4.38	428	4.15
316	561.0	633.5	7.28	6.81	1.67	4.13	487	4.42
317	532.9	641.1	7.79	6.54	1.58	3.89	538	4.63
318	504.0	649.0	8.05	6.26	1.54	3.65	571	4.69
319	475.9	657.1	7.98	5.98	1.57	3.44	575	4.57
320	449.7	664.9	7.63	5.71	1.66	3.25	553	4.35
321	426.2	672.2	7.10	5.46	1.80	3.10	516	4.03
322	405.7	679.1	6.52	5.24	1.98	2.97	472	3.70
323	387.9	685.3	5.96	5.04	2.18	2.87	429	3.39
324	372.3	691.0	5.46	4.87	2.40	2.78	3[illegible]	3.12
325	358.8	696.2	5.01	4.72	2.62	2.71	354	2.88
326	346.8	701.1	4.63	4.59	2.86	2.66	324	2.68

Table 15 (continued)

T, K	ρ, kg/m^3	i, kJ/kg	c_p, kJ/kg·°C	$\lambda\times10^2$, W/m·°C	$a\times10^8$, m^2/s	$\mu\times10^6$, Pa·s	$\beta\times10^4$, K^{-1}	Pr
327	336.3	705.5	4.30	4.47	3.09	2.61	297	2.51
328	326.8	709.7	4.02	4.37	3.33	2.56	275	2.36
329	318.3	713.6	3.77	4.28	3.56	2.53	255	2.23
330	310.5	717.2	3.56	4.19	3.80	2.50	238	2.12
331	303.4	720.7	3.37	4.12	4.03	2.47	223	2.02
332	296.9	724.0	3.21	4.05	4.25	2.44	210	1.94
333	290.9	727.1	3.07	3.99	4.47	2.42	198	1.86
334	285.4	730.1	2.94	3.94	4.70	2.40	188	1.79
335	280.2	733.0	2.82	3.88	4.91	2.39	179	1.73
336	275.4	735.8	2.72	3.84	5.12	2.37	170	1.68
337	270.8	738.4	2.63	3.79	5.33	2.36	163	1.63
338	266.5	741.0	2.54	3.75	5.54	2.34	156	1.59
339	262.5	743.5	2.47	3.71	5.74	2.33	150	1.55
340	258.7	746.0	2.40	3.68	5.94	2.32	144	1.51
341	255.1	748.3	2.33	3.65	6.13	2.31	139	1.48
342	251.6	750.6	2.27	3.62	6.33	2.30	134	1.45
343	248.3	752.9	2.22	3.59	6.51	2.30	128	1.42
344	245.2	755.1	2.17	3.56	6.70	2.29	125	1.39
345	242.2	757.2	2.12	3.54	6.90	2.28	121	1.37
346	239.3	759.3	2.08	3.51	7.07	2.28	118	1.35
347	236.6	761.4	2.04	3.49	7.25	2.27	114	1.33
348	233.9	763.4	2.00	3.47	7.43	2.27	111	1.31
349	231.4	765.3	1.96	3.45	7.60	2.26	108	1.29
350	228.9	767.3	1.93	3.43	7.77	2.26	105	1.27
352	224.3	771.1	1.87	3.40	8.11	2.25	100	1.24
354	219.9	774.8	1.82	3.37	8.44	2.25	95.5	1.21
356	215.9	778.4	1.77	3.35	8.76	2.24	91.3	1.19
358	212.0	781.9	1.73	3.32	9.08	2.24	87.6	1.16
360	208.4	785.3	1.69	3.30	9.39	2.24	84.2	1.14
370	193.0	801.4	1.54	3.23	10.9	2.24	71.0	1.07
380	180.6	816.2	1.44	3.20	12.3	2.25	61.8	1.02
390	170.4	830.3	1.38	3.19	13.6	2.27	55.0	0.979
400	161.7	843.8	1.33	3.20	14.9	2.29	49.7	0.950

Table 15 (concluded)

T, K	ρ, kg/m³	i, kJ/kg	c_p, kJ/kg·°C	$\lambda \times 10^2$, W/m·°C	$a \times 10^8$, m²/s	$\mu \times 10^5$, Pa·s	$\beta \times 10^4$, K⁻¹	Pr
450	131.7	906.6	1.21	3.39	21.3	2.44	34.6	0.871
500	113.1	965.8	1.17	3.69	27.9	2.62	27.2	0.830
550	99.9	1024	1.16	4.04	34.5	2.80	22.7	0.803
600	89.9	1082	1.16	4.40	42.2	2.97	19.6	0.784
650	82.0	1140	1.17	4.77	49.8	3.15	17.4	0.770
700	75.5	1199	1.18	5.14	57.8	3.31	15.7	0.760
750	70.0	1258	1.19	5.51	66.0	3.47	14.3	0.752
800	65.4	1318	1.21	5.87	74.5	3.63	13.2	0.746

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* $p_{cr} = 73.9 \times 10^5$ Pa; $T_{cr} = 304$ K. At $p = 100 \times 10^5$ Pa; $T_m \approx 318$ K.

Table 16

Physical Properties of Flue Gas

($B = 760$ mm Hg $\approx 1.01 \times 10^5$ Pa; $\bar{p}_{CO_2} = 0.13$; $\bar{p}_{H_2O} = 0.11$; $\bar{p}_{N_2} = 0.76$) [13]

t, °C	ρ, kg/m³	c_p, kJ/kg·°C	$\lambda \times 10^2$, W/m·°C	$a \times 10^6$, m²/s	$\mu \times 10^6$, Pa·s	$\nu \times 10^6$, m²/s	Pr
0	1.295	1.042	2.28	16.9	15.8	12.20	0.72
100	0.950	1.068	3.13	30.8	20.4	21.54	0.69
200	0.748	1.097	4.01	48.9	24.5	32.80	0.67
300	0.617	1.122	4.84	69.9	28.2	45.81	0.65
400	0.525	1.151	5.70	94.3	31.7	60.38	0.64
500	0.457	1.185	6.56	121.1	34.8	76.30	0.63
600	0.405	1.214	7.42	150.9	37.9	93.61	0.62
700	0.363	1.239	8.27	183.8	40.7	112.1	0.61
800	0.330	1.264	9.15	219.7	43.4	131.8	0.60
900	0.301	1.290	10.0	258.0	45.9	152.5	0.59
1000	0.275	1.306	10.90	303.4	48.4	174.3	0.58
1100	0.257	1.323	11.75	345.5	50.7	197.1	0.57
1200	0.240	1.340	12.62	392.4	53.0	221.0	0.56

Table 17

Dependence of Properties of Transformer Oil on Temperature [27]

t, °C	ρ, kg/m^3	c_p, kJ/kg·°C	λ, W/m·°C	$\mu \times 10^4$, Pa·s	$\nu \times 10^6$, m^2/s	$a \times 10^8$, m^2/s	$\beta \times 10^4$, K^{-1}	Pr
0.0	892.5	1.549	0.1123	629.8	70.5	8.14	6.80	866
10	886.4	1.620	0.1115	335.5	37.9	7.83	6.85	484
20	880.3	1.666	0.1106	198.2	22.5	7.56	6.90	298
30	874.2	1.729	0.1098	128.5	14.7	7.28	6.95	202
40	868.2	1.788	0.1090	89.4	10.3	7.03	7.00	146
50	862.1	1.846	0.1082	65.3	7.58	6.80	7.05	111
60	856.0	1.905	0.1072	49.5	5.78	6.58	7.10	87.8
70	850.0	1.964	0.1064	38.6	4.54	6.36	7.15	71.3
80	843.9	2.026	0.1056	30.8	3.66	6.17	7.20	59.3
90	837.8	2.085	0.1047	25.4	3.03	6.00	7.25	50.5
100	831.8	2.144	0.1038	21.3	2.56	5.83	7.30	43.9
110	825.7	2.202	0.1030	18.1	2.20	5.67	7.35	38.8
120	819.6	2.261	0.1022	15.7	1.92	5.50	7.40	34.9

Table 18

Dependence of Physical Properties of Oil, Grade MC-20, on Temperature [24]

t, °C	ρ, kg/m³	c_p, kJ/kg·°C	λ, W/m·°C	$\mu\times10^4$, Pa·s	$\nu\times10^6$, m²/s	$a\times10^8$, m²/s	$\beta\times10^4$, K⁻¹	Pr
−10	990.3	1.951	0.136	—	—	7.75	6.24	—
0	903.6	1.980	0.135	—	—	7.58	6.24	—
+10	897.9	2.010	0.135	—	—	7.44	6.31	—
20	892.3	2.043	0.134	10 026	1125	7.30	6.35	15 400
30	886.6	2.072	0.132	4 670	526	7.19	6.38	7 310
40	881.0	2.106	0.131	2 433	276	7.08	6.42	3 890
50	875.3	2.135	0.130	1 334	153	7.00	6.46	2 180
60	869.6	2.165	0.129	798.5	91.9	6.86	6.51	1 340
70	864.0	2.198	0.128	498.3	58.4	6.75	6.55	865
80	858.3	2.227	0.127	336.5	39.2	6.67	6.60	538
90	852.7	2.261	0.126	234.4	27.5	6.56	6.64	420
100	847.0	2.290	0.126	171.7	20.3	6.44	6.69	315
110	841.3	2.320	0.124	132.4	15.7	6.36	6.73	247
120	835.7	2.353	0.123	101.0	12.1	6.25	6.77	193
130	830.0	2.382	0.122	79.76	9.61	6.17	6.82	156
140	824.4	2.420	0.121	61.80	7.50	6.08	6.87	123
150	818.7	2.445	0.120	53.17	6.50	6.00	6.92	108

Table 19

Physical Properties of Mercury and Some Molten Metals [9]

Name of metal	t, °C	ρ, kg/m³	λ, W/m·°C	c_p, kJ/kg·°C	$a\times10^6$, m²/s	$\nu\times10^8$, m²/s	$\mathrm{Pr}\times10^2$
Mercury Hg	20	13 550	7.90	0.1390	4.36	11.4	2.72
$t_m=-38.9$ °C; $t_b=357$ °C;	100	13 350	8.95	0.1373	4.89	9.4	1.92
$r_m=11.72$ kJ/kg;	150	13 230	9.65	0.1373	5.30	8.6	1.62
$r_{ev}=291.8$ kJ/kg.	200	13 120	10.3	0.1373	5.72	8.0	1.40
	300	12 880	11.7	0.1373	6.64	7.1	1.07
Tin Sn	250	6 980	34.1	0.255	19.2	27.0	1.41
$t_m=231.9$ °C; $t_b=2270$ °C;	300	6 940	33.7	0.255	19.0	24.0	1.26
$r_m=58.2$ kJ/kg;	400	6 865	33.1	0.255	18.9	20.0	1.06
$r_{ev}=2015$ kJ/kg.	500	6 790	32.6	0.255	18.8	17.3	0.92
Bismuth Bi	300	10 030	13.0	0.151	8.61	17.1	1.98
$t_m=271$ °C; $t_b=1477$ °C;	400	9 910	14.4	0.151	9.72	14.2	1.46
$r_m=50.2$ kJ/kg;	500	9 785	15.8	0.151	10.8	12.2	1.13
$r_{ev}=855.4$ kJ/kg	600	9 660	17.2	0.151	11.9	10.8	0.91
Lithium Li	200	515	37.2	4.187	17.2	111.0	6.43
$t_m=179$ °C; $t_b=1317$ °C;	300	505	39.0	4.187	18.3	92.7	5.03

$r_m = 661.5$ kJ/kg $r_{ev} = 19\,595$ kJ/kg.	400	495	41.9	4.187	20.3	81.7	4.04
	500	484	45.3	4.187	22.3	73.4	3.28
Alloy 56 5% Bi+43.5% Pb $t_m = 123.5$°C; $t_b = 1670$°C	150	10 550	9.8	0.146	6.39	28.9	4.50
	200	10 490	10.3	0.146	6.67	24.3	3.64
	300	10 360	11.4	0.146	7.50	18.7	2.50
	400	10 240	12.6	0.146	8.33	15.7	1.87
	500	10 120	14.0	0.146	9.44	13.6	1.44
Alloy 25% Na+75% K $t_m = -11$ °C; $t_b = 784$ °C.	100	852	23.2	1.143	23.9	60.7	2.51
	200	828	24.5	1.072	27.6	45.2	1.64
	300	808	25.8	1.038	31.0	36.6	1.18
	400	778	27.1	1.005	34.7	30.8	0.89
	500	753	28.4	0.967	39.0	26.7	0.69
	600	729	29.6	0.934	43.6	23.7	0.54
	700	704	30.9	0.900	48.8	21.4	0.44
Sodium Na $t_m = 97.8$ °C; $t_b = 883$ °C; $r_m = 113.26$ kJ/kg, $r_{ev} = 3208$ kJ/kg.	150	916	84.9	1.356	68.3	59.4	0.87
	200	903	81.4	1.327	67.8	50.6	0.75
	300	878	70.9	1.281	63.0	39.4	0.63
	400	854	63.9	1.273	58.9	33.0	0.56
	500	829	57.0	1.273	54.2	28.9	0.53

Table 20

Dependence of Physical Properties of Grade MK Oil on Temperature [24]

t, °C	ρ, kg/m³	c_p, kJ/kg·°C	λ, W/m·°C	$\mu\times10^4$, Pa·s	$\nu\times10^6$, m²/s	$a\times10^8$, m²/s	$\beta\times10^4$, K⁻¹	Pr
10	911.0	1.645	0.1510	35 414	3883	9.94	8.56	39 000
20	903.0	1.712	0.1485	18 560	1514	9.58	8.64	15 800
30	894.5	1.758	0.1461	6 180	691.2	9.28	8.71	7 450
40	887.5	1.804	0.1437	3 031	342.0	8.97	8.79	3 810
50	879.0	1.851	0.1413	1 638	186.2	8.69	8.86	2 140
60	871.5	1.897	0.1389	961.4	110.7	8.39	8.95	1 320
70	864.0	1.943	0.1363	603.3	69.3	8.14	9.03	858
80	856.0	1.989	0.1340	399.3	46.6	7.89	9.12	591
90	848.2	2.035	0.1314	273.7	32.3	7.61	9.20	424
100	840.7	2.081	0.1290	202.1	24.0	7.33	9.28	327
110	838.0	2.127	0.1264	145.2	17.4	7.11	9.37	245
120	825.0	2.173	0.1240	110.4	13.4	8.92	9.46	193.5
130	817.0	2.219	0.1214	87.31	10.7	6.69	9.54	160.0
140	809.2	2.265	0.1188	70.34	8.70	6.53	9.65	133.3
150	801.6	2.311	0.1168	56.90	7.10	6.25	9.73	113.5

Table 21

Emissivity of Various Materials [24]

Material and kind of surface	t, °C	ε
Metals		
Aluminium:		
polished	225-575	0.039-0.057
rough	26	0.055
oxidised at 600 °C	200-600	0.11-0.19
Tungsten	230-2230	0.053-0.31
Tungsten filament	3300	0.39
Iron:		
electrolytic, thoroughly polished	175-225	0.052-0.064
welding (wrought) thoroughly polished	40-250	0.28
polished	425-1020	0.14-0.377
freshly emeried	20	0.242
cast, not machined	925-1115	0.87-0.95
Steel castings, polished	770-1040	0.52-0.56
Steel:		
plate (sheet) ground	940-1100	0.52-0.61
oxidised at 600 °C	200-600	0.79
oxidised, rough	40-370	0.94-0.97
Cast iron		
polished	200	0.21
turned on a lathe	830-990	0.60-0.70
oxidized at 600 °C	200-600	0.64-0.78
rough, heavily oxidised	40-250	0.96
Gold, thoroughly polished	225-625	0.018-0.035
Brass:		
thoroughly polished, composition (by mass) 73.2% Cu, 26.7% Zn	245-355	0.028-0.031

Table 21 (continued)

Material and kind of surface	t, °C	ε
rolled with natural surface	22	0.06
dull surface	50-350	0.22
oxidised while heated to 600 °C	200-600	0.61-0.59
Copper:		
thoroughly polished, electrolytic	80	0.018
polished	115	0.023
long-time heated, coated with a thick oxide layer	25	0.78
oxidised while heated to 600 °C	200-600	0.57-0.55
Platinum, clean, polished	225-625	0.054-0.105
Platinum:		
strip	925-1115	0.12-0.17
filament	25-1230	0.036-0.192
Chromium	38-538	0.08-0.26
Refractory, building, heat-insulating and other materials		
Asbestos cardboard	24	0.96
Refractories:		
weakly emitting	500-1000	0.65-0.75
strongly emitting	500-1000	0.80-0.90
Dinas brick, rough:		
nonglazed	1000	0.8
glazed	1100	0.85
Fireclay brick, glazed	1100	0.75
Red brick, rough	20	0.93
Porcelain, glazed	22	0.92
Plaster, lime, rough	10-90	0.91
Lamp black, layer 0.075 mm thick and more	40-370	0.95

Table 22

Bessel Functions of the First Kind Zero and First Orders $J_0(x)$ and $J_1(x)$

x	$J_0(x)$	$J_1(x)$	x	$J_0(x)$	$J_1(x)$
0.0	1.0000	0.0000	2.5	—0.0484	0.4871
0.1	0.9975	0.0499	2.6	—0.0968	0.4708
0.2	0.9900	0.0995	2.7	—0.1424	0.4416
0.3	0.9776	0.1483	2.8	—0.1850	0.4097
0.4	0.9604	0.1960	2.9	—0.2243	0.3754
0.5	0.9385	0.2423	3.0	—0.2600	0.3391
0.6	0.9120	0.2867	3.1	—0.2921	0.3009
0.7	0.8812	0.3290	3.2	—0.3202	0.2613
0.8	0.8463	0.3688	3.3	—0.3443	0.2207
0.9	0.8075	0.4059	3.4	—0.3643	0.1792
1.0	0.7652	0.4400	3.5	—0.3801	0.1374
1.1	0.7196	0.4709	3.6	—0.3918	0.0055
1.2	0.6711	0.4983	3.7	—0.3992	0.0538
1.3	0.6201	0.5220	3.8	—0.4026	0.0128
1.4	0.5669	0.5419	3.9	—0.4018	—0.0272
1.5	0.5118	0.5579	4.0	—0.3971	—0.0660
1.6	0.4554	0.5699	4.1	—0.3887	—0.1033
1.7	0.3980	0.5778	4.2	—0.3766	—0.1386
1.8	0.3400	0.5815	4.3	—0.3610	—0.1719
1.9	0.2818	0.5812	4.4	—0.3423	—0.2028
2.0	0.2239	0.5767	4.5	—0.3205	—0.2311
2.1	0.1666	0.5683	4.6	—0.2961	—0.2666
2.2	0.1104	0.5560	4.7	—0.2693	—0.2791
2.3	0.0555	0.5399	4.8	—0.2404	—0.2985
2.4	0.0025	0.5202	4.9	—0.2097	—0.3147

Table 22 (continued)

x	$J_0(x)$	$J_1(x)$	x	$J_0(x)$	$J_1(x)$
5.0	—0.1776	—0.3276	7.5	0.2663	0.1352
5.1	—0.1443	—0.3371	7.6	0.2516	0.1592
5.2	—0.1103	—0.3432	7.7	0.2346	0.1813
5.3	—0.0758	—0.3460	7.8	0.2154	0.2014
5.4	—0.0412	—0.3453	7.9	0.1944	0.2192
5.5	—0.0068	—0.3414	8.0	0.1716	0.2346
5.6	0.0270	—0.3343	8.1	0.1475	0.2476
5.7	0.0599	—0.3241	8.2	0.1222	0.2580
5.8	0.0917	—0.3110	8.3	0.0960	0.2657
5.9	0.1220	—0.2951	8.4	0.0692	0.2708
6.0	0.1506	—0.2767	8.5	0.0419	0.2731
6.1	0.1773	—0.2559	8.6	0.0146	0.2728
6.2	0.2017	—0.2329	8.7	—0.0125	0.2697
6.3	0.2238	—0.2081	8.8	—0.0392	0.2641
6.4	0.2433	—0.1816	8.9	—0.0652	0.2559
6.5	0.2601	—0.1538	9.0	—0.0903	0.2453
6.6	0.2740	—0.1250	9.1	—0.1142	0.2324
6.7	0.2851	—0.0953	9.2	—0.1368	0.2174
6.8	0.2931	—0.0652	9.3	—0.1577	0.2004
6.9	0.2981	—0.0349	9.4	—0.1768	0.1816
7.0	0.3001	—0.0047	9.5	—0.1939	0.1613
7.1	0.2991	0.0252	9.6	—0.2090	0.1395
7.2	0.2951	0.0543	9.7	—0.2218	0.1116
7.3	0.2882	0.0826	9.8	—0.2323	0.0928
7.4	0.2786	0.1096	9.9	—0.2403	0.0684

Table 22 (concluded)

x	$J_0(x)$	$J_1(x)$	x	$J_0(x)$	$J_1(x)$
10.0	—0.2459	0.0435	12.5	0.1469	—0.1655
10.1	—0.2490	0.0184	12.6	0.1626	—0.1437
10.2	—0.2496	—0.0066	12.7	0.1766	—0.1307
10.3	—0.2477	—0.0312	12.8	0.1887	—0.1114
10.4	—0.2434	—0.0555	12.9	0.1988	—0.0912
10.5	—0.2366	—0.0788	13.0	0.2069	—0.0703
10.6	—0.2276	—0.1012	13.1	0.2129	—0.0488
10.7	—0.2164	—0.1224	13.2	0.2167	—0.0271
10.8	—0.2032	—0.1422	13.3	0.2183	—0.0052
10.9	—0.1881	—0.1604	13.4	0.2177	0.0166
11.0	—0.1712	—0.1768	13.5	0.2150	0.0380
11.1	—0.1528	—0.1913	13.6	0.2101	0.0590
11.2	—0.1330	—0.2038	13.7	0.2032	0.0791
11.3	—0.1121	—0.2143	13.8	0.1943	0.0984
11.4	—0.0902	—0.2224	13.9	0.1836	0.1165
11.5	—0.0677	—0.2284	14.0	0.1711	0.1334
11.6	—0.0446	—0.2320	14.1	0.1570	0.1488
11.7	—0.0213	—0.2333	14.2	0.1414	0.1626
11.8	0.0020	—0.2323	14.3	0.1245	0.1747
11.9	0.0250	—0.2290	14.4	0.1065	0.1850
12.0	0.0477	—0.2234	14.5	0.0875	0.1934
12.1	0.0696	—0.2158	14.6	0.0679	0.1989
12.2	0.0908	—0.2060	14.7	0.0476	0.2043
12.3	0.1108	—0.1943	14.8	0.0271	0.2066
12.4	0.1296	—0.1807	14.9	0.0064	0.2060
			15.0	—0.0142	0.2051

Table 23

Modified Bessel Functions of the First Kind Zero and First Orders $I_0(x)$ and $I_1(x)$

x	$I_0(x)$	$I_1(x)$	x	$I_0(x)$	$I_1(x)$
0.0	1.000	0.0000	3.0	4.881	3.953
0.1	1.0025	0.0501	3.1	5.294	4.316
0.2	1.0100	0.1005	3.2	5.747	4.734
0.3	1.0226	0.1517	3.3	6.243	5.181
0.4	1.0404	0.2040	3.4	6.785	5.670
0.5	1.0635	0.2579	3.5	7.378	6.206
0.6	1.0920	0.3137	3.6	8.028	6.793
0.7	1.1263	0.3719	3.7	8.739	7.436
0.8	1.1665	0.4329	3.8	9.517	8.140
0.9	1.2130	0.4971	3.9	10.369	8.913
1.0	1.2661	0.5652	4.0	11.30	9.76
1.1	1.3262	0.6375	4.1	12.32	10.69
1.2	1.3937	0.7174	4.2	13.44	11.71
1.3	1.4693	0.7973	4.3	14.67	12.82
1.4	1.5534	0.8861	4.4	16.01	14.05
1.5	1.6467	0.9817	4.5	17.48	15.39
1.6	1.7500	1.0848	4.6	19.09	16.86
1.7	1.8640	1.1963	4.7	20.86	18.48
1.8	1.9896	1.3172	4.8	22.79	20.25
1.9	2.1277	1.4482	4.9	24.91	22.20
2.0	2.280	1.591	5.0	27.24	24.34
2.1	2.446	1.746	5.1	29.79	26.68
2.2	2.629	1.914	5.2	32.58	29.25
2.3	2.830	2.098	5.3	35.65	32.08
2.4	3.049	2.298	5.4	39.01	35.18
2.5	3.290	2.517	5.5	42.70	38.59
2.6	3.553	2.755	5.6	46.74	42.33
2.7	3.842	3.016	5.7	51.17	46.44
2.8	4.157	3.301	5.8	56.04	50.95
2.9	4.503	3.613	5.9	61.38	55.90

Table 24

Modified Bessel Functions of the Second Kind Zero and First Orders $K_0(x)$ and $K_1(x)$

x	$K_0(x)$	$K_1(x)$	x	$K_0(x)$	$K_1(x)$
0.0	∞	∞	2.2	0.089	0.108
0.1	2.447	9.854	2.3	0.078	0.0942
0.2	1.753	4.776	2.4	0.071	0.0832
0.3	1.373	3.056	2.5	0.062	0.0739
0.4	1.115	2.184	2.6	0.055	0.0660
0.5	0.924	1.656	2.7	0.049	0.0581
0.6	0.775	1.303	2.8	0.044	0.0503
0.7	0.661	1.050	2.9	0.039	0.0456
0.8	0.565	0.862	3.0	0.0347	0.0402
0.9	0.487	0.717	3.1	0.0314	0.0361
1.0	0.421	0.602	3.2	0.0283	0.0314
1.1	0.366	0.509	3.3	0.0251	0.0283
1.2	0.318	0.435	3.4	0.0220	0.0251
1.3	0.278	0.372	3.5	0.0196	0.0222
1.4	0.244	0.320	3.6	0.0173	0.0204
1.5	0.214	0.278	3.7	0.0157	0.0173
1.6	0.188	0.241	3.8	0.0141	0.0157
1.7	0.165	0.209	3.9	0.0126	0.0141
1.8	0.146	0.183	4.0	0.0112	0.0125
1.9	0.129	0.160	4.5	0.0064	0.00708
2.0	0.114	0.140	5.0	0.0037	0.00404
2.1	0.100	0.122			

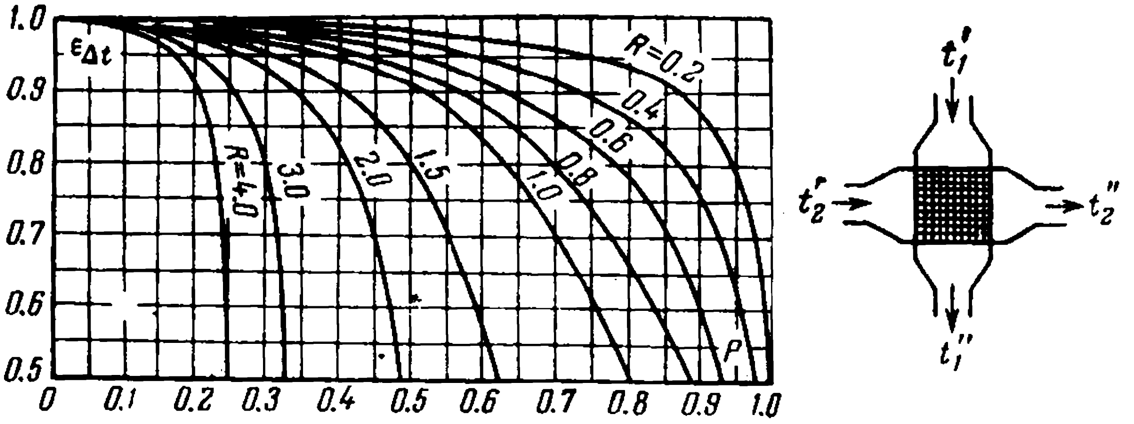


Fig. A-1

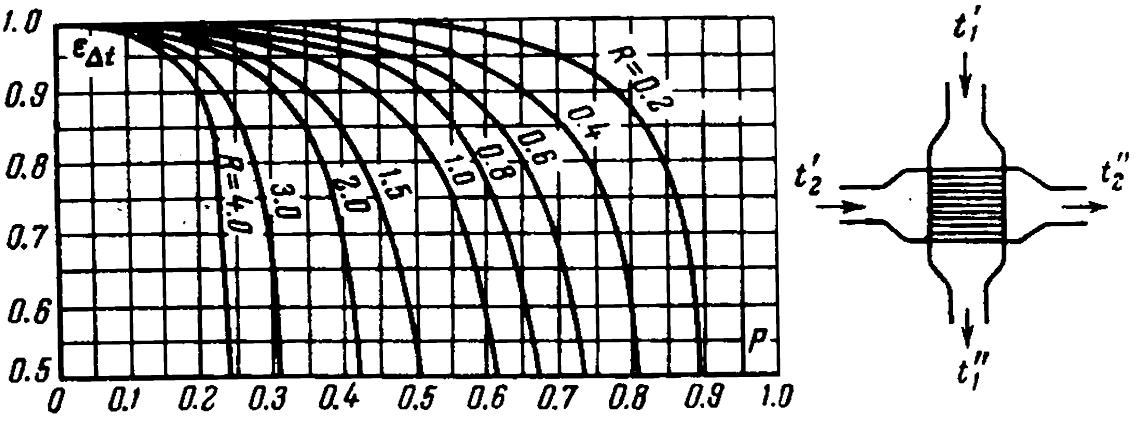


Fig A-2

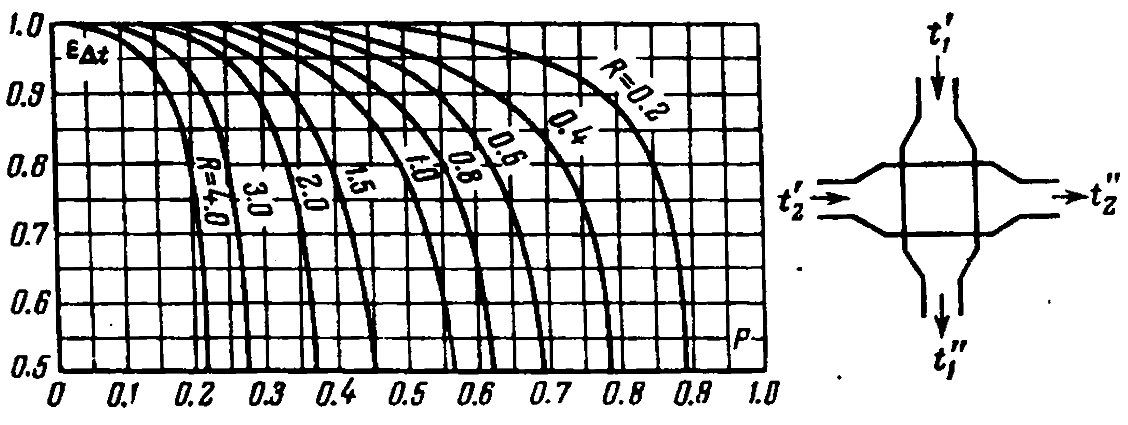
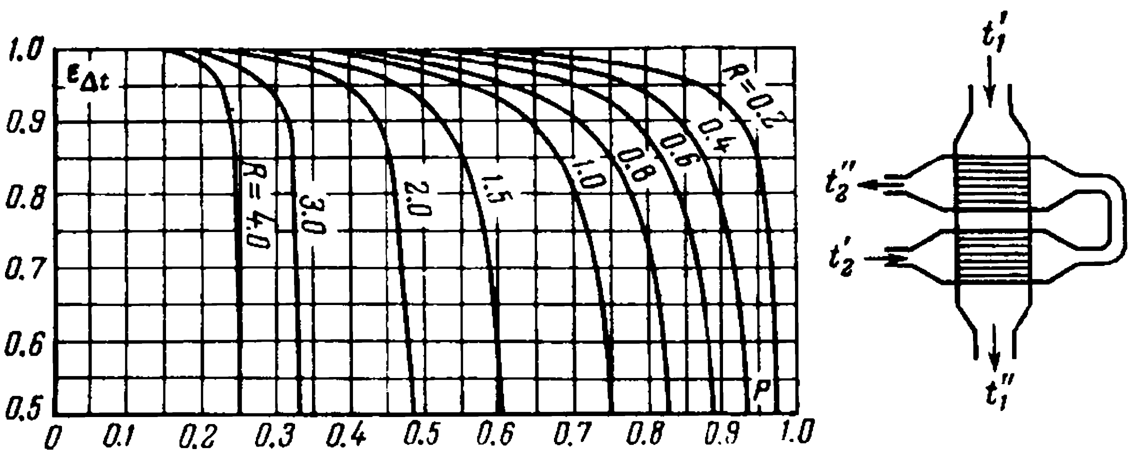


Fig. A-3



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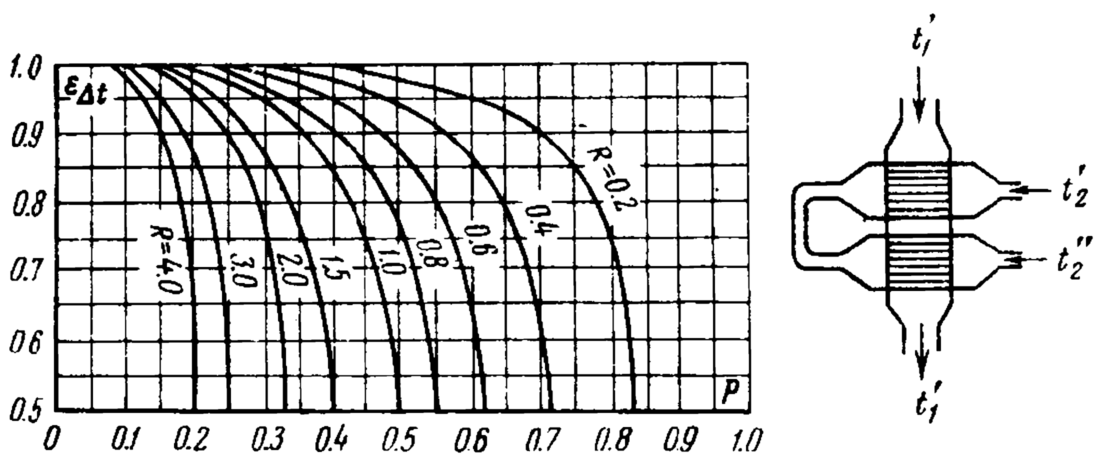

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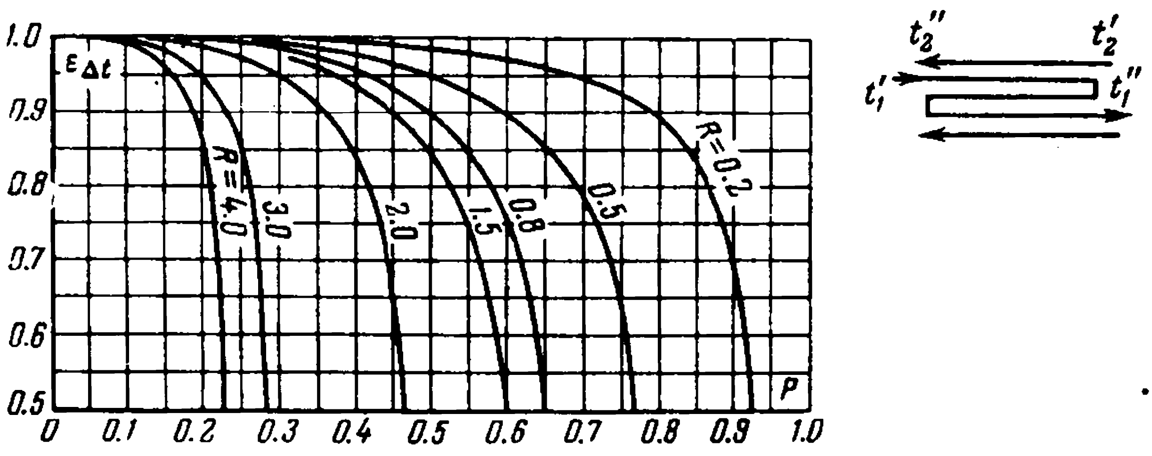

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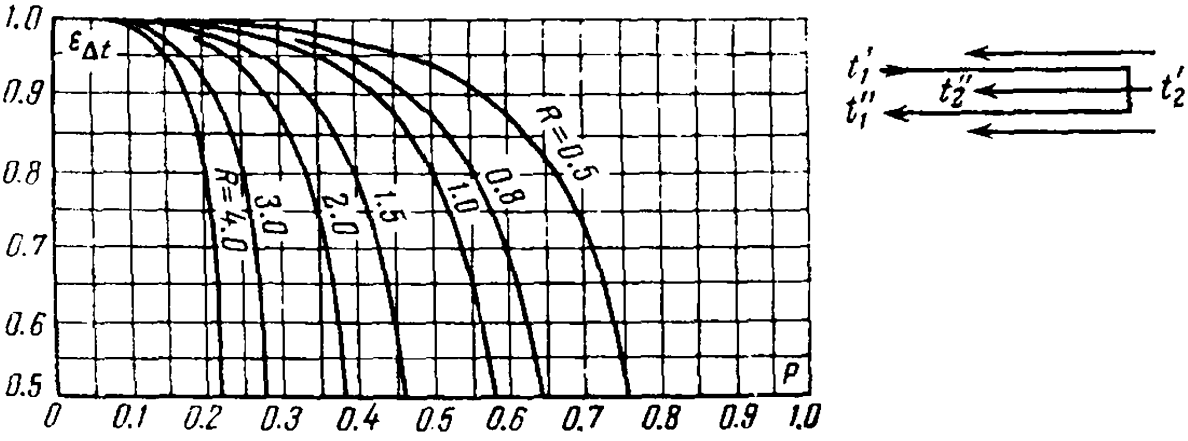

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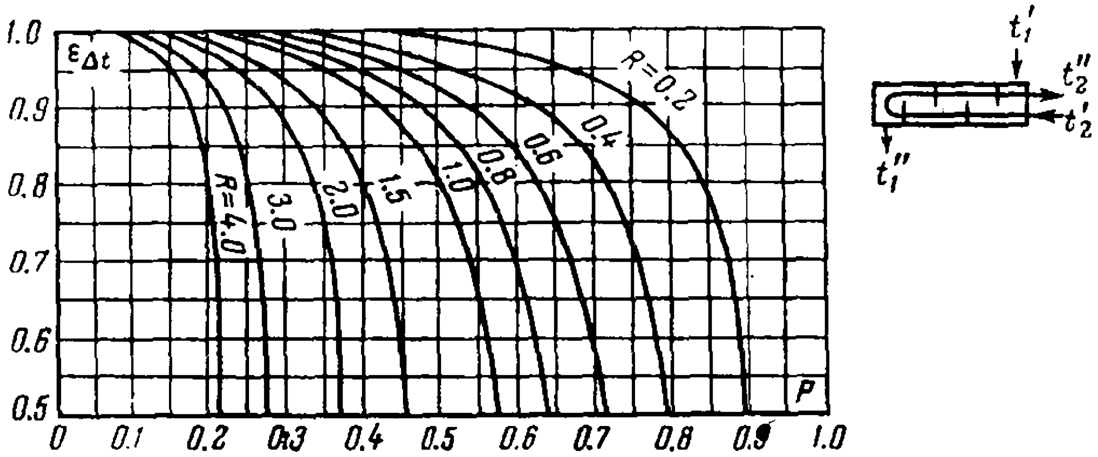

Fig. A-8

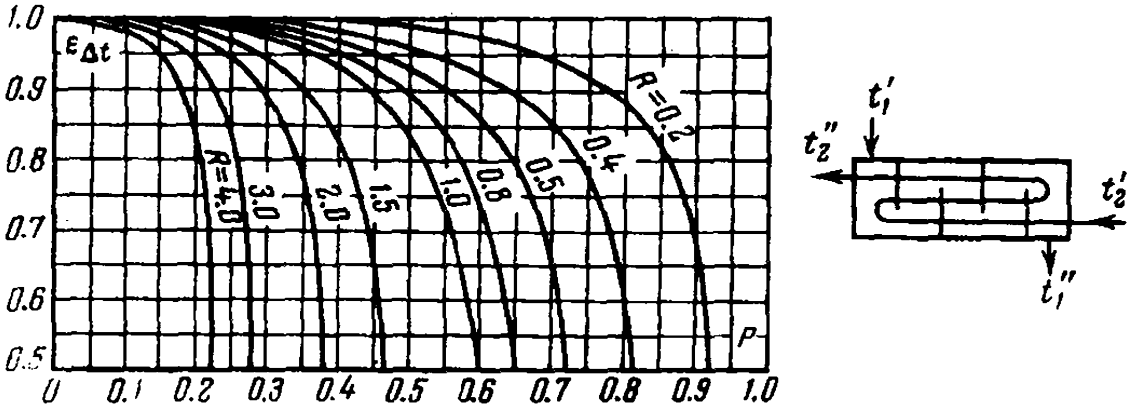

Fig. A-9

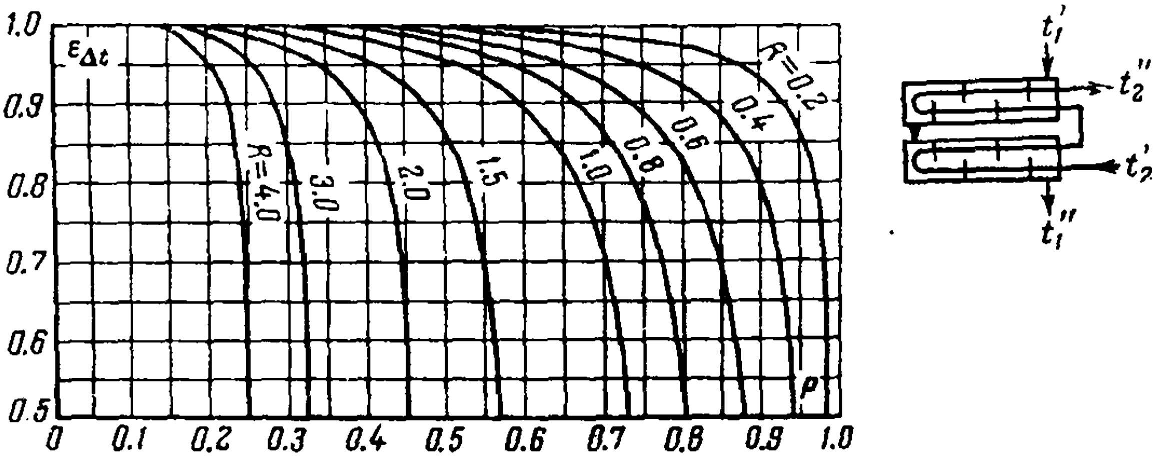

Fig. A-10

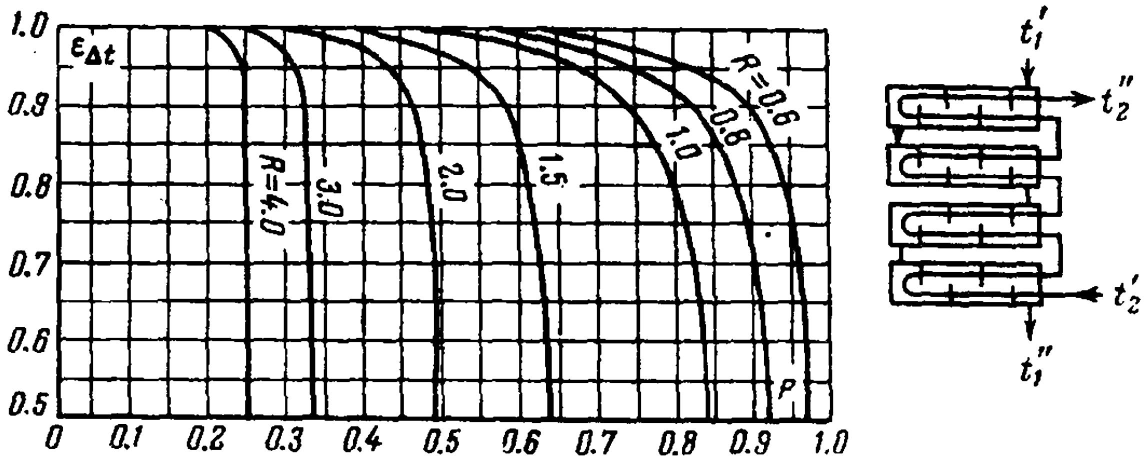

Fig. A-11

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